PHYS 1P22/92

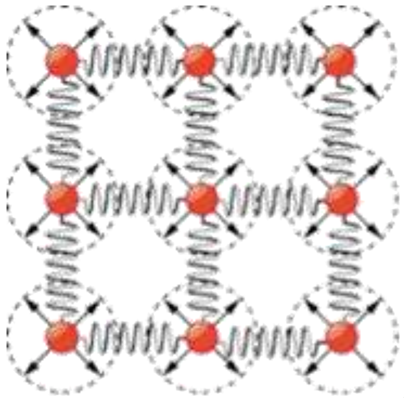
Prof. Barak Shoshany

Spring 2024

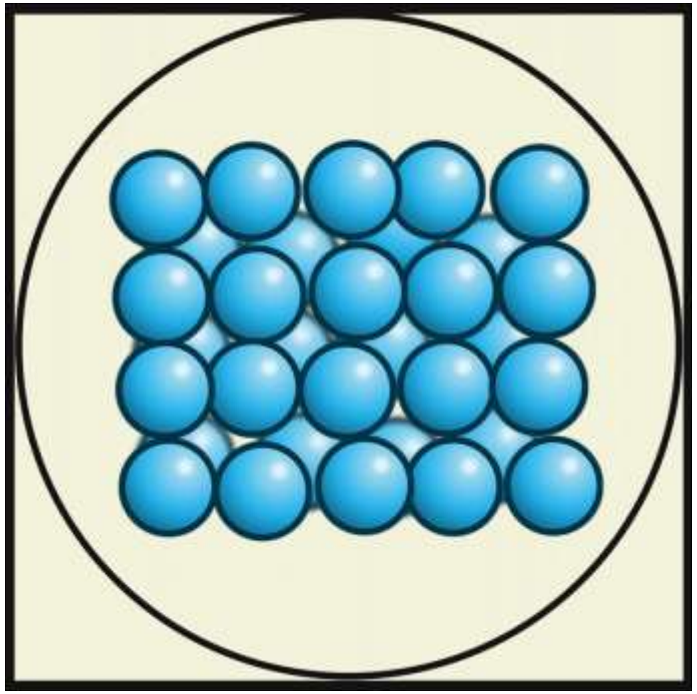
11. Fluid Statics

11.1 What Is a Fluid?

Phases of matter

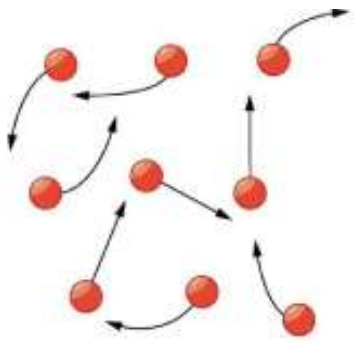


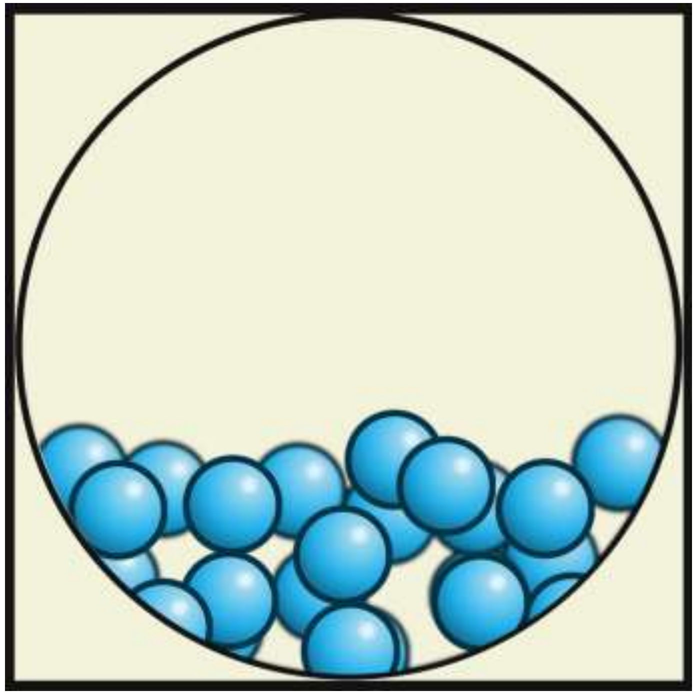
- Matter has 4 phases or states: solid, liquid, gas, and plasma.
- Solids have a definite volume and shape.
- The particles (atoms or molecules) are closely packed together.
- The forces between the particles are strong, so they are fixed in place.
- The particles can vibrate, but not move.



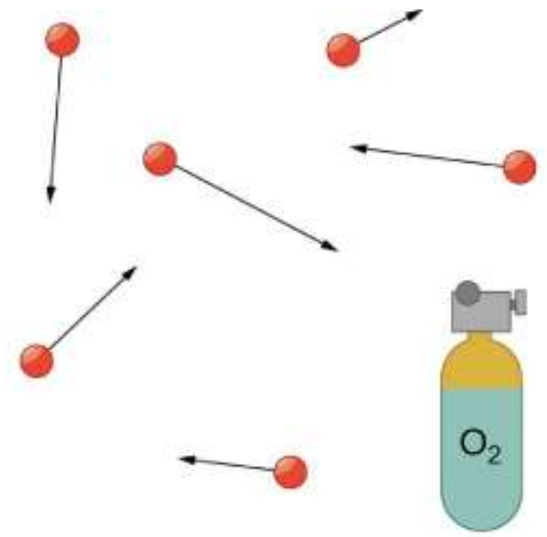
Phases of matter

- Liquids have a definite volume, but their shape can change.
- The shape will be determined by the container.
- The forces between the particles are weaker, so they can move around.

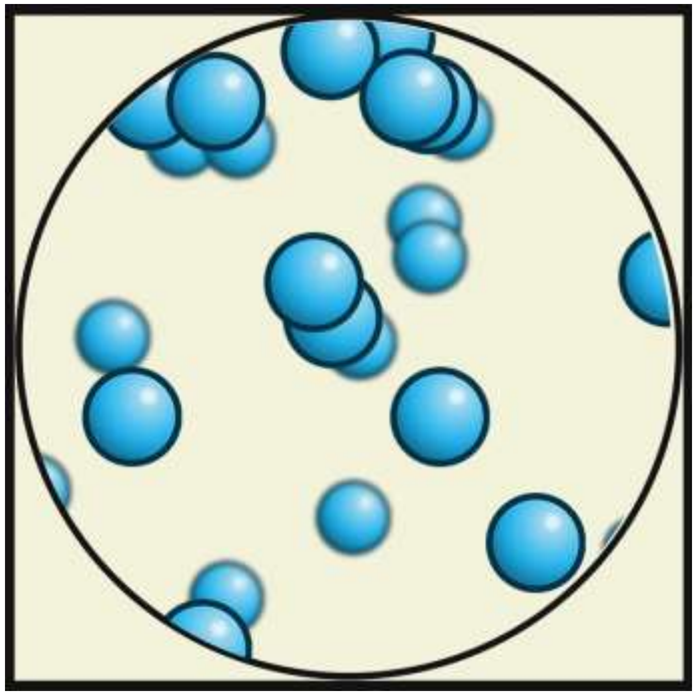




Phases of matter

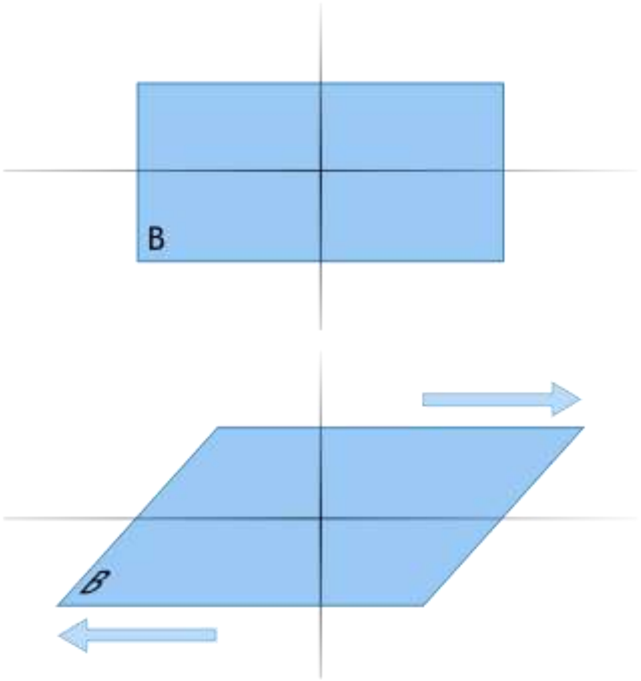


- Gases and plasmas do not have a definite volume or shape.
- They expand to fill their container.
- The particles are very far from each other, and the forces between them are very weak.
- Plasmas are gases, but hot enough that their atoms ionize (the electrons separate from the nuclei).



Fluids

- Liquid, gas, and plasma are considered fluids because they yield to shearing forces.
  - Recall from section 5.3: Shearing is when you apply forces on two parts of an object in two different directions.
- Solids resist shearing forces. They are very hard to deform or compress.
- Fluids can flow, solids can't.



Shearing force

Fluids

- Liquids deform easily, and do not restore their original shape once the force is removed.
- However, they resist compression.
- A liquid in a container with no lid will remain inside.

Fluids

- Gases deform easily and are also easy to compress, because the particles are very far from each other.
- A gas in a container with no lid will escape, as the gas will expand, and its particles are moving fast in all directions.
- Note: When we say “fluids” in this course that can mean liquid, gas, or both.

Simulation

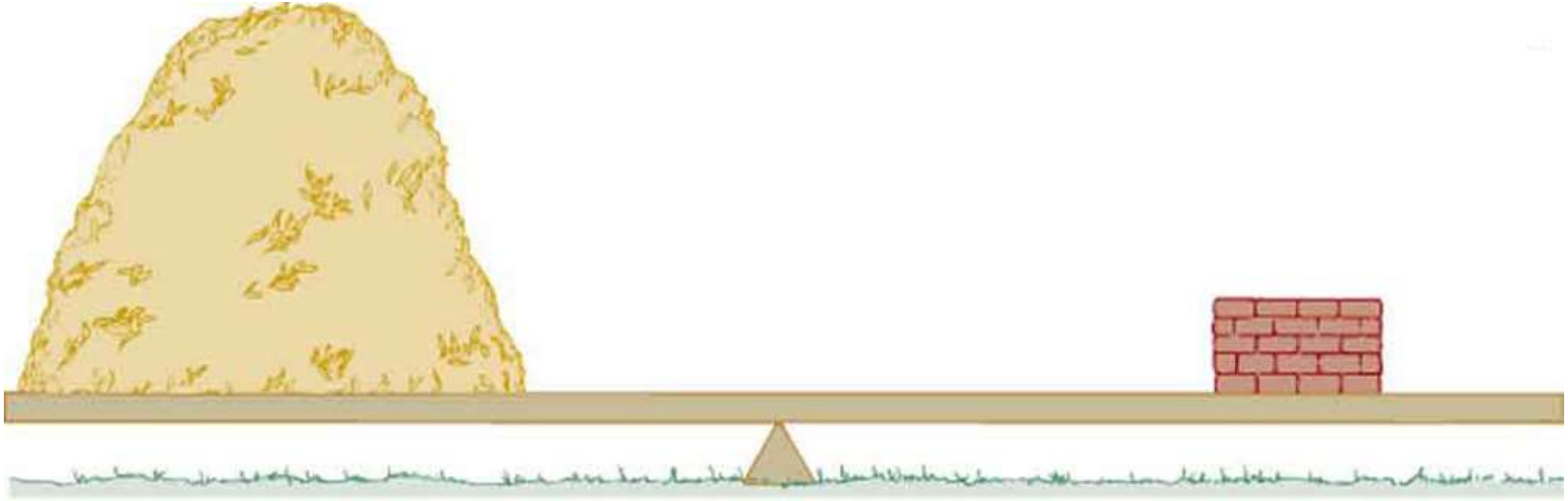
- Simulation of the different states of matter:

  https://phet.colorado.edu/sims/html/states-of-matter-basics/latest/states-of-matter-basics_all.html

11.2 Density

Definition of density

- Pop Quiz: Which weighs more, a ton of feathers or a ton of bricks?
- Answer: Both weigh the same: a ton. But bricks are more dense.



Definition of density

- Density ρ (Greek rho) is defined as mass m per unit volume V:

$$\rho \equiv \frac{m}{V}$$

- Pop Quiz: What are the units of density?
- Answer: mass is kg, volume is m^3, so density is $\mathrm{kg/m}^3$.

Definition of density

- The average density of a brick is 2,000 kg/m^3.
- The average density of a feather is 2 kg/m^3, 1,000 times smaller.

$$\rho \equiv \frac{m}{V} \quad \Longrightarrow \quad m = \rho V$$

- To get a ton of feathers, we need a 1,000 times larger volume than a ton of bricks!

Substance	ρ ($\times 10^3$ kg/m^3)	Substance	ρ ($\times 10^3$ kg/m^3)	Substance	ρ ($\times 10^3$ kg/m^3)	Substance	ρ (kg/m^3)
Solids				**Liquids**		**Gases**	
Aluminum	2.7	Cork	0.24	Water (4 °C)	1.000	Air	1.29
Brass	8.44	Glass	2.6	Blood	1.05	Carbon dioxide	1.98
Copper	8.8	Granite	2.7	Sea water	1.025	Carbon monoxide	1.25
Gold	19.32	Earth's crust	3.3	Mercury	13.6	Hydrogen	0.090
Iron or steel	7.8	Wood	0.3–0.9	Ethyl alcohol	0.79	Helium	0.18
Lead	11.3	Ice (0 °C)	0.917	Gasoline	0.68	Methane	0.72
Polystyrene	0.10	Bone	1.7–2.0	Glycerin	1.26	Nitrogen	1.25
Tungsten	19.30	Silver	10.49	Olive oil	0.92	Nitrous oxide	1.98
Uranium	18.70					Oxygen	1.43
Concrete	2.30–3.0					Steam (100 °C)	0.60

11.3 Pressure

Definition of pressure

- If a force F is applied to an area A perpendicular to the force, the pressure P is defined as the force per unit area:

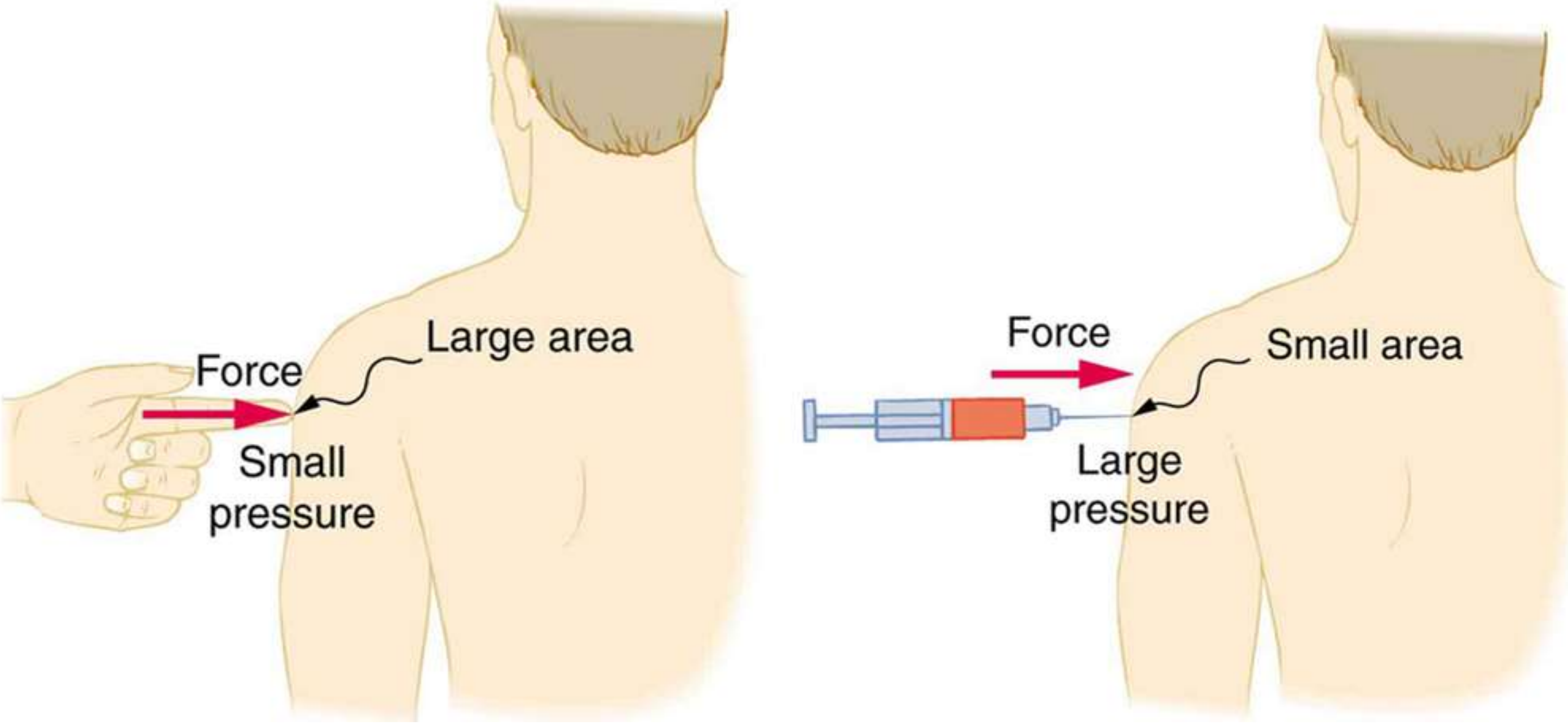
$$P \equiv \frac{F}{A}$$

- The SI units of pressure are $\mathrm{N/m^2}$, called "pascal" (Pa) for short:

$$1\ \mathrm{Pa} \equiv 1\ \mathrm{N/m^2}$$

- In terms of the base units:

$$1\ \mathrm{N} = 1\ \mathrm{kg\cdot m/s^2} \quad \Longrightarrow \quad 1\ \mathrm{Pa} = 1\ \frac{\mathrm{kg\cdot m}}{\mathrm{s^2\cdot m^2}} = 1\ \frac{\mathrm{kg}}{\mathrm{s^2\cdot m}}$$

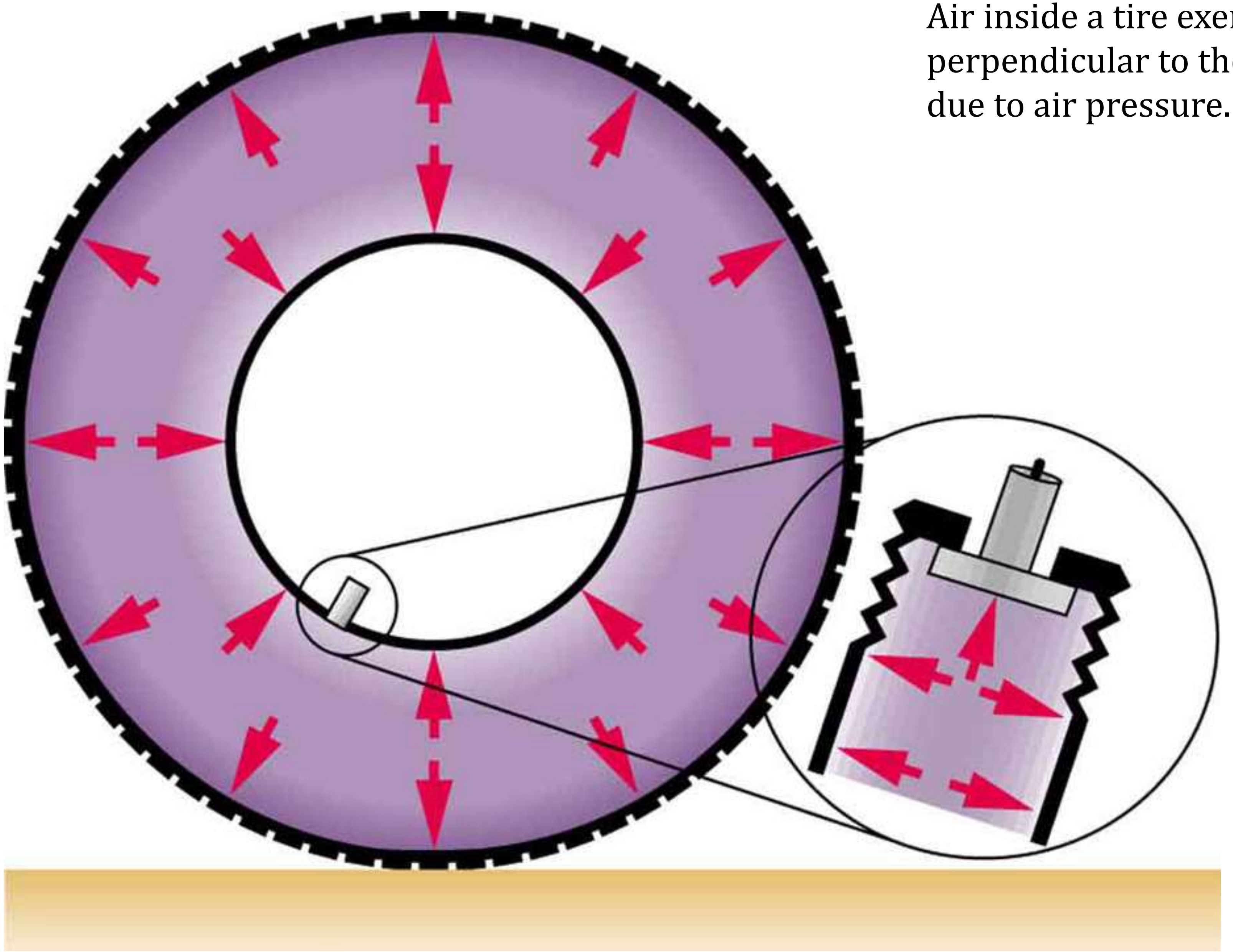

Force
Large area
Small
pressure
Force
Small area
Large
pressure

Force due to pressure

- Pressure is a scalar, not a vector, so it has no direction, only magnitude.
- We can rearrange the equation:

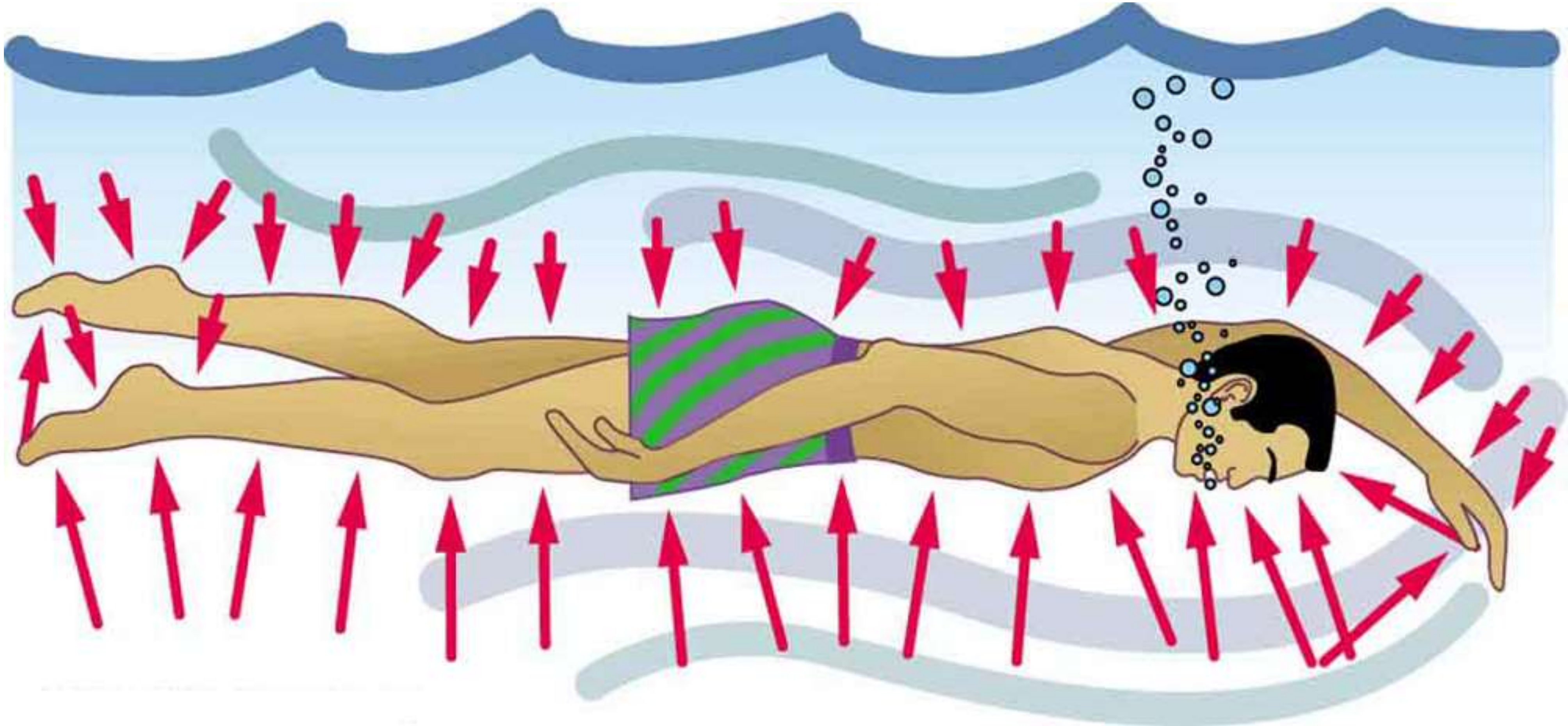
$$P = \frac{F}{A} \implies F = PA$$

- A fluid with pressure P will exert a force perpendicular to the surface, on any surface it comes into contact with.
- Note: A fluid cannot withstand a shearing force, and cannot exert it either.



Air inside a tire exerts forces perpendicular to the tire everywhere due to air pressure.

Water exerts a force on all sides of a swimmer due to water pressure. The forces are larger at lower depths. (We'll see why in the next section.)



11.4 Variation of Pressure with Depth in a Fluid

Pressure and weight

- Why do our ears “pop” on a plane or when diving?
- Pressure is caused by the weight of the fluid above you.
- On Earth’s surface: more air above you compared to higher altitudes, so more pressure.
- When you dive deeper: more water above you compared to shallower depths, so more pressure.
  - On top of the weight of the air above the water, but water is much denser.

- A fluid with mass m has weight $W = mg$.
- The pressure due to the fluid on an area A is:

$$P = \frac{W}{A} = \frac{mg}{A}$$

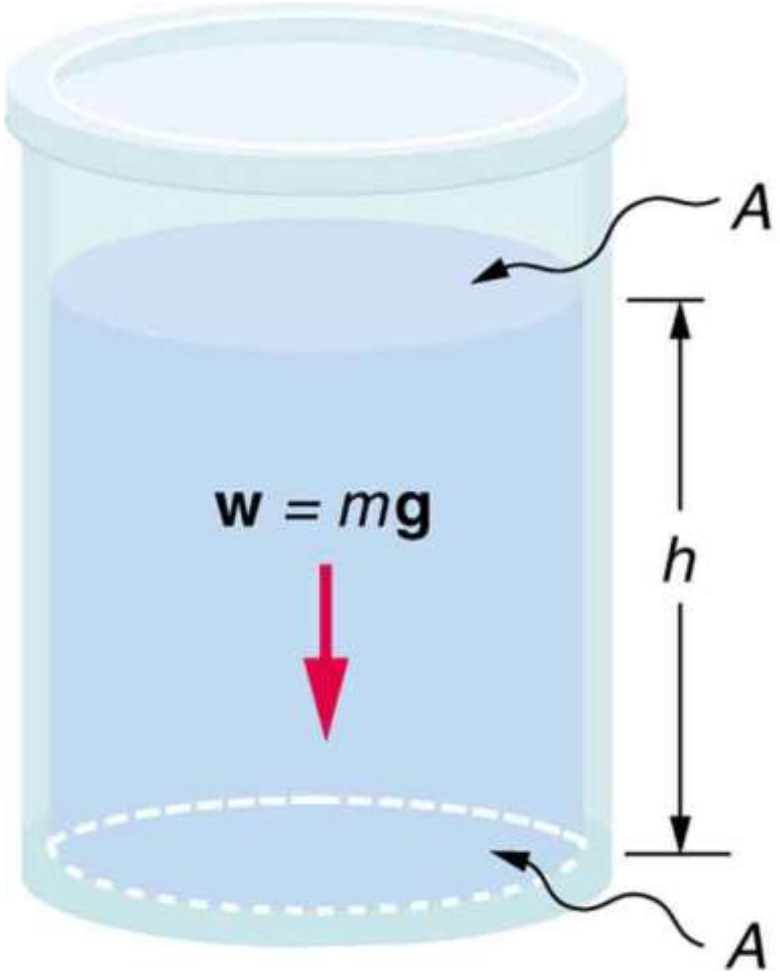
- Volume is $V = Ah$. From definition of density:

$$\rho = \frac{m}{V} \implies m = \rho V = \rho A h$$

- Plug into P:

$$P = \frac{mg}{A} = \frac{(\rho A h)g}{A} = \rho h g$$

- The pressure on bottom of container is $P = \rho h g$.
- This result is always true, even if there is no container!



Problem: A reservoir has depth h. What is the average force F exerted by the water on a dam with width L?

Analytical solution:

- The pressure on the dam is not $\rho h g$, because the dam is not below the water.
- The pressure is zero on the surface and $\rho h g$ at the bottom, so the **average** pressure is

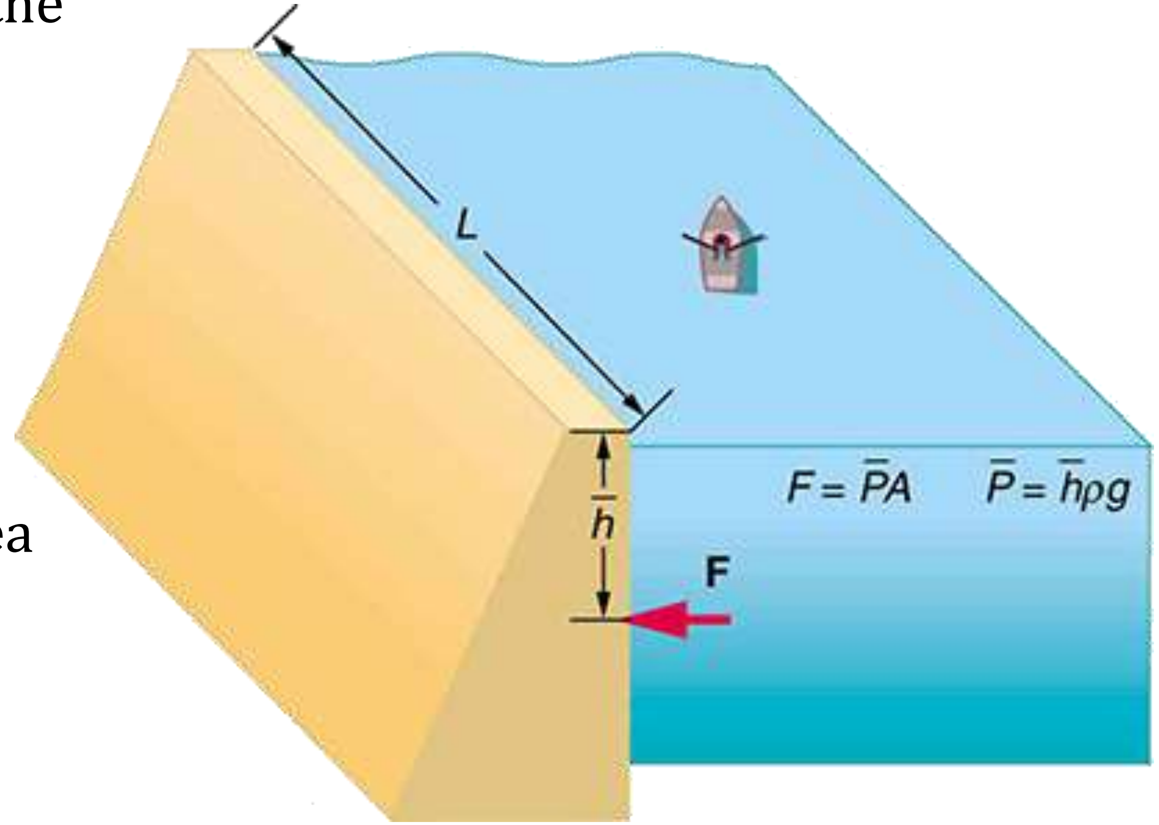
$$P = \frac{\rho h g + 0}{2} = \frac{1}{2}\rho h g$$

- The average force it exerts will be

$$F = PA = \frac{1}{2}\rho h g A$$

- The area A that the force is exerted on is the area of the dam, $A = Lh$, so:

$$F = \frac{1}{2}\rho h g A = \frac{1}{2}\rho h g L h = \frac{1}{2}\rho h^2 g L$$



Problem: Find the numerical value of the force if $h \approx 80.0$ m and $L \approx 500$ m.

Numerical solution: Using $\rho \approx 1000\ \text{kg/m}^3$:

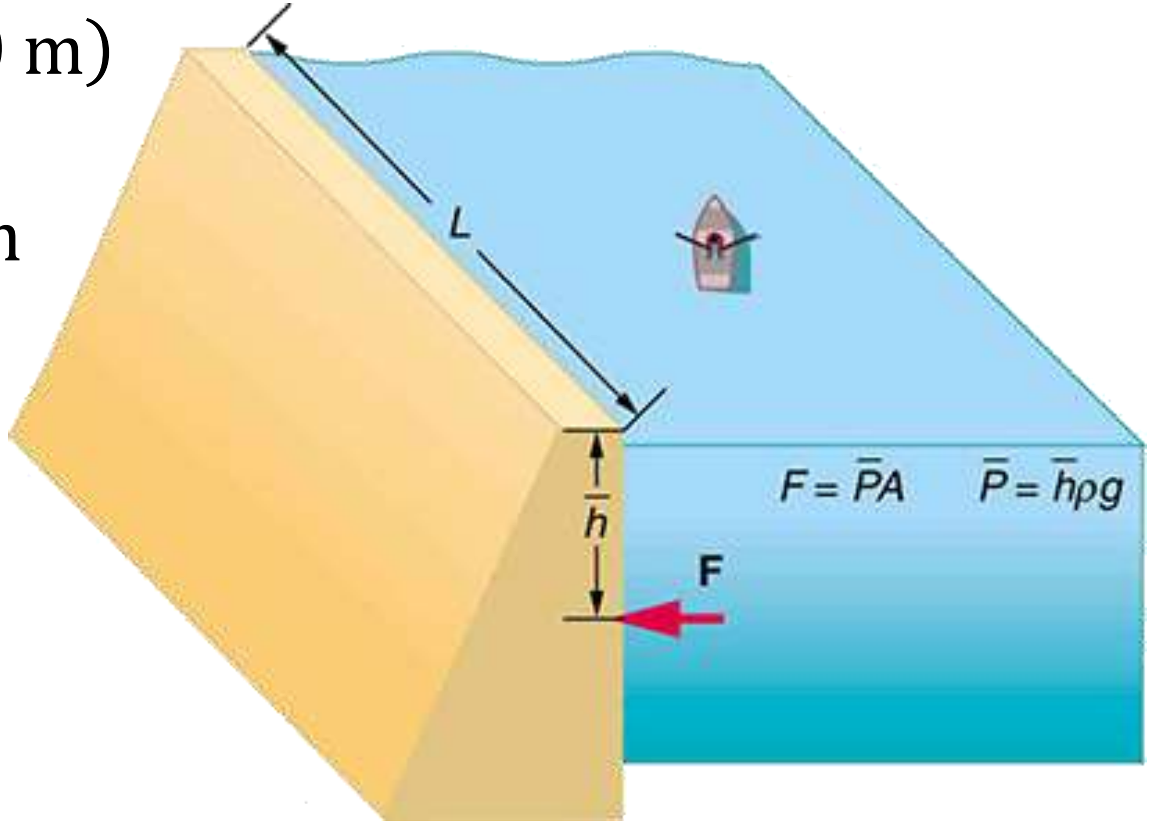
$$F = \frac{1}{2}\rho h^2 g L$$

$$\approx \frac{1}{2}(1000\ \text{kg/m}^3)(80\ \text{m})^2(9.8\ \text{m/s}^2)(500\ \text{m})$$

$$= \frac{1}{2}\cdot 1000 \cdot 80^2 \cdot 9.8 \cdot 500\ \frac{\text{kg}}{\text{m}^3}\cdot \text{m}^2 \cdot \frac{\text{m}}{\text{s}^2}\cdot \text{m}$$

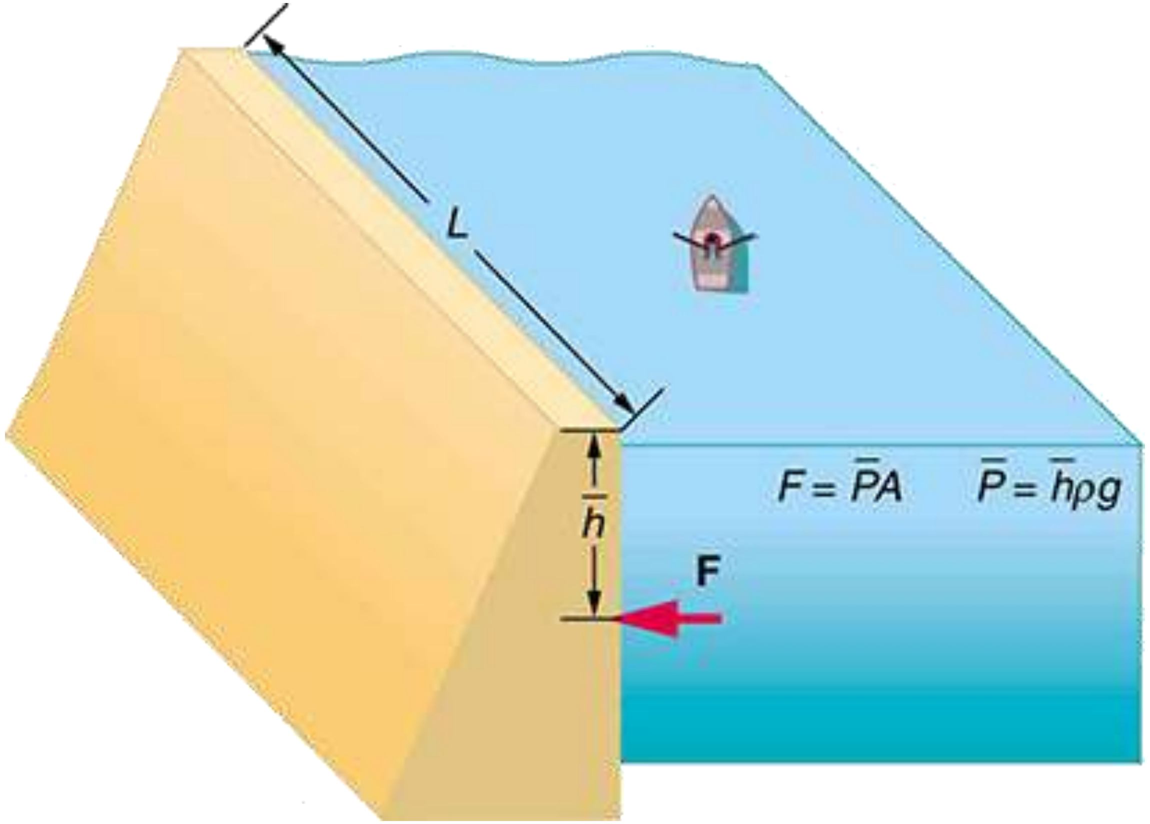
$$\approx 1.58 \times 10^{10}\ \text{kg}\cdot\text{m/s}^2$$

$$= 1.58 \times 10^{10}\ \text{N}$$



Pop Quiz: Why does the thickness of the dam increase with depth?

Answer: Because the pressure, and therefore the force that the dam needs to withstand, increases with depth.



Atmospheric pressure

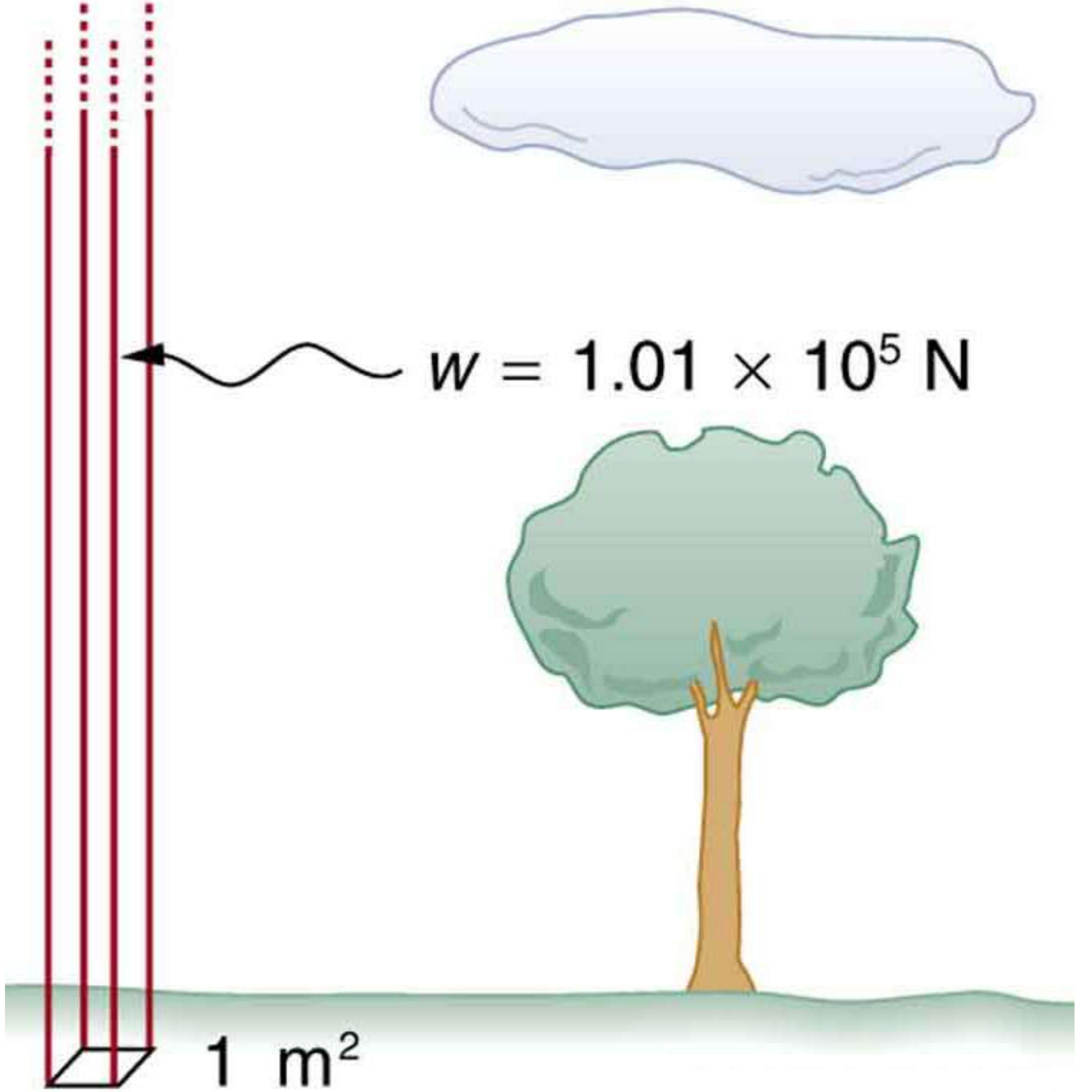
- Atmospheric pressure is due to the weight of the air above a given altitude.
- The standard atmosphere (atm) is the approximate average atmospheric pressure at sea level:

$$1 \text{ atm} \equiv 101{,}325 \text{ Pa}$$

  - Note: This is a definition, not a measurement.

- **Problem:** What is the weight of a column of air above 1 m^2 of Earth's surface?
- **Solution:**

$$W = PA \approx 101{,}325 \text{ N}$$



- **Problem:** What is the average density of air in the atmosphere, assuming it extends to altitude $h \approx 120$ km?
- **Analytical solution:**

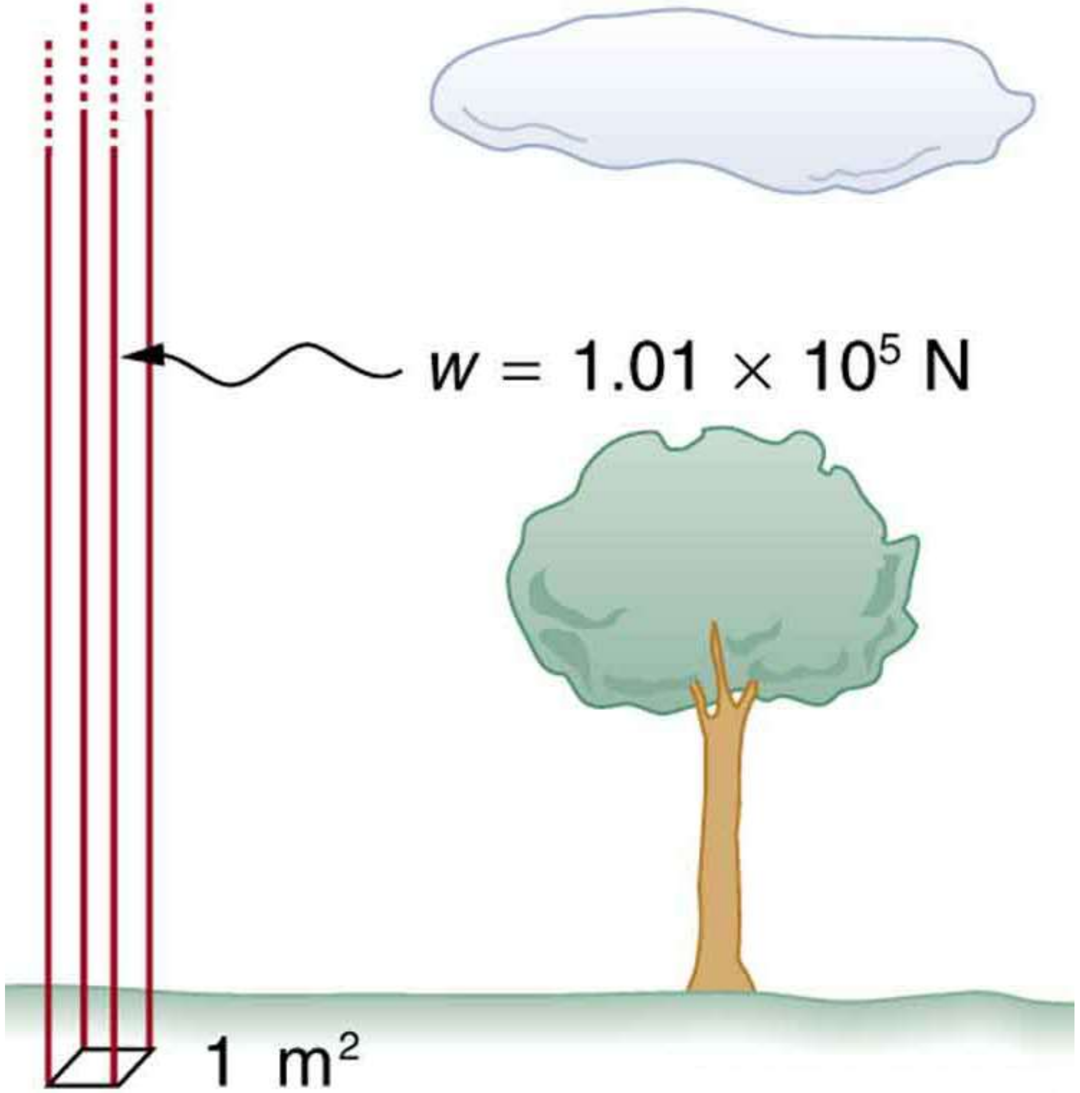
$$P = \rho h g \quad \Longrightarrow \rho = \frac{P}{hg}$$

- **Numerical solution:**

$$\rho \approx \frac{101{,}325 \text{ Pa}}{(120{,}000 \text{ m})(9.80 \text{ m/s}^2)}$$

$$\approx 0.0862 \; \frac{\text{kg/s}^2 \cdot \text{m}}{\text{m} \cdot \text{m/s}^2}$$

$$\approx 0.0862 \text{ kg/m}^3$$



11.5 Pascal’s Principle

Pascal’s principle

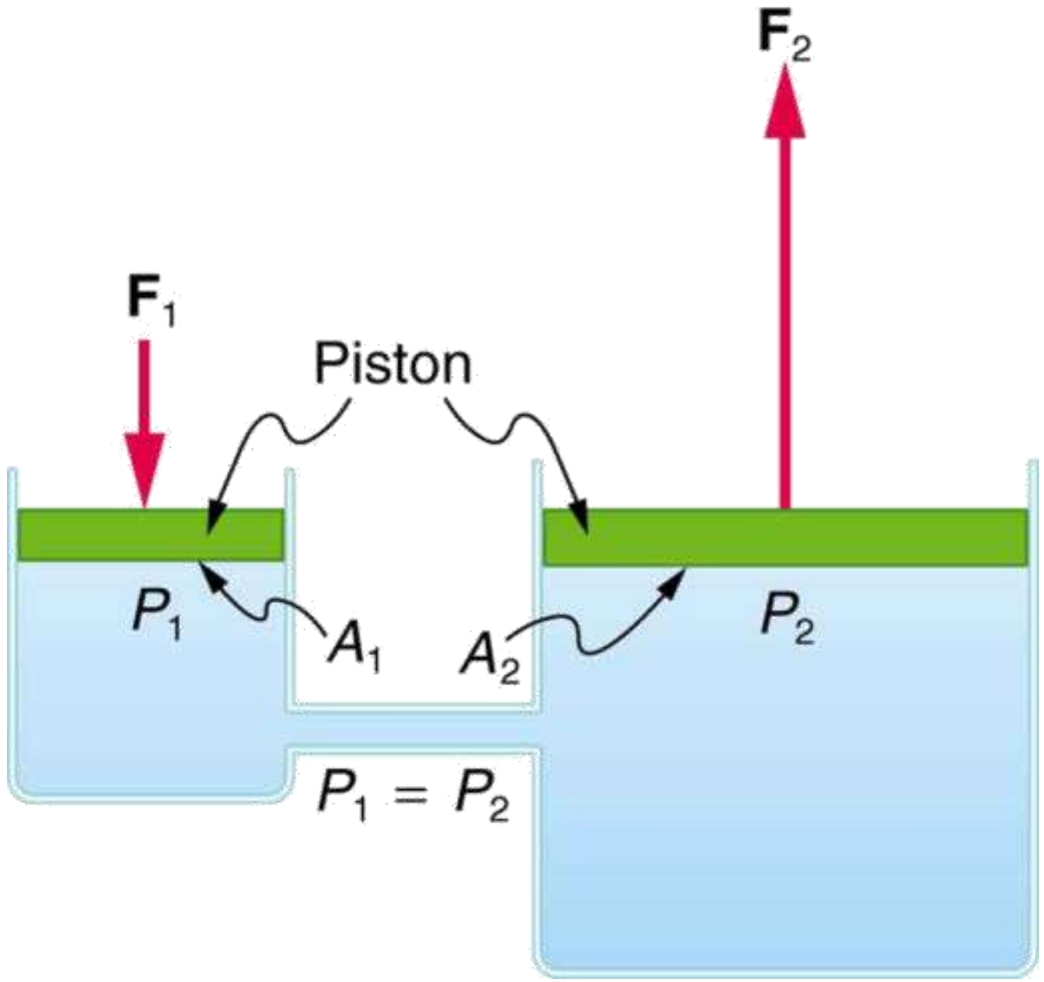
- Pascal’s principle (or Pascal’s law) says that a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- This only works for an enclosed fluid. Not if you push on fluid in an open system, e.g. a river.
- The fluid is transmitted undiminished, so pressure is not “lost”.
- Pressures add up: the total pressure is the sum of all pressures applied.

- Pascal's principle is applied in hydraulic systems.
- Force F_1 acts on area A_1, so pressure $P_1 = F_1/A_1$. Similarly, $P_2 = F_2/A_2$.
- If we push on piston 1, the pressure P_1 is transmitted undiminished to piston 2, so:

$$P_1 = P_2 \quad \Longrightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

- We can use this to increase or decrease the applied force. For example, if $A_2 = 5A_1$:

$$F_2 = F_1 \frac{A_2}{A_1} = F_1 \frac{5A_1}{A_1} = 5F_1$$



Conservation of energy

- Pop Quiz: A hydraulic system can increase the applied force by any amount, given by the ratio of areas A_2/A_1. Doesn't that give us "something from nothing", and violate conservation of energy?
- Answer: Piston 2 applies more force, but does the same work (and thus conserves energy). Work = force × distance, so

$$W_1 = W_2 \quad \Longrightarrow \quad F_1 d_1 = F_2 d_2 \quad \Longrightarrow \quad d_2 = d_1 \frac{F_1}{F_2} = d_1 \frac{A_1}{A_2}$$

So for example, if $A_2 = 5A_1$, we have $d_2 = d_1/5$. You need to apply a smaller force on piston 1, but for a longer distance.

11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

Gauge pressure

- Absolute pressure P_{abs} is the actual pressure on a system.
- Gauge pressure P_{gauge} is the pressure after we subtract atmospheric pressure P_{atm}:

$$P_{gauge} = P_{abs} - P_{atm}$$

- Gauge pressure is negative if $P_{abs} < P_{atm}$ or positive if $P_{abs} > P_{atm}$.
- Absolute pressure cannot be negative; pressure can only push, not pull (unlike a force).
  - If it's zero then there is no pressure of any kind.

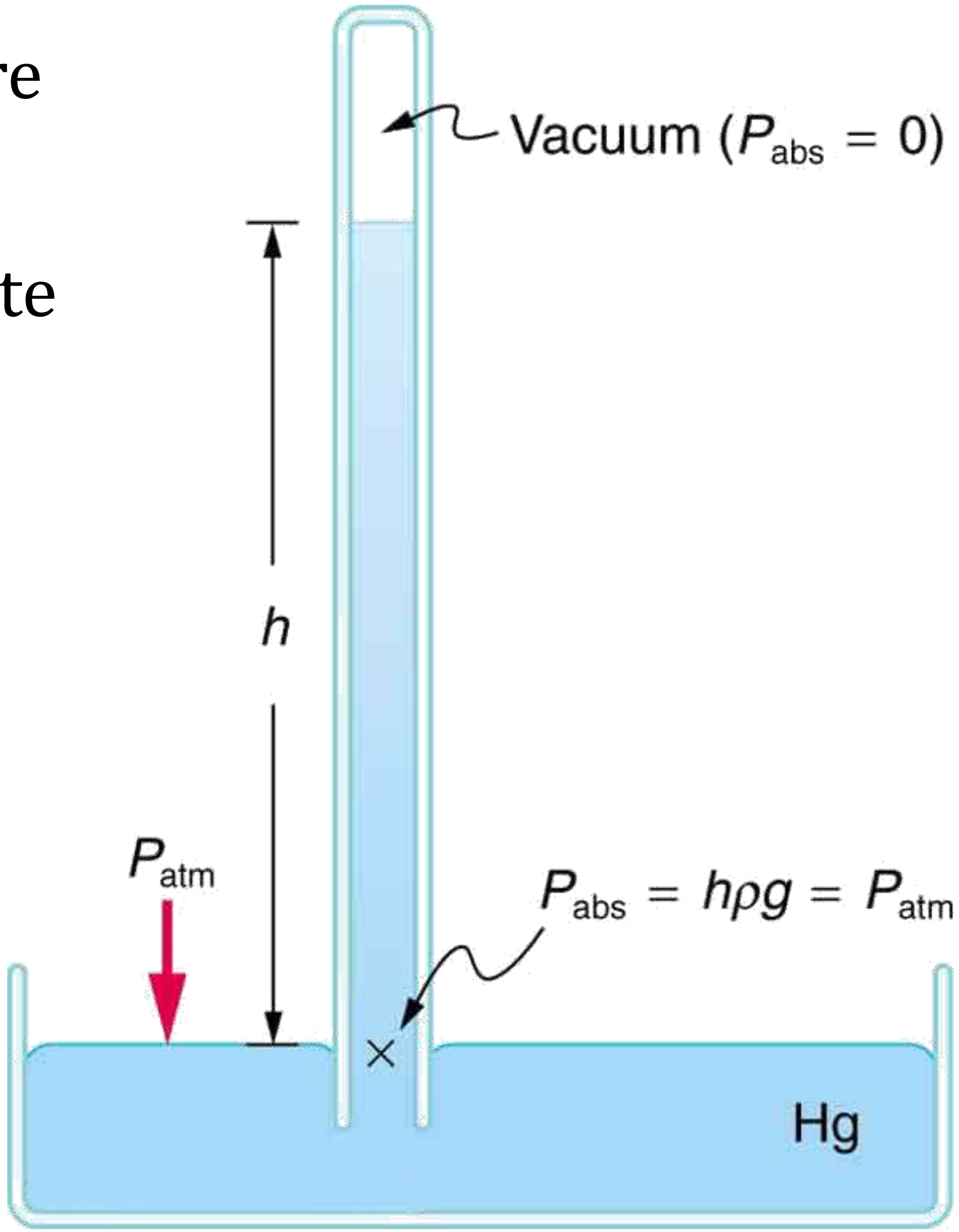
Gauge pressure

- Pop Quiz: If a tire has a hole in it, what will be its gauge pressure?
- Answer: Zero, because it will be the same as atmospheric pressure.
- If the tire doesn't have a hole, then it has positive gauge pressure, because the total pressure will be atmospheric pressure + air pressure inside the tire.

- A barometer measures atmospheric pressure.
- In a mercury barometer, atmospheric pressure pushes the mercury up the tube to height h.
- If we know ρ for mercury, then we can calculate P_{atm} based on h:

$$P_{atm} = h\rho g$$

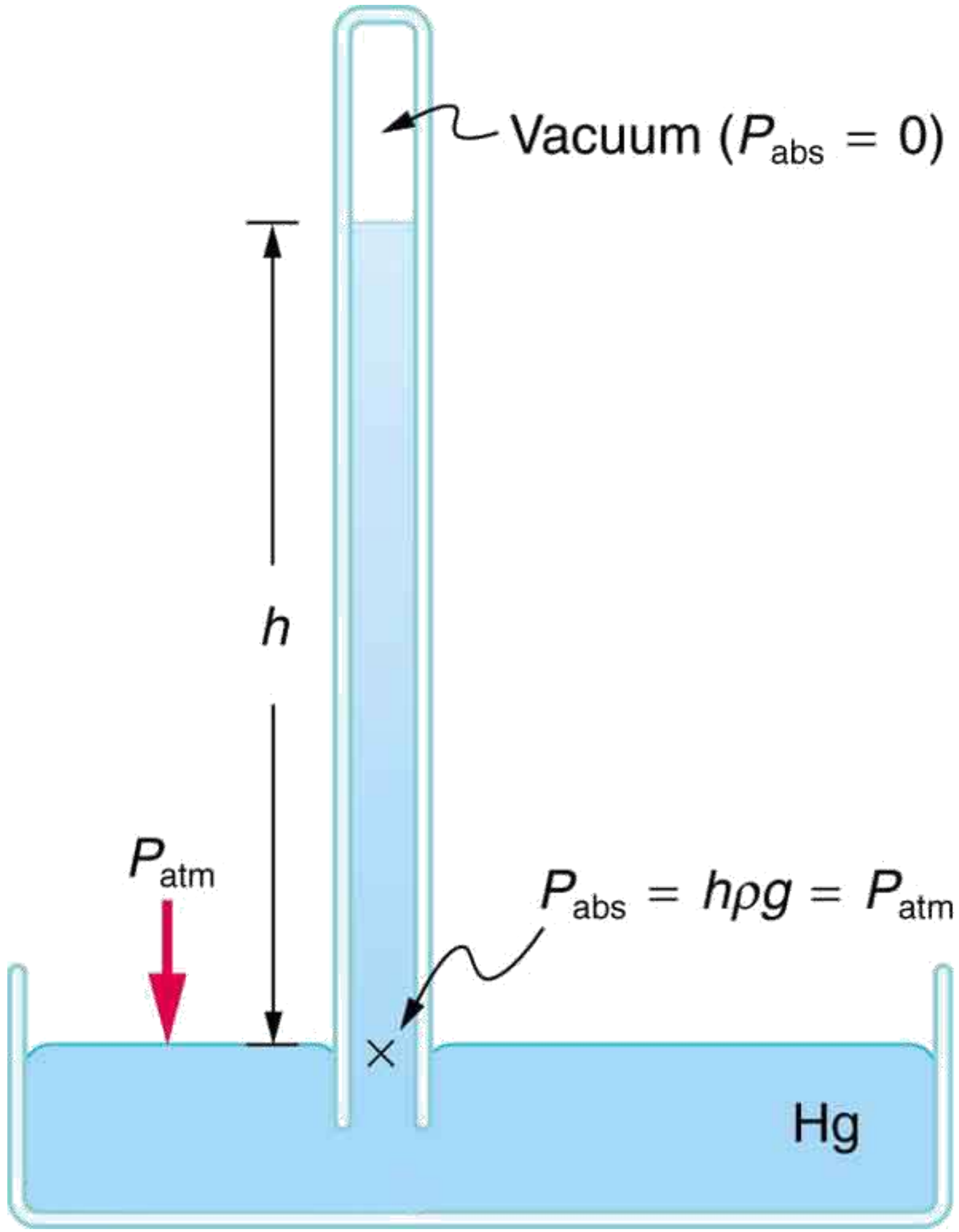
- We can also use it as an altimeter (altitude meter), because P_{atm} decreases with altitude.



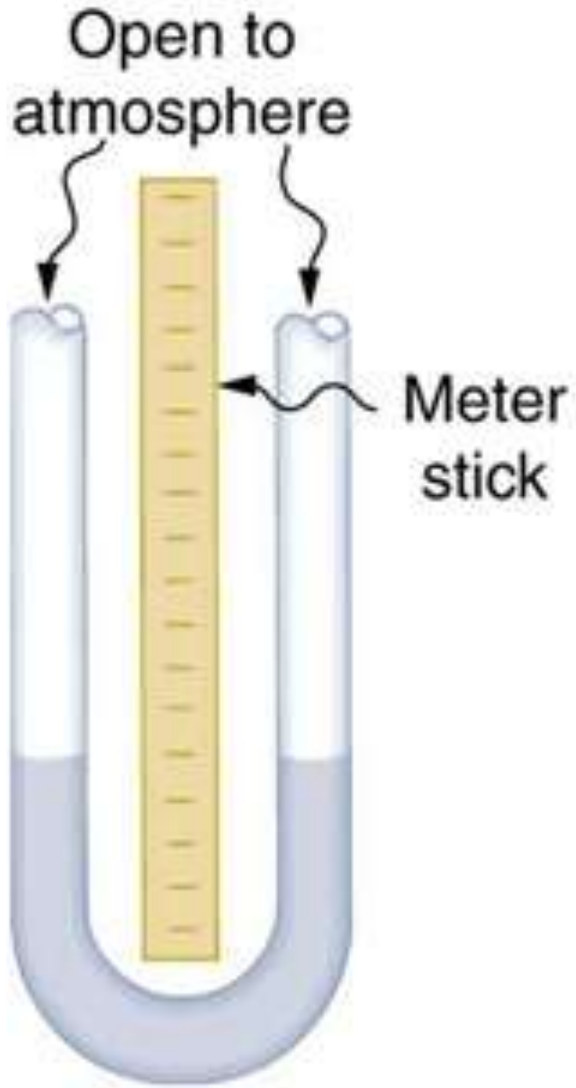
- This is the origin of the units “mm Hg” (millimeters of mercury), used e.g. for blood pressure.
- 1 mm Hg was formerly defined as the gauge pressure generated by a 1 mm column of mercury.
- In modern SI units it is defined exactly as:

$$1 \text{ mm Hg} \equiv 133.322 \text{ Pa}$$

- There are other units for pressure: bar, torr, pound per square inch, cm H_2O (centimeter of water), etc… But in this course we will only use SI units!



- A manometer measures gauge pressure.
- If both sides are open to the atmosphere, the same pressure applies to both, so the level is the same.

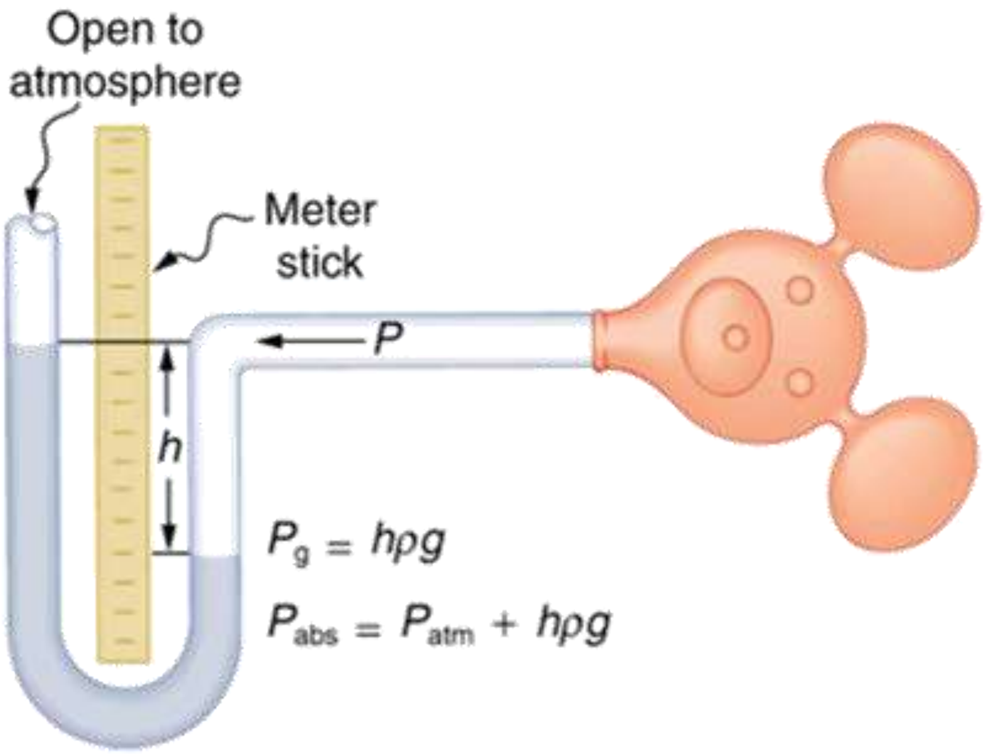


- If one side is connected to a source of pressure, the fluid levels are no longer equal.
- In this example, the balloon applies pressure $P_{abs} > P_{atm}$.
- The fluid on the open side is pushed up with a height difference h.
- We can calculate P_{gauge} if we know h:

$$P_{gauge} = h\rho g$$

- Then the balloon pressure is

$$P_{abs} = P_{atm} + P_{gauge} = P_{atm} + h\rho g$$

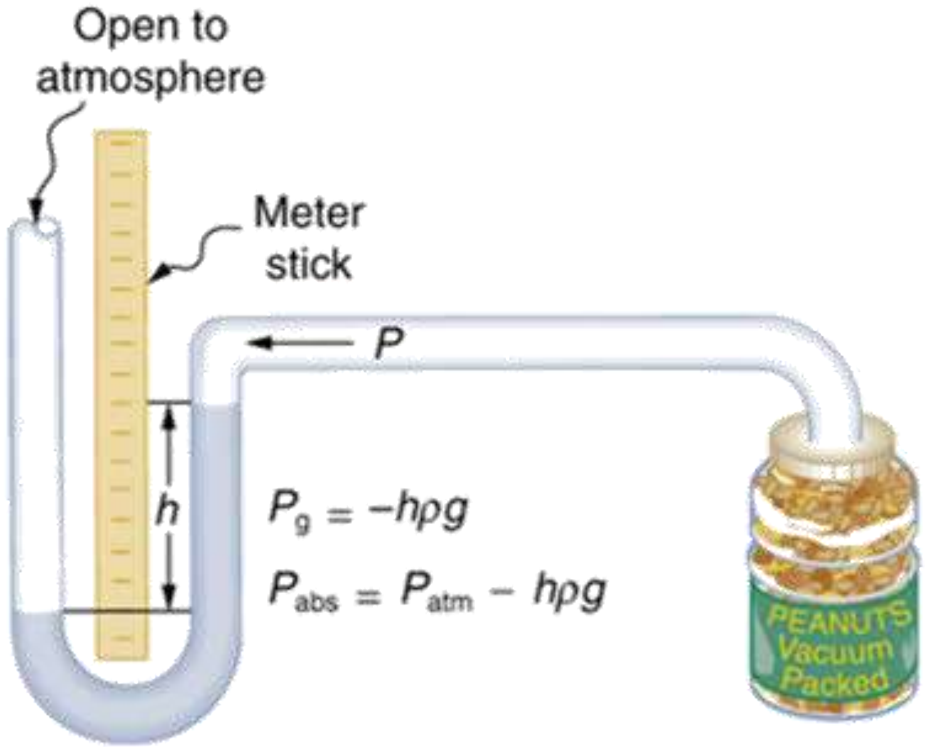


- In this example, the vacuum-packed jar applies pressure $P_{abs} < P_{atm}$.
- The fluid on the open side is pushed down with a height difference h.
- We can calculate P_{gauge} if we know h:

$$P_{gauge} = -h\rho g$$

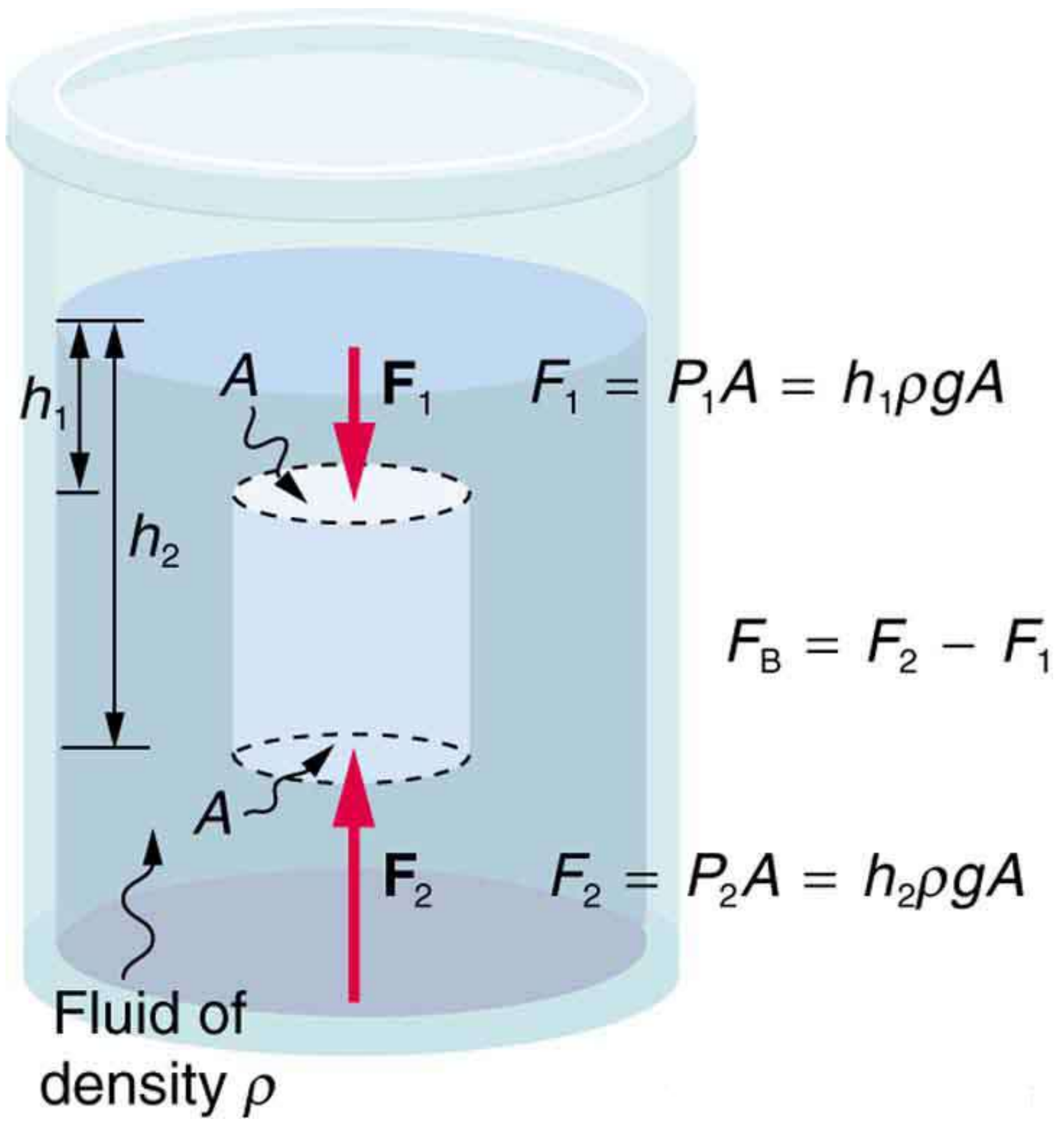
- Then the jar pressure is

$$P_{abs} = P_{atm} + P_{gauge} = P_{atm} - h\rho g$$

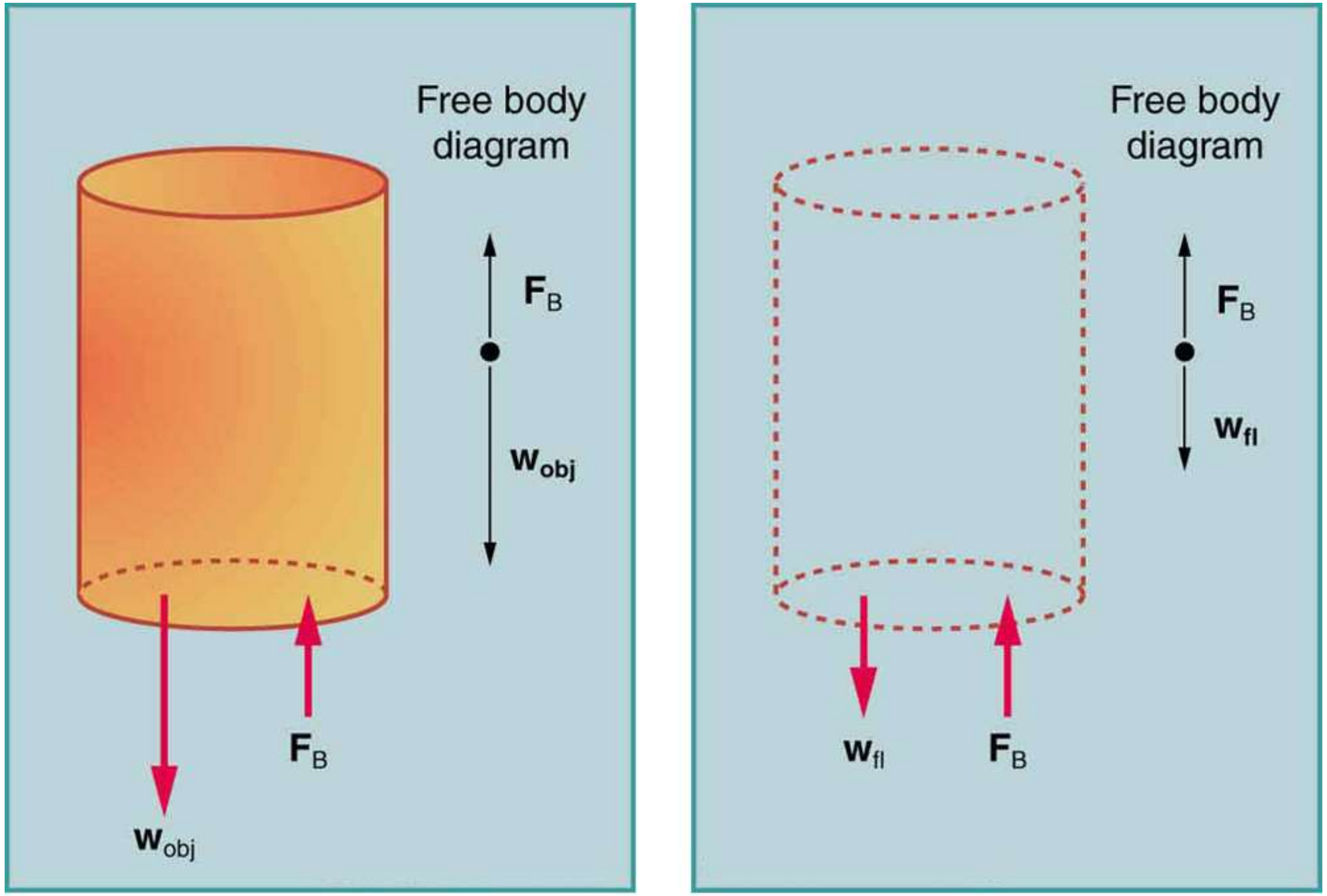


11.7 Archimedes' Principle

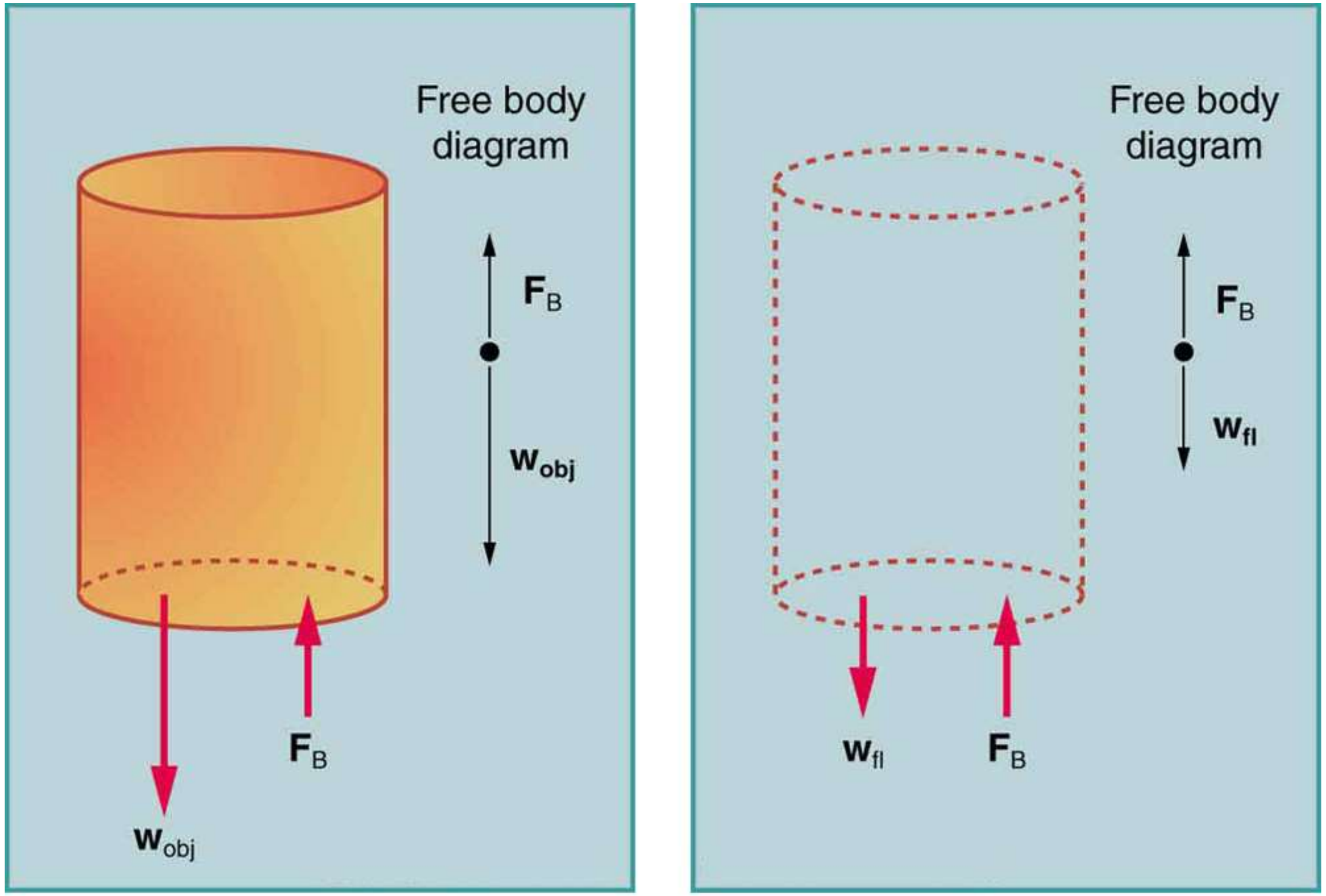
- Buoyant force is the net upward force on an object in a fluid.
- It is caused by the pressure difference between the top and bottom of the object.
- There is more pressure at the bottom, so more force pushing up than down.
- Since $F_2 > F_1$, there is a net buoyant force F_B.
- There is no horizontal force because there is no horizontal pressure difference.



- If the object has weight W_{obj}:
- If $F_B > W_{obj}$, the object will rise.
- If $F_B < W_{obj}$, the object will sink.



- How big is the buoyant force?
- Let's remove the submerged object.
- The space is filled by the fluid, with weight W_{fl}.
- The fluid doesn't rise or sink within itself. So we must have $F_B = W_{fl}$.
- This is Archimedes' principle: the buoyant force on an object equals the weight of the fluid it displaces.



Problem: Calculate the buoyant force on $m_{steel} \approx 10{,}000$ tons (1.00×10^7 kg) of solid steel completely submerged in water, and compare this with the steel's weight.

Analytical solution: If the steel has density ρ_{steel} then its volume is

$$V_{steel} = \frac{m_{steel}}{\rho_{steel}}$$

The steel displaces the same volume of water, $V_{water} = V_{steel}$. So the mass of the water is

$$m_{water} = \rho_{water} V_{water} = \frac{\rho_{water}}{\rho_{steel}} m_{steel}$$

The buoyant force is the weight of the displaced water:

$$F_B = W_{water} = m_{water} g = \frac{\rho_{water}}{\rho_{steel}} m_{steel} g$$

Problem: Calculate the buoyant force on $m_{steel} \approx$ 10,000 tons (1.00×10^7 kg) of solid steel completely submerged in water, and compare this with the steel's weight.

Numerical solution:

$$F_B = \frac{\rho_{water}}{\rho_{steel}} m_{steel} g$$
$$\approx \frac{10^3 \text{ kg/m}^3}{7.8 \times 10^3 \text{ kg/m}^3} (10^7 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= \frac{10^3}{7.8 \times 10^3} \cdot 10^7 \cdot 9.8 \text{ kg} \cdot \text{m/s}^2$$
$$\approx 1.26 \times 10^7 \text{ N}$$

The steel's weight is $m_{steel} g \approx 9.8 \times 10^7$ N, much greater than F_B, so the steel will sink.

Problem: What is the maximum buoyant force that water could exert on the steel if it were shaped into a boat that could displace $\mathrm{V_{water}} \approx 1.00 \times 10^5\ \mathrm{m}^3$ of water?

Analytical solution: The weight of the displaced water will be the buoyant force:

$$F_B = m_{water} g = \rho_{water} V_{water} g$$

Numerical solution:

$$F_B \approx (10^3\ \mathrm{kg/m^3})(10^5\ \mathrm{m^3})(9.8\ \mathrm{m/s^2})$$

$$= 10^3 \cdot 10^5 \cdot 9.8\ \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot \mathrm{m}^3 \cdot \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$= 9.8 \times 10^8\ \mathrm{N}$$

This is 10 times the weight of the steel, so the ship can easily float, even with a load 9 times its own weight.

Density and Archimedes' principle

- Generalizing the problem's solution to any object and any fluid:

$$F_B = \frac{\rho_{fluid}}{\rho_{obj}} W_{obj}$$

- If $\rho_{obj} < \rho_{fluid}$, we have $F_B > W_{obj}$, so the object will float.
- If $\rho_{obj} > \rho_{fluid}$, we have $F_B < W_{obj}$, so the object will sink.

Problem: An object with volume V_{obj} and density ρ_{obj} is partially submerged in a fluid with density ρ_{fluid}. The submerged volume is V_{sub}. What is the fraction f of the object that is submerged?

Solution: The submerged part displaces $V_{fluid} = V_{sub}$ of fluid. The fraction of the object that is submerged is

$$f = \frac{V_{sub}}{V_{obj}} = \frac{V_{fluid}}{V_{obj}} = \frac{m_{fluid}/\rho_{fluid}}{m_{obj}/\rho_{obj}} = \frac{m_{fluid}}{m_{obj}}\frac{\rho_{obj}}{\rho_{fluid}}$$

Since the object floats, we know its weight exactly cancels the buoyant force, so $F_B = W_{obj} = m_{obj}g$. But also, $F_B = m_{fluid}g$ from Archimedes' principle, so $m_{obj} = m_{fluid}$. Therefore:

$$f = \frac{\rho_{obj}}{\rho_{fluid}}$$

Specific gravity

- Specific gravity is the ratio of the densities of an object and a fluid:

$$S_g \equiv \frac{\rho_{obj}}{\rho_{fluid}}$$

  - This is exactly the same as f, the submerged fraction.
  - ρ_{obj} is the **average** density, since most objects have components with different densities.
  - The fluid considered is almost always water at 4 °C.
- If the object floats, $\rho_{obj} < \rho_{fluid}$, so $S_g < 1$.
- If the object sinks, $\rho_{obj} > \rho_{fluid}$, so $S_g > 1$.

Measuring density

- We can measure the density of an object by submerging it in water:

$$f = \frac{\rho_{obj}}{\rho_{water}} \quad \Longrightarrow \quad \rho_{obj} = f\rho_{water}$$

- This can be used, for example, to measure the percent body fat of a person based on their density.