PHYS 1P22/92

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14. Heat and Heat Transfer Methods

14.1 Heat

Temperature and energy

- Recall that temperature is proportional to kinetic energy:

$$T = \frac{2}{3k_B} E_k$$

- This kinetic energy is the thermal energy of the system.
- Internal energy is the total energy "stored" in the system.
  - This includes thermal energy, but also e.g. potential energy between particles.
  - This does not include external energy, e.g. kinetic energy due to movement of the system as a whole.
  - For an ideal gas, thermal energy is equal to internal energy, since there are no other internal sources of energy.

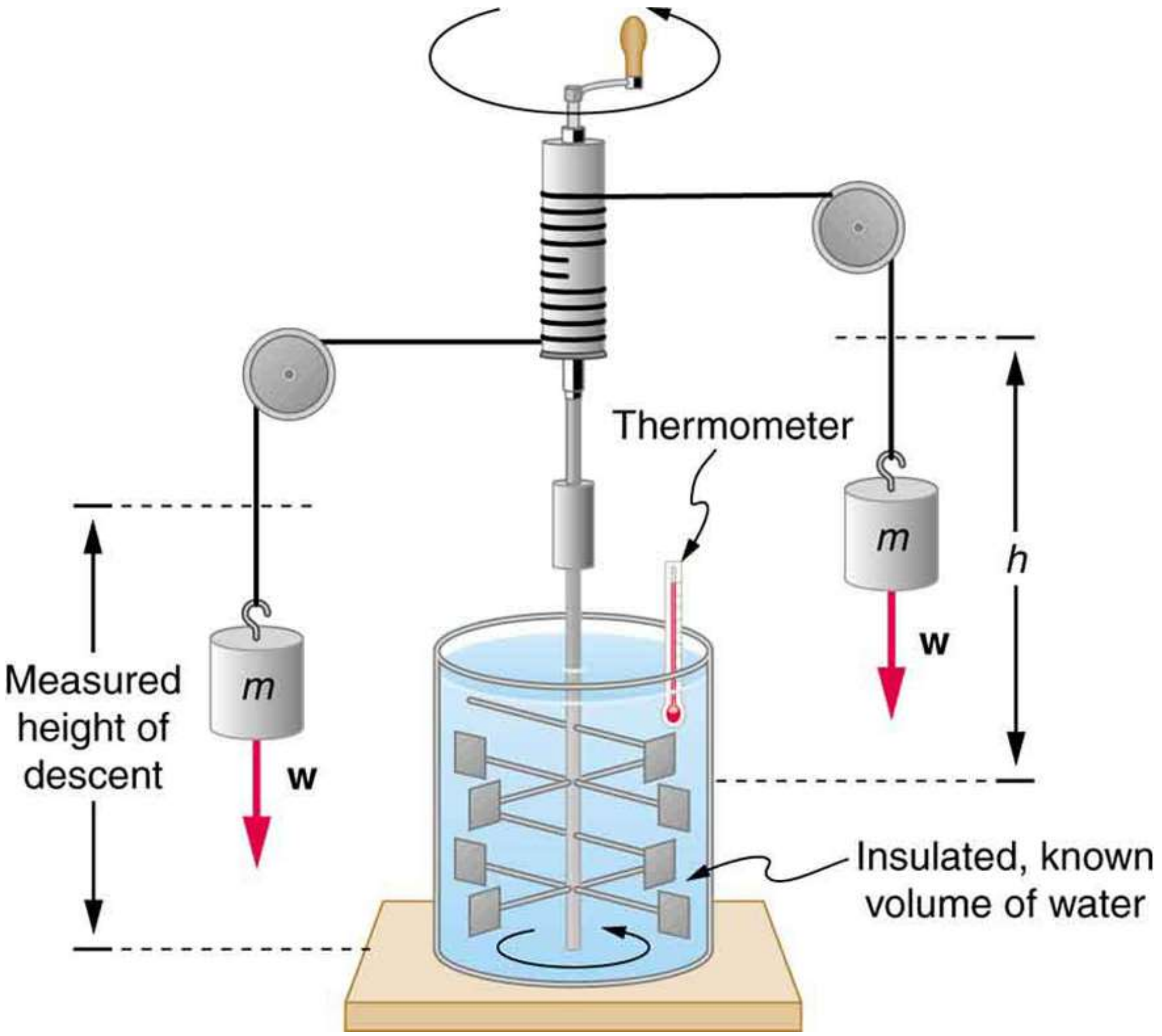
Heat

- When two objects are in contact, energy transfers from the hotter to the colder object until thermal equilibrium is reached.
  - **Pop Quiz:** Is work done by or on the objects?
  - **Answer:** No, since no force is acting over a distance ($W = Fx$).
- Heat is the thermal energy spontaneously transferred between systems due to a temperature difference.
- Misconception: Heat should not be confused with temperature or thermal energy. It is only the energy transferred.

Heat units

- Heat is transferred energy, so its units are joules (J).
- Calories (cal) are often used in non-scientific contexts.
- 1 calorie is the energy needed to change the temperature of 1 g of water by 1 °C.
- In SI units, 1 cal $\equiv$ 4.184 J (exact definition).
  - Note: This is the definition we will use in this course, even though the textbook uses a slightly different one.
- Food calories are actually kilocalories (kcal = 1,000 cal = 4,184 J).

- This experiment by Joule demonstrated the mechanical equivalent of heat.
- Gravitational potential energy does work, which is used to stir the water and increase its temperature.
    - **Pop Quiz:** Why does this increase the temperature? (Hint: Remember the relation between temperature and energy.)
    - Answer: The stirring moves atoms around, so it gives them kinetic energy.
- The energy is converted to heat, so heat is a form of energy.

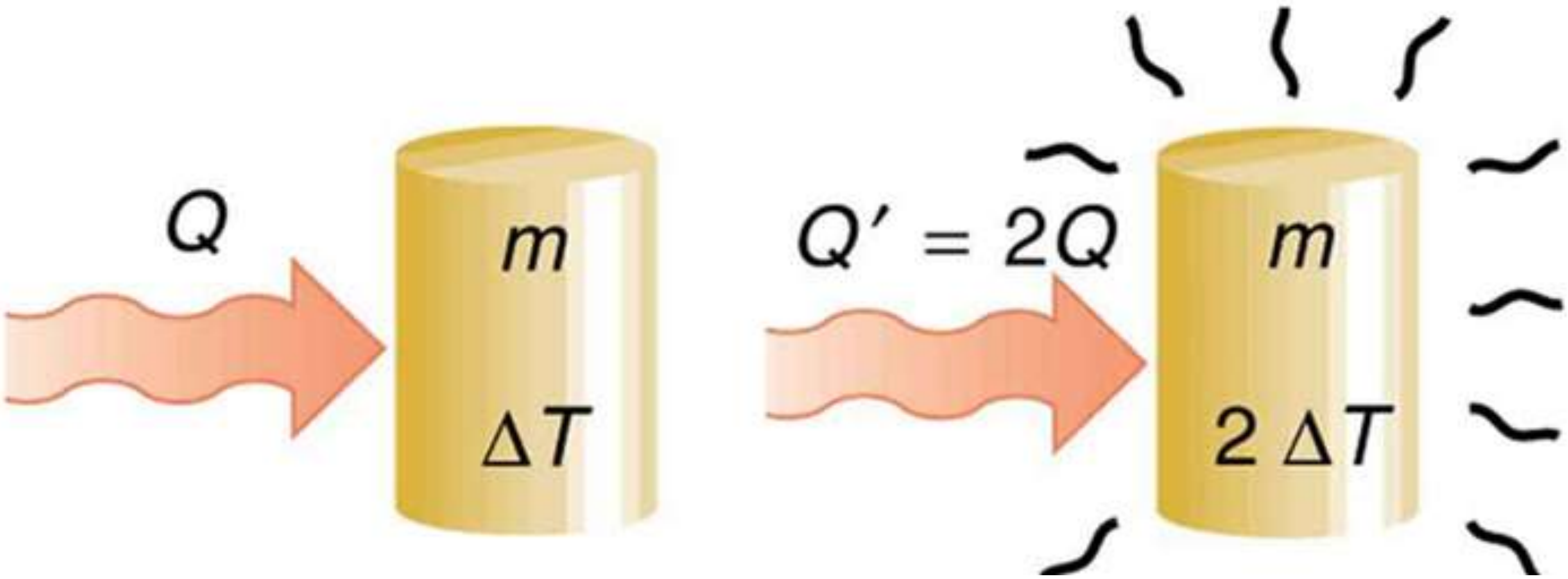


14.2 Temperature Change and Heat Capacity

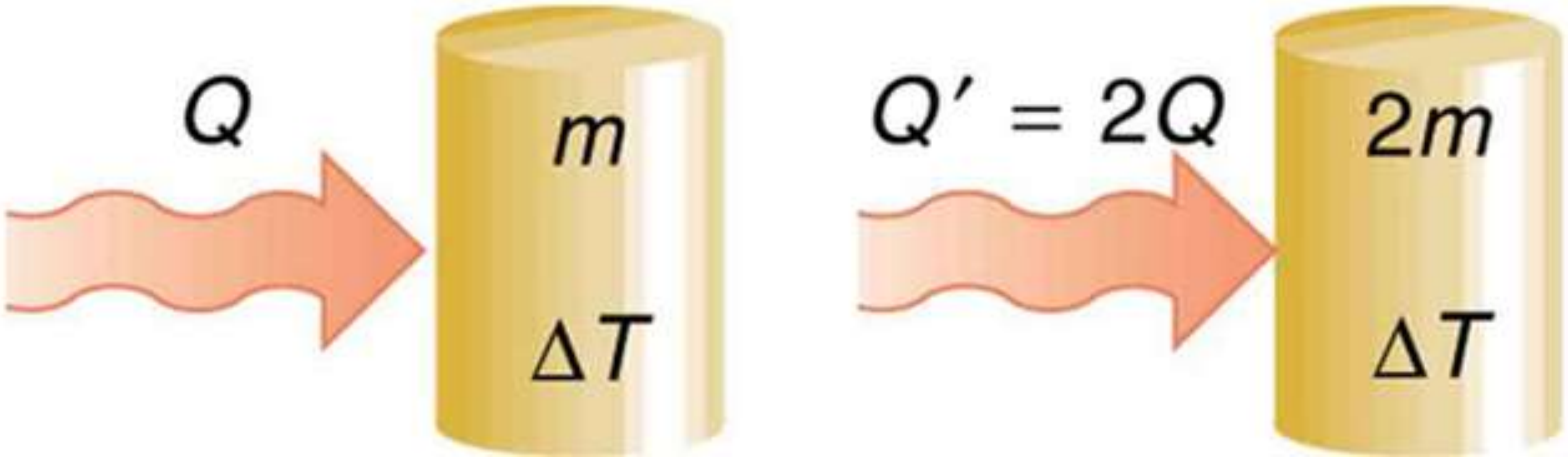
Temperature change

- If there is no phase change and no work is being done, heat transfer will cause a temperature change.
- Heating increases temperature, cooling decreases it.
- The transferred heat depends on three factors:
  - The change in temperature.
  - The mass of the system.
  - The substance and its phase.

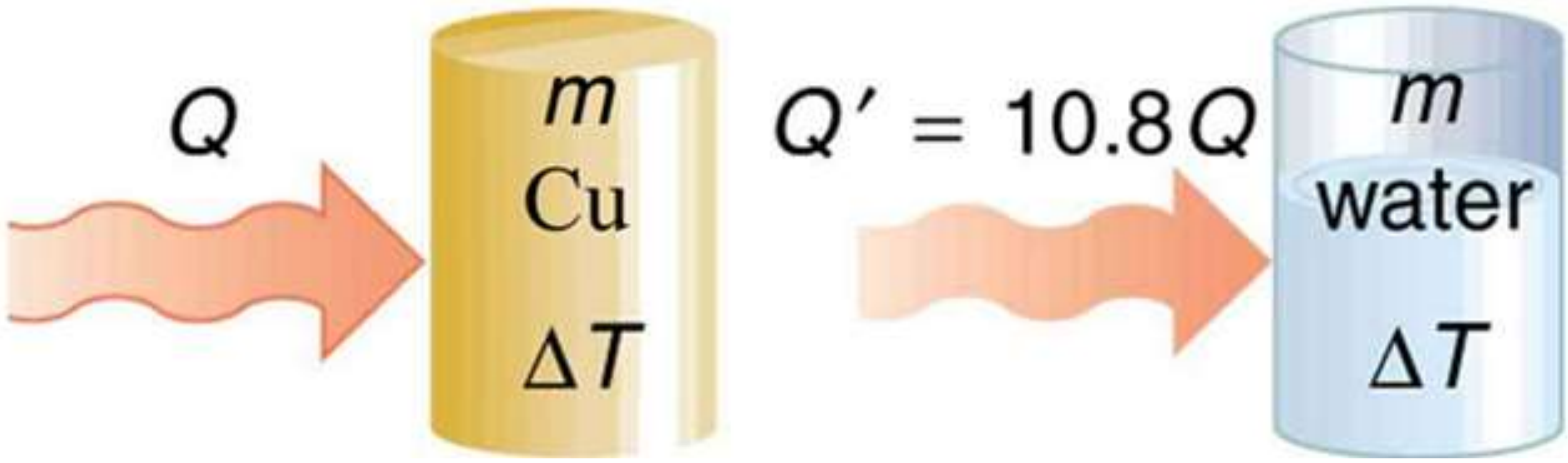
- The amount Q of heat transferred is directly proportional to the temperature change ΔT.



- The amount Q of heat transferred is also directly proportional to the mass m.



- The amount Q of heat transferred is different for different substances and phases.
- E.g.: Water needs 10.8 times Q for the same ΔT compared to copper.



Specific heat

- The heat Q needed for a temperature change ΔT of a mass m is:

$$Q = mc\Delta T$$

- c is specific heat capacity (or just specific heat) and depends on the specific substance.
- **Pop Quiz:** What are the units of c?
- Answer:

$$c = \frac{Q}{m\Delta T} \quad \Longrightarrow \quad [c] = \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$$

Values at constant volume at 25 °C, unless otherwise noted.

Substance (solid)	Specific heat ($J \cdot kg^{-1} \cdot K^{-1}$)
Aluminum	900
Asbestos	800
Concrete, granite (average)	840
Copper	387
Glass	840
Gold	129
Human body (average at 37 °C)	3500
Ice (average, −50 °C to 0 °C)	2090
Iron, steel	452
Lead	128
Silver	235
Wood	1700

Values at constant volume at 25 °C, unless otherwise noted.

Substance (liquid)	Specific heat ($J \cdot kg^{-1} \cdot K^{-1}$)
Benzene	1740
Ethanol	2450
Glycerin	2410
Mercury	139
Water (15 °C)	4186

c_V and c_P

- Raising temperature generally also increases volume and/or pressure.
- c_P is specific heat at constant pressure (or isobaric).
  - Example: at atmospheric pressure.
  - V will change, so work will be done.
- c_V is specific heat at constant volume (or isochoric).
  - Example: inside a rigid container.
  - P will change, so internal energy will change.
  - No work is done, more energy goes to temperature, so usually $c_V < c_P$.

Values at 20.0 °C, unless otherwise noted.

c_V at constant volume.

c_P at 1.00 atm.

Substance (gas)	c_V ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)	c_P ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)
Air (dry)	721	1015
Ammonia	1670	2190
Carbon dioxide	638	833
Nitrogen	739	1040
Oxygen	651	913
Steam (100°C)	1520	2020

- **Problem:** A truck controls speed when going downhill using its brakes, converting gravitational potential energy to heat instead of speed. Calculate the temperature increase ΔT of the brake material with mass m and specific heat c if the material retains a fraction f of the energy from a truck with mass M descending a height h at a constant speed.
- **Solution:** The truck needs to cancel out potential energy $E_p = Mgh$. This is converted into heat $Q = E_p$. The heat is transferred mostly to the environment, but a fraction fQ is retained and causes a temperature increase ΔT:

$$fQ = mc\Delta T \quad \Longrightarrow \quad \Delta T = \frac{fQ}{mc} = \frac{fMgh}{mc}$$

- **Problem:** Calculate ΔT if $m \approx 100$ kg, $c \approx 800\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}}$, $f \approx 10\%$, $M \approx 10{,}000$ kg, $h \approx 75.0$ m.
- **Solution:**

$$\Delta T = \frac{fMgh}{mc} = \frac{(10\%)(10{,}000\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(75\ \mathrm{m})}{(100\ \mathrm{kg})(800\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}})}$$

$$\approx \frac{0.1 \cdot 10{,}000 \cdot 9.8 \cdot 75}{100 \cdot 800}\ \frac{\mathrm{kg \cdot m/s^2 \cdot m}}{\mathrm{kg \cdot J \cdot kg^{-1} \cdot K^{-1}}}$$

$$\approx 9.2\ \frac{\mathrm{m^2 \cdot kg \cdot K}}{\mathrm{J \cdot s^2}} \qquad (\mathrm{J = kg \cdot m^2/s^2})$$

$$= 9.2\ \mathrm{K}$$

Fun fact: Hybrid cars work by storing this as electrical energy in a battery.

- **Problem:** You pour a mass m_w of water at temperature T_w into a pan off the stove with a mass m_p and temperature T_p. Assume that the pan is placed on an insulated pad and that no water boils off. What is the temperature T_f when the water and pan reach thermal equilibrium? Use c_w and c_p for the specific heats.

- **Solution:** $Q_w = m_w c_w (T_f - T_w)$, $\quad Q_p = m_p c_p (T_f - T_p)$

Heat is transferred from the hot pan to the cold water, so $Q_p < 0$ and $Q_w > 0$. There is no loss of energy, so $Q_w + Q_p = 0$:

$$m_w c_w (T_f - T_w) = -m_p c_p (T_f - T_p)$$

- **Class Problem:** Isolate T_f.

- **Solution:** $T_f = \frac{m_w c_w T_w + m_p c_p T_p}{m_w c_w + m_p c_p}$

- **Class Problem:** Calculate $T_f = \frac{m_w c_w T_w + m_p c_p T_p}{m_w c_w + m_p c_p}$ if:
  - $m_w \approx 0.250\ \mathrm{kg}$, $T_w \approx 20.0\ °\mathrm{C}$, $c_w \approx 4186\ \mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}$
  - $m_p \approx 0.500\ \mathrm{kg}$, $T_p \approx 150\ °\mathrm{C}$, $c_p \approx 900\ \mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}$
  - $\mathrm{K} = °\mathrm{C} + 273.15$
- **Solution:** Numerical value:

$$T_f \approx \frac{0.25 \cdot 4186 \cdot (20 + 273.15) + 0.5 \cdot 900 \cdot (150 + 273.15)}{0.25 \cdot 4186 + 0.5 \cdot 900}$$

$$\approx 332\ \mathrm{K} \approx 58.9\ °\mathrm{C}$$

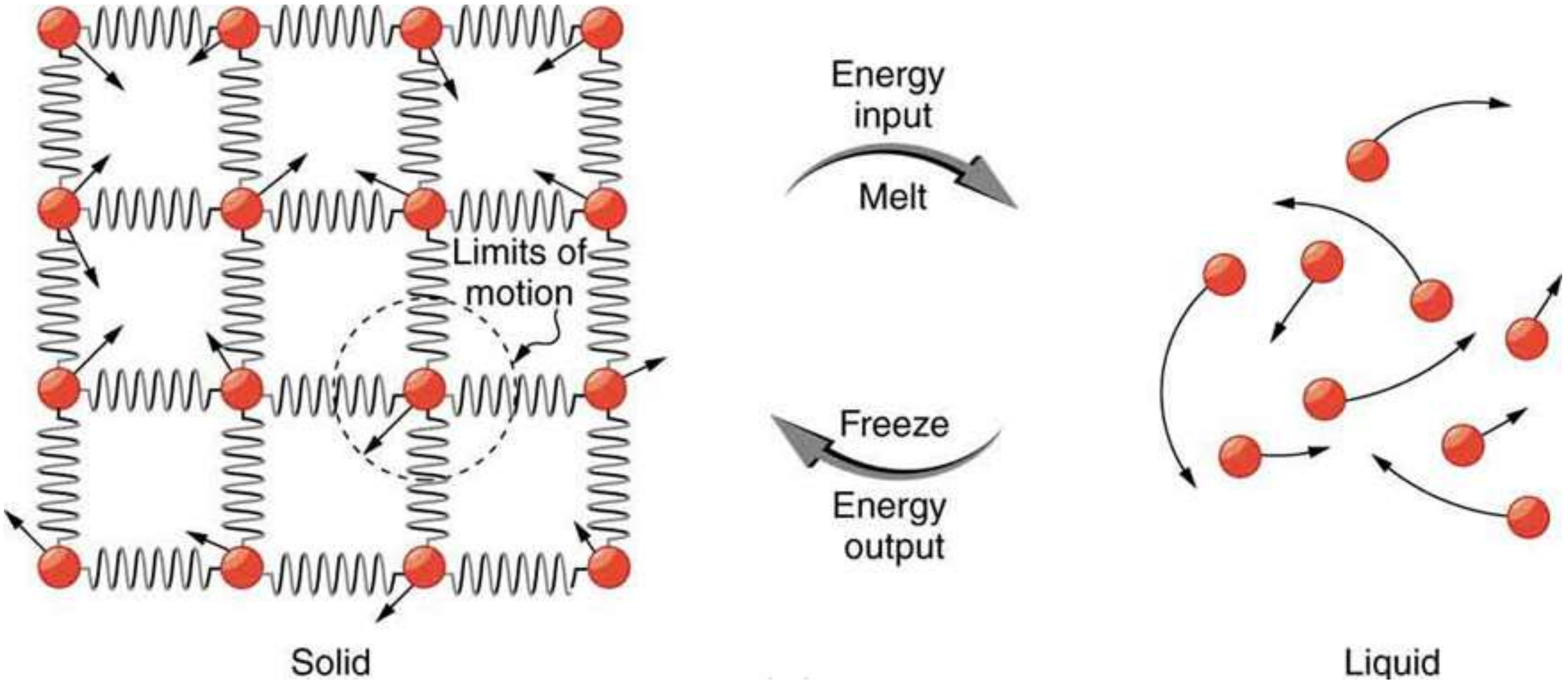
Units:

$$\frac{\mathrm{kg} \cdot \mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1} \cdot \mathrm{K}}{\mathrm{kg} \cdot \mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}} = \mathrm{K}$$

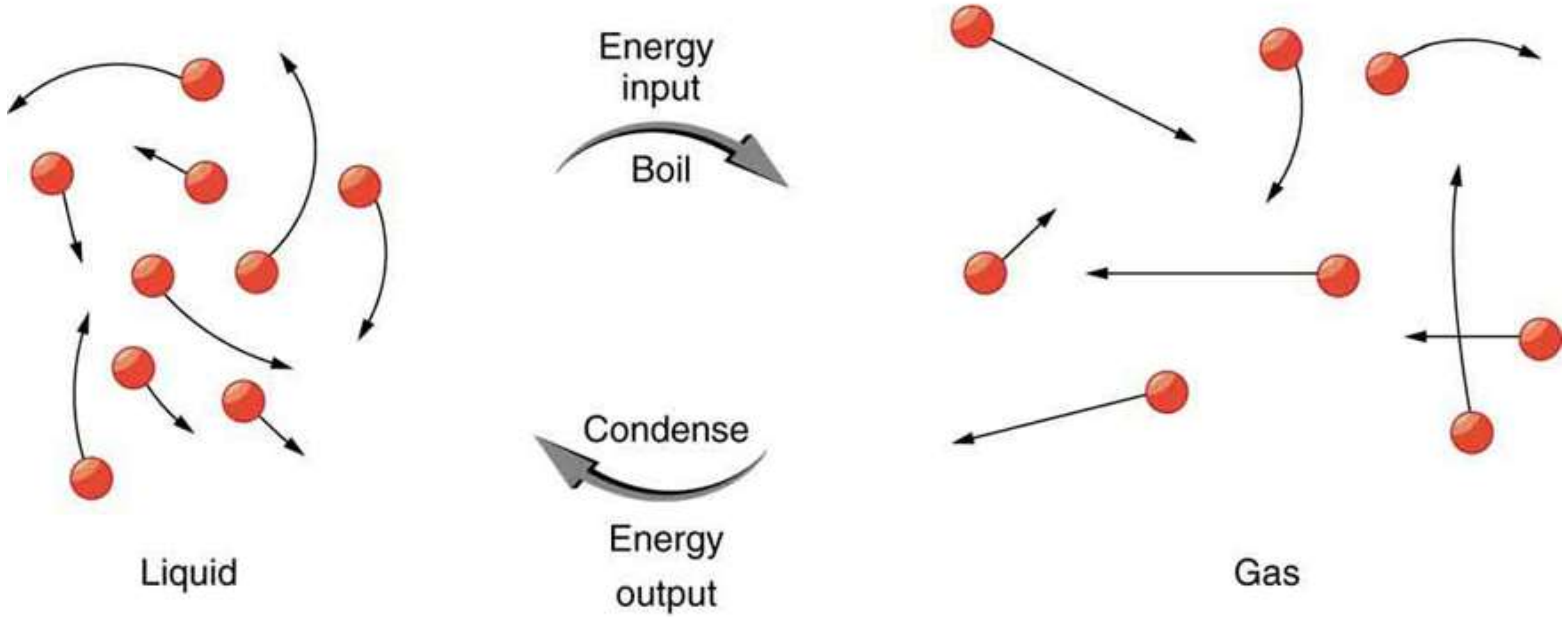
- **Pop Quiz:** If 25 kJ is necessary to raise the temperature of a block from 25 °C to 30 °C, how much heat is necessary to heat the block from 45 °C to 50 °C?
- **Answer:** $Q = mc\Delta T$ depends only on temperature **difference**, so 25 kJ.

14.3 Phase Change and Latent Heat

- When we heat a solid to its melting temperature, any further heat energy will be used to break the bonds between the particles.
- Only once the solid turns into a liquid, the temperature will start increasing again.
- Same applies to freezing, in the other direction.



- This also applies to vaporization (boiling) or condensation.



Latent heat

- The heat required to change the phase of a sample of mass m is
$$Q = mL$$
- L is called latent heat. There are two values:
  - L_f: latent heat of fusion (melting/freezing)
  - L_v: latent heat of vaporization (boiling/condensation)
- These are constants determined experimentally for each substance.
- "Latent" = hidden; has a "hidden" effect instead of changing temperature.
- Pop Quiz: What are the units of L?
  - Answer: $L = Q/m$ so $[L] = \mathrm{J/kg}$. L is the energy needed to melt/boil 1 kg.

Values of L at 1 atm for various substances.

Substance	Melting point (°C)	L_f (kJ/kg)	Boiling point (°C)	L_v kJ/kg
Helium	−269.7	5.23	−268.9	20.9
Hydrogen	−259.3	58.6	−252.9	452
Nitrogen	−210.0	25.5	−195.8	201
Oxygen	−218.8	13.8	−183.0	213
Ethanol	−114	104	78.3	854
Ammonia	−75	452	−33.4	1370
Mercury	−38.9	11.8	357	272
Water	0.00	334	100.0	2256
Sulfur	119	38.1	444.6	326
Lead	327	24.5	1750	871
Antimony	631	165	1440	561
Aluminum	660	380	2450	11400
Silver	961	88.3	2193	2336
Gold	1063	64.5	2660	1578
Copper	1083	134	2595	5069
Uranium	1133	84	3900	1900
Tungsten	3410	184	5900	4810

- Class Problem: Let Q be the energy required to melt a mass m of ice. If we used the same energy to heat a mass m of water, what will be the temperature difference ΔT?
- Answer:

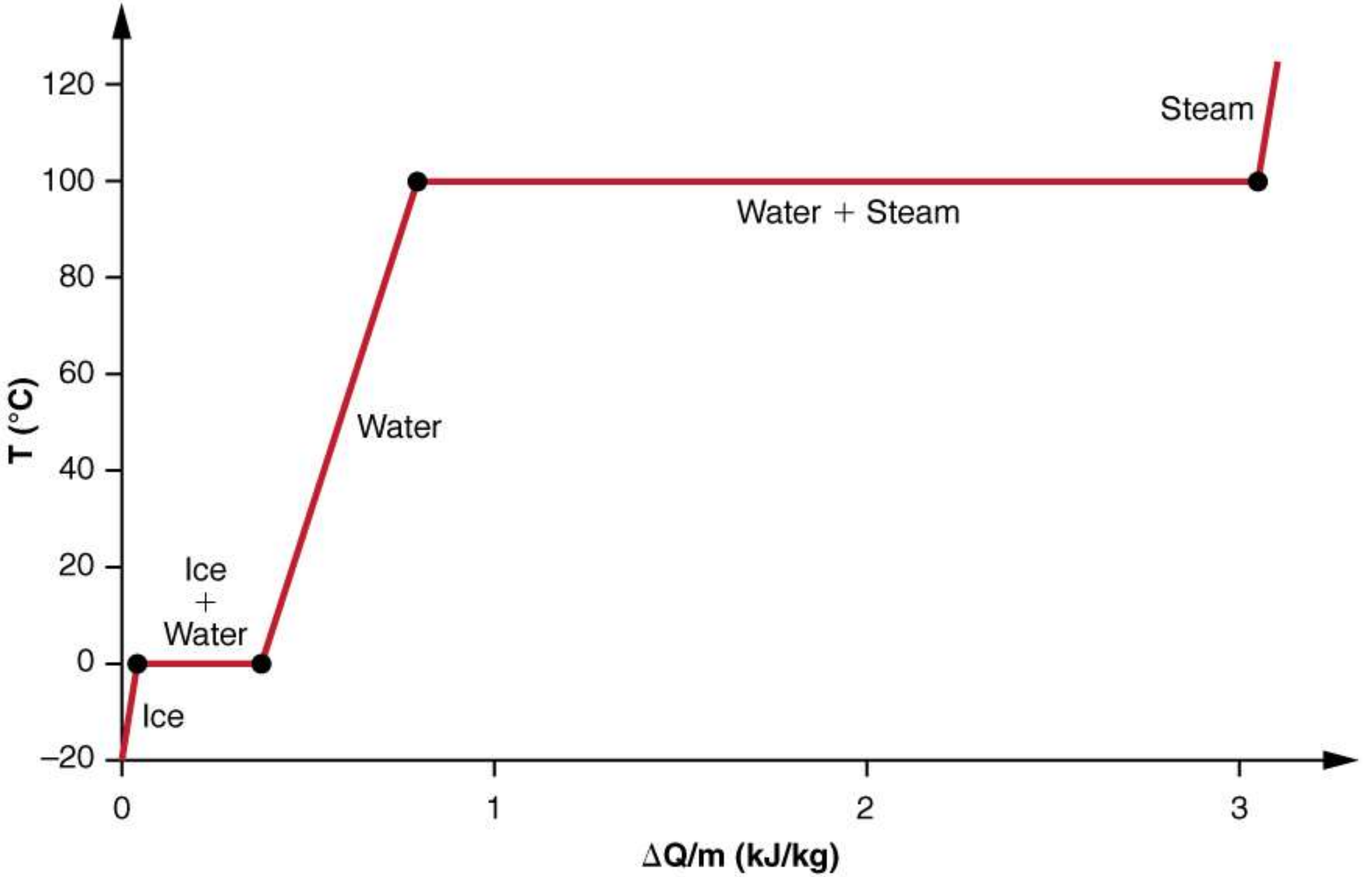
$$Q = mL_f = mc\Delta T \quad \Longrightarrow \quad \Delta T = \frac{L_f}{c}$$

- Class Problem: Calculate ΔT for water, given:
  - $L_f \approx 334\ \mathrm{kJ \cdot kg^{-1}}$
  - $c \approx 4186\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}}$.
- Answer:

$$\Delta T \approx \frac{334{,}000\ \mathrm{J \cdot kg^{-1}}}{4186\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}}} \approx 79.8\ \mathrm{K}$$

- The energy required to melt 1 kg of ice is huge – can be used to heat 1 kg of water from 0 to 79.8 °C!

- This graph shows how temperature changes as ice is heated from 20 °C.
- Note the constant temperature at the phase transitions.



- **Problem:** N ice cubes are used to chill a soda at temperature T_s with mass m_s. The ice is at freezing temperature T_i ($\approx 0.0\ °C$) and each ice cube has mass m_i. Find the final temperature T when all the ice has melted.
- **Solution:** When the ice melts, it first changes phase from solid to liquid ($Q = mL$). Then the liquid heats up ($Q = mc\Delta T$) until thermal equilibrium.
- Total heat transferred to the ice is:

$$Q_i \equiv Nm_iL + Nm_ic(T - T_i) = Nm_i(L + c(T - T_i))$$

- Total heat transferred from the soda is:

$$Q_s \equiv m_sc(T_s - T)$$

- $Q_i, Q_s > 0$ since $T_s > T > T_i$. They are equal from conservation of energy:

$$Nm_i\big(L + c(T - T_i)\big) = m_sc(T_s - T)$$

- Class Problem: Isolate T.

Answer: $$T = \frac{m_sT_s + Nm_i(T_i - L/c)}{m_s + Nm_i}$$

- **Problem:** Calculate T for:
  - $N = 3, T_i \approx 0.0\ °\text{C}, m_i \approx 6.0\ \text{g}$
  - $T_s \approx 20\ °\text{C}, m_s \approx 0.25\ \text{kg},$
  - $L \approx 334\ \text{kJ} \cdot \text{kg}^{-1}, c \approx 4186\ \text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
  - $\text{K} = °\text{C} + 273.15$
- **Solution:**

$$T = \frac{m_s T_s + N m_i (T_i - L/c)}{m_s + N m_i}$$

$$\approx \frac{(0.25\ \text{kg})(293.15\ \text{K}) + 3(0.006\ \text{kg})\big(273.15\ \text{K} - (334{,}000\ \text{J} \cdot \text{kg}^{-1})/(4186\ \text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1})\big)}{0.25\ \text{kg} + 3(0.006\ \text{kg})}$$

$$= \frac{0.25 \cdot 293.15 + 3 \cdot 0.006 \cdot (273.15 - 334{,}000/4186)}{0.25 + 3 \cdot 0.006}\ \frac{\text{kg} \cdot \text{K}}{\text{kg}}$$

$$\approx 286.45\ \text{K} \approx 13\ °\text{C}$$

- Pop Quiz: 1. If $N = 3$, why not 1 s.f.? 2. Why not round up the K amount to 2 s.f.?
  1. This is a discrete number of items = infinite precision.
  2. Different orders of magnitude; 290 K ≈ 17 °C is very imprecise!

Sublimation

- Sublimation is direct transition from solid to gas without passing through the liquid phase.
  - Example: Dry ice.
- The reverse process is deposition (or desublimation).
  - Example: Frost.
- This occurs via the usual equation $Q = mL_s$, with L_s the latent heat of sublimation.

- **Pop Quiz:** Why do mounds of snow on the ground not melt even if the temperature is above freezing?
- **Answer:** Heat will be transferred from the air, but it takes a lot of heat to cause a phase change.
- Recall: Energy to melt 1 kg of ice $=$ heat 1 kg of water from 0 to 79.8 °C.
- To melt the snow, the air must be hot enough to transfer all that energy over the day, before night falls and it goes below freezing again.

14.4 Heat Transfer Methods

- Conduction: Heat transfer through stationary matter by physical contact.
  - Examples: Cooking on a stove, holding a hot cup of coffee.
- Convection: Heat transfer by the macroscopic movement of a fluid.
  - Examples: Furnace, weather systems.
- Radiation: Heat transfer by emitting or absorbing electromagnetic radiation.
  - Examples: Warming of the Earth by the Sun, microwave oven.
- We won't learn about this in any more detail; you can read sections 14.5-14.7.