PHY 256: Introduction To Quantum Physics
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Practice Questions for the Final Assessment

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1. Write the complex number $z = 3\,\mathrm{e}^{\mathrm{i}\,\pi}$ in Cartesian coordinates.
Answer:
$$z = 3 \cdot (-1) = -3. \tag{0.1}$$
Write the complex number $w = 1 + \mathrm{i}$ in polar coordinates.
Answer:
$$w = \sqrt{2}\,\mathrm{e}^{\mathrm{i}\,\pi/4}. \tag{0.2}$$
Calculate $|z + w|$.
Answer:
$$|-3 + 1 + \mathrm{i}| = |-2 + \mathrm{i}| = \sqrt{5}. \tag{0.3}$$
What is the complex conjugate of $v = -5\,\mathrm{e}^{-7\,\mathrm{i}}$?
Answer:
$$v^* = -5\,\mathrm{e}^{7\,\mathrm{i}}. \tag{0.4}$$

2. Let
$$|A\rangle \equiv \begin{pmatrix} 1 \\ 2\,\mathrm{i} \end{pmatrix}, \qquad |B\rangle \equiv \begin{pmatrix} -3 \\ 5 - \mathrm{i} \end{pmatrix}. \tag{0.5}$$
Calculate $\langle A|B\rangle$.
Answer:
$$\langle A|B\rangle = \begin{pmatrix} 1 & -2\,\mathrm{i} \end{pmatrix} \begin{pmatrix} -3 \\ 5 - \mathrm{i} \end{pmatrix} = -3 - 2\,\mathrm{i}\,(5 - \mathrm{i}) = -5 - 10\,\mathrm{i}. \tag{0.6}$$

Calculate $\langle B|A\rangle$.
Answer:

$$\langle B|A\rangle = \langle A|B\rangle^* = -5 + 10\,\mathrm{i}\,. \tag{0.7}$$

Calculate $|\langle A|B\rangle|$.
Answer:

$$|\langle A|B\rangle| = \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}. \tag{0.8}$$

Calculate $|A\rangle + |B\rangle$.
Answer:

$$|A\rangle + |B\rangle = \begin{pmatrix} 1 \\ 2\,\mathrm{i} \end{pmatrix} + \begin{pmatrix} -3 \\ 5 - \mathrm{i} \end{pmatrix} = \begin{pmatrix} -2 \\ 5 + \mathrm{i} \end{pmatrix}. \tag{0.9}$$

Calculate $\||A\rangle + |B\rangle\|$.
Answer:

$$\||A\rangle + |B\rangle\| = \sqrt{2^2 + 5^2 + 1^2} = \sqrt{30}. \tag{0.10}$$

3. Let

$$|A\rangle = \begin{pmatrix} 1 \\ 2\,\mathrm{i} \end{pmatrix}, \qquad |B\rangle = \begin{pmatrix} 2\,\mathrm{i} \\ 1 \end{pmatrix}. \tag{0.11}$$

Do these vectors form an orthonormal basis?
Answer: They are linearly independent, span $\mathbb{C}^2$, and orthogonal since

$$\langle A|B\rangle = \begin{pmatrix} 1 & -2\,\mathrm{i} \end{pmatrix} \begin{pmatrix} 2\,\mathrm{i} \\ 1 \end{pmatrix} = 0, \tag{0.12}$$

but they are not normalized to 1. So it's an orthogonal basis, but not an orthonormal basis. We can normalize it as follows:

$$|A\rangle \mapsto \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2\,\mathrm{i} \end{pmatrix}, \qquad |B\rangle \mapsto \frac{1}{\sqrt{5}} \begin{pmatrix} 2\,\mathrm{i} \\ 1 \end{pmatrix}. \tag{0.13}$$

4. Let

$$A = \begin{pmatrix} 1 & 2\,\mathrm{i} \\ 0 & -1 \end{pmatrix}, \qquad |B\rangle = \begin{pmatrix} 0 \\ \mathrm{i} \end{pmatrix}. \tag{0.14}$$

Calculate $A\,|B\rangle$ and $\langle B|\,A^\dagger$.
Answer:

$$A\,|B\rangle = \begin{pmatrix} 1 & 2\,\mathrm{i} \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ \mathrm{i} \end{pmatrix} = \begin{pmatrix} -2 \\ -\,\mathrm{i} \end{pmatrix} \tag{0.15}$$

$$\langle B|\,A^\dagger = \begin{pmatrix} 0 & -\,\mathrm{i} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2\,\mathrm{i} & -1 \end{pmatrix} = \begin{pmatrix} -2 & \mathrm{i} \end{pmatrix}, \tag{0.16}$$

or we can just calculate

$$\langle B| A^\dagger = (A|B\rangle)^\dagger . \tag{0.17}$$

5. Let

$$|A\rangle = \begin{pmatrix} 1 \\ \mathsf{i} \end{pmatrix}, \qquad |B\rangle = \begin{pmatrix} -\mathsf{i} \\ 2 \end{pmatrix}. \tag{0.18}$$

Calculate $|A\rangle\langle B|$.
Answer:

$$|A\rangle\langle B| = \begin{pmatrix} 1 \\ \mathsf{i} \end{pmatrix} \begin{pmatrix} \mathsf{i} & 2 \end{pmatrix} = \begin{pmatrix} \mathsf{i} & 2 \\ -1 & 2\mathsf{i} \end{pmatrix}. \tag{0.19}$$

Calculate $\langle B|A\rangle\langle B|A\rangle$.
Answer:

$$\langle B|A\rangle\langle B|A\rangle = \begin{pmatrix} \mathsf{i} & 2 \end{pmatrix} \begin{pmatrix} \mathsf{i} & 2 \\ -1 & 2\mathsf{i} \end{pmatrix} \begin{pmatrix} 1 \\ \mathsf{i} \end{pmatrix} = \begin{pmatrix} \mathsf{i} & 2 \end{pmatrix} \begin{pmatrix} 3\mathsf{i} \\ -3 \end{pmatrix} = -3-6 = -9 \tag{0.20}$$

or just note that

$$\langle B|A\rangle = \begin{pmatrix} \mathsf{i} & 2 \end{pmatrix} \begin{pmatrix} 1 \\ \mathsf{i} \end{pmatrix} = \mathsf{i} + 2\mathsf{i} = 3\mathsf{i}, \tag{0.21}$$

so

$$\langle B|A\rangle\langle B|A\rangle = \langle B|A\rangle^2 = -9. \tag{0.22}$$

6. Let

$$|A\rangle = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ 3\mathsf{i} \end{pmatrix}, \qquad |B\rangle = \frac{1}{\sqrt{13}} \begin{pmatrix} 3\mathsf{i} \\ 2 \end{pmatrix}. \tag{0.23}$$

Calculate $|A\rangle\langle A| + |B\rangle\langle B|$.
Answer: No need to calculate, it's an orthonormal basis, so $|A\rangle\langle A| + |B\rangle\langle B|$ is just the identity matrix by the completeness relation.

7. A 2×2 matrix A satisfies for some vector $|B\rangle$

$$A|B\rangle = \mathsf{i}|B\rangle . \tag{0.24}$$

Can this matrix be Hermitian?
Answer: No, since it has a non-real eigenvalue. Hermitian matrices can only have real eigenvalues.
Can this matrix be unitary?
Answer: Possibly, since its eigenvalues should have unit magnitude.
The matrix also satisfies for some other vector $|C\rangle$

$$A|C\rangle = \mathsf{i}|C\rangle . \tag{0.25}$$

Is it diagonalizable?
Answer: Not necessarily, since $|C\rangle$ might be just a scalar multiple of $|B\rangle$.
The matrix also satisfies for some vector $|D\rangle$

$$A\,|D\rangle = -\mathsf{i}\,|D\rangle\,. \tag{0.26}$$

Is it diagonalizable?
Answer: Yes, now we know it has two different eigenvalues, so it is diagonalizable to

$$\begin{pmatrix} \mathsf{i} & 0 \\ 0 & -\mathsf{i} \end{pmatrix}. \tag{0.27}$$

8. Consider the matrix

$$A = y \begin{pmatrix} 1 & x \\ \mathsf{i} & 1 \end{pmatrix}. \tag{0.28}$$

For which values of x and y can this matrix represent evolution in quantum mechanics?
Answer: It has to be unitary, so its rows (or columns) must form an orthonormal basis. We see that we should take

$$x = \mathsf{i}, \qquad y = \frac{1}{\sqrt{2}}, \qquad A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathsf{i} \\ \mathsf{i} & 1 \end{pmatrix}. \tag{0.29}$$

Another way to solve this would be to calculate $AA^\dagger$ and demand that it's the identity matrix, but that would be a much longer calculation.

9. Write down a normal 2×2 matrix that has the number $7\,\mathsf{i}$ in the **top-left** element.
Answer: For example

$$A = \begin{pmatrix} 7\,\mathsf{i} & 0 \\ 0 & 0 \end{pmatrix} \quad \Longrightarrow \quad A^\dagger = \begin{pmatrix} -7\,\mathsf{i} & 0 \\ 0 & 0 \end{pmatrix}, \qquad A^\dagger A = AA^\dagger = \begin{pmatrix} 49 & 0 \\ 0 & 0 \end{pmatrix}. \tag{0.30}$$

Write down a normal 2×2 matrix that has the number $7\,\mathsf{i}$ in the **top-right** element.
Answer: The simplest solution is to make a Hermitian matrix, for example

$$A = \begin{pmatrix} 0 & 7\,\mathsf{i} \\ -7\,\mathsf{i} & 0 \end{pmatrix} = A^\dagger, \qquad A^\dagger A = AA^\dagger = \begin{pmatrix} -49 & 0 \\ 0 & -49 \end{pmatrix}. \tag{0.31}$$

10. What is the probability distribution for the sum of three coin tosses, with heads = 0 and tails = 1?
Answer: There are $2^3 = 8$ possible rolls, and the results can be numbers

between 0 and 3. We count the number of rolls that make up each possible result:

$$0 = 0 + 0 + 0, \qquad 1 = 1 + 0 + 0 = 0 + 1 + 0 = 0 + 0 + 1, \tag{0.32}$$

$$2 = 1 + 1 + 0 = 1 + 0 + 1 = 0 + 1 + 1, \qquad 3 = 1 + 1 + 1. \tag{0.33}$$

So the answer is:

$$P(0) = \frac{1}{8}, \qquad P(1) = \frac{3}{8}, \qquad P(2) = \frac{3}{8}, \qquad P(3) = \frac{1}{8}. \tag{0.34}$$

What is the expected value?
Answer:

$$0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = \frac{3}{2}. \tag{0.35}$$

Or we can recall that the expected value for one coin toss is $1/2$, so for three coin tosses it will be $3/2$.

11. A qubit is in the state

$$|A\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2\,\mathsf{i} \end{pmatrix}. \tag{0.36}$$

Write it as a superposition of the state $|+\rangle$ and $|-\rangle$, where

$$|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad |-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{0.37}$$

Answer: We calculate that

$$\langle +|A\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2\,\mathsf{i} \end{pmatrix} = \frac{1}{\sqrt{10}} (1 + 2\,\mathsf{i}), \tag{0.38}$$

$$\langle +|A\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2\,\mathsf{i} \end{pmatrix} = \frac{1}{\sqrt{10}} (1 - 2\,\mathsf{i}). \tag{0.39}$$

Therefore

$$\begin{aligned} |A\rangle &= |+\rangle \langle +|A\rangle + |-\rangle \langle -|A\rangle \\ &= \frac{1}{\sqrt{10}} \Big((1 + 2\,\mathsf{i})\,|+\rangle + (1 - 2\,\mathsf{i})\,|-\rangle \Big). \end{aligned}$$

12. Find four **different** states of a qubit such that the measurement of σ_z will yield 0 with probability $2/3$ and 1 with probability $1/3$.
Answer: For example

$$\sqrt{\frac{2}{3}}\,|0\rangle + \sqrt{\frac{1}{3}}\,|1\rangle, \tag{0.40}$$

$$\sqrt{\frac{2}{3}}\,|0\rangle - \sqrt{\frac{1}{3}}\,|1\rangle\,, \tag{0.41}$$

$$\sqrt{\frac{2}{3}}\,|0\rangle + \mathsf{i}\sqrt{\frac{1}{3}}\,|1\rangle\,, \tag{0.42}$$

$$\sqrt{\frac{2}{3}}\,|0\rangle - \mathsf{i}\sqrt{\frac{1}{3}}\,|1\rangle\,. \tag{0.43}$$

The trick is that the **relative** phase between the two states in the superposition should be different. But a vector such as

$$-\sqrt{\frac{2}{3}}\,|0\rangle + \sqrt{\frac{1}{3}}\,|1\rangle \tag{0.44}$$

will represent the same state as (0.41), since they are related by an **overall** phase of -1.

13. A qubit is in the state

$$|A\rangle = \frac{1}{\sqrt{10}}\begin{pmatrix} \mathsf{i} \\ 3\,\mathsf{i} \end{pmatrix}. \tag{0.45}$$

What is the expectation value for a measurement of spin along the x axis? You can use the fact that

$$S_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{0.46}$$

Answer:

$$\begin{aligned}
\langle A|S_x|A\rangle &= \frac{1}{\sqrt{10}}\begin{pmatrix} -\mathsf{i} & -3\,\mathsf{i} \end{pmatrix}\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{10}}\begin{pmatrix} \mathsf{i} \\ 3\,\mathsf{i} \end{pmatrix} \\
&= \frac{1}{20}\begin{pmatrix} -\mathsf{i} & -3\,\mathsf{i} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \mathsf{i} \\ 3\,\mathsf{i} \end{pmatrix} \\
&= \frac{1}{20}\begin{pmatrix} -\mathsf{i} & -3\,\mathsf{i} \end{pmatrix}\begin{pmatrix} 3\,\mathsf{i} \\ \mathsf{i} \end{pmatrix} \\
&= \frac{1}{20}(3+3) \\
&= \frac{3}{10}
\end{aligned}$$

14. The measurement of spin of a particle along the z axis was 2. What can you say about the other possibilities for a measurement of spin for that particle? **Answer:** For a particle of spin s, where

$$s \in \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, ...\right\}, \tag{0.47}$$

the measurement of the spin along any axis can yield any of the values

$$\{-s, -s+1, \ldots, s-1, s\}\,. \tag{0.48}$$

Since 2 was the result of the measurement, we know that for this particle $s \geq 2$, and furthermore, it must be an integer (since half-integer s only allows half-integer measurement results). The other possibilities for a measurement of spin for that particle will thus be $-2, 1, 0, 1$, but there could of course be other (integer) options if $s > 2$.

15. Let

$$|A\rangle = \begin{pmatrix} 2 \\ \mathsf{i} \end{pmatrix}, \qquad |B\rangle = \begin{pmatrix} \mathsf{i} \\ 3 \end{pmatrix}. \tag{0.49}$$

Calculate the tensor product $|A\rangle \otimes |B\rangle$.
Answer:

$$|A\rangle \otimes |B\rangle = \begin{pmatrix} 2 \\ \mathsf{i} \end{pmatrix} \otimes \begin{pmatrix} \mathsf{i} \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \cdot \begin{pmatrix} \mathsf{i} \\ 3 \end{pmatrix} \\ \mathsf{i} \cdot \begin{pmatrix} \mathsf{i} \\ 3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 2\,\mathsf{i} \\ 6 \\ -1 \\ 3\,\mathsf{i} \end{pmatrix}. \tag{0.50}$$

Let

$$C = \begin{pmatrix} 1 & 0 \\ 0 & \mathsf{i} \end{pmatrix}, \qquad D = \begin{pmatrix} 0 & 2 \\ 2\,\mathsf{i} & 0 \end{pmatrix}. \tag{0.51}$$

Calculate the tensor product $C \otimes D$.
Answer:

$$C \otimes D = \begin{pmatrix} \begin{pmatrix} 0 & 2 \\ 2\,\mathsf{i} & 0 \end{pmatrix} & 0 \\ 0 & \mathsf{i} \begin{pmatrix} 0 & 2 \\ 2\,\mathsf{i} & 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2\,\mathsf{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,\mathsf{i} \\ 0 & 0 & -2 & 0 \end{pmatrix}. \tag{0.52}$$

Calculate the action of $C \otimes D$ on $|A\rangle \otimes |B\rangle$.
Answer:

$$(C \otimes D)\,(|A\rangle \otimes |B\rangle) = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2\,\mathsf{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,\mathsf{i} \\ 0 & 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} 2\,\mathsf{i} \\ 6 \\ -1 \\ 3\,\mathsf{i} \end{pmatrix} = \begin{pmatrix} 12 \\ -4 \\ -6 \\ 2 \end{pmatrix}. \tag{0.53}$$

16. Consider the composite state of two qubits

$$|A\rangle = \frac{1}{\sqrt{6}} \left(|00\rangle + \mathsf{i}\, |01\rangle + 2\, |10\rangle \right). \tag{0.54}$$

Is it entangled?

Answer: $\alpha_{00}\alpha_{11} - \alpha_{01}\alpha_{10} = 1 \cdot 0 - \mathsf{i} \cdot 2 = -2\,\mathsf{i} \neq 0$, so it is entangled. Or instead of calculating, we can just notice by inspection that there is a correlation between the qubits, since 1 for first qubit implies that we **must** have 0 for second qubit.

Consider the composite state of two qubits

$$|B\rangle = \frac{1}{\sqrt{6}} \left(\mathsf{i}\, |00\rangle + 2\, |01\rangle - 2\,\mathsf{i}\, |10\rangle + 4\, |11\rangle \right). \tag{0.55}$$

Is it entangled?

Answer: $\alpha_{00}\alpha_{11} - \alpha_{01}\alpha_{10} = \mathsf{i} \cdot 4 - 2 \cdot (-2\,\mathsf{i}) = 4\,\mathsf{i} + 4\,\mathsf{i} = 8\,\mathsf{i} \neq 0$, so it is entangled. Here it is not immediately obvious that the qubits are correlated, but in fact they are!

Consider the composite state of two qubits

$$|C\rangle = \frac{1}{5} \left(\mathsf{i}\, |00\rangle - 2\, |01\rangle - 2\,\mathsf{i}\, |10\rangle + 4\, |11\rangle \right). \tag{0.56}$$

Is it entangled?

Answer: $\alpha_{00}\alpha_{11} - \alpha_{01}\alpha_{10} = \mathsf{i} \cdot 4 - (-2) \cdot (-2\,\mathsf{i}) = 4\,\mathsf{i} - 4\,\mathsf{i} = 0$, so it is in fact **not** entangled.

Write this state as a tensor product.

Answer:

$$\begin{aligned}
|C\rangle &= \frac{1}{5} \Big(\mathsf{i}\, |0\rangle \otimes |0\rangle - 2\, |0\rangle \otimes |1\rangle - 2\,\mathsf{i}\, |1\rangle \otimes |0\rangle + 4\, |1\rangle \otimes |1\rangle \Big) \\
&= \frac{1}{5} \Big(|0\rangle \otimes \Big(\mathsf{i}\, |0\rangle - 2\, |1\rangle \Big) - 2\, |1\rangle \otimes \Big(\mathsf{i}\, |0\rangle - 2\, |1\rangle \Big) \Big) \\
&= \frac{1}{\sqrt{5}} \left(|0\rangle - 2\, |1\rangle \right) \otimes \frac{1}{\sqrt{5}} \left(\mathsf{i}\, |0\rangle - 2\, |1\rangle \right).
\end{aligned}$$

17. The system was in the state

$$|A\rangle = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ 3\,\mathsf{i} \end{pmatrix}. \tag{0.57}$$

Now it's in the state

$$U\, |A\rangle = \frac{1}{\sqrt{13}} \begin{pmatrix} 3\,\mathsf{i} \\ 2\,\mathsf{i} \end{pmatrix}. \tag{0.58}$$

What was the unitary operator responsible for the evolution?

Answer: The components were flipped and a phase of i was added to the lower component, so

$$U = \begin{pmatrix} 0 & 1 \\ \mathsf{i} & 0 \end{pmatrix}. \tag{0.59}$$

What will be the state after the same amount of time has passed?
Answer:

$$U^2 |A\rangle = \frac{1}{\sqrt{13}} \begin{pmatrix} 2\,\mathsf{i} \\ -3 \end{pmatrix}. \tag{0.60}$$

18. A qubit is given by

$$|A\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ \mathsf{i} \end{pmatrix}. \tag{0.61}$$

What is the action of the NOT gate on this qubit?
Answer: The NOT gate just flips the components, so

$$\mathsf{NOT}\,|A\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} \mathsf{i} \\ 2 \end{pmatrix}. \tag{0.62}$$

19. The composite state of two qubits is given by

$$|A\rangle = \frac{1}{3} \left(|00\rangle + 2\,|01\rangle - 2\,\mathsf{i}\,|11\rangle \right). \tag{0.63}$$

What is the probability to measure 0 for the **first** qubit?
Answer:

$$\left|\frac{1}{3}\right|^2 + \left|\frac{2}{3}\right|^2 = \frac{5}{9}. \tag{0.64}$$

What will be the state of the two qubits after the measurement?
Answer:

$$|A\rangle = \frac{1}{\sqrt{5}} \left(|00\rangle + 2\,|01\rangle \right). \tag{0.65}$$

What is the probability to measure 1 for the **first** qubit?
Answer:

$$\left|\frac{-2\,\mathsf{i}}{3}\right|^2 = \frac{4}{9}, \qquad \text{or just: } 1 - \frac{5}{9} = \frac{4}{9}. \tag{0.66}$$

What will be the state of the two qubits after the measurement?
Answer:

$$|A\rangle = |11\rangle\,. \tag{0.67}$$

What is the probability to measure 0 for the **second** qubit?
Answer:

$$\left|\frac{1}{3}\right|^2 = \frac{1}{9}. \tag{0.68}$$

What will be the state of the two qubits after the measurement?
Answer:

$$|A\rangle = |00\rangle\,. \tag{0.69}$$

What is the probability to measure 1 for the **second** qubit?
Answer:

$$\left|\frac{2}{3}\right|^2 + \left|\frac{-2\,\mathsf{i}}{3}\right|^2 = \frac{8}{9}, \qquad \text{or just: } 1 - \frac{1}{9} = \frac{8}{9}. \tag{0.70}$$

What will be the state of the two qubits after the measurement?
Answer:

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - \mathsf{i}\,|11\rangle\right). \tag{0.71}$$