ASTR 1P02

Brock University
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Lecture 16: Einstein’s theory of relativity

We will learn about…

- Einstein's theories of special and general relativity.
- Consequences of special relativity, such as time dilation and length contraction.
- Consequences of general relativity, such as gravitational time dilation, gravitational lensing, and gravitational waves.
- Experimental tests of the theories.
- The possibility of "sci-fi" concepts such as faster-than-light travel and time travel.

In the background: A photo from the Hubble Space Telescope. A bit to the right of the center, we see an example of an Einstein ring – light warped around a galaxy due to gravitational lensing.
Credits: ESA/Hubble & NASA

Special relativity

- Isaac Newton published his theory of Newtonian mechanics in 1687. It was considered correct for more than 200 years.
  - We learned about Newtonian mechanics in lecture 6.
- In 1865, James Clerk Maxwell published Maxwell’s equations, which form the foundation of the theory of electromagnetism.
- Maxwell combined previous ideas by many other physicists into one set of equations, describing electricity, magnetism, and the relation between them.

Special relativity

- As we previously learned, light is an electromagnetic wave.
- Using Maxwell's equation, it is possible to derive the speed of an electromagnetic wave, and therefore the speed of light.
- It turns out that light always moves in vacuum at a constant speed of $\sim$300,000,000 m/s.
  - When passing through a material medium like glass, light can move slower, but that won't be relevant in this lecture.
- When we say "constant speed", we mean that light always moves at this speed in vacuum, for all observers.

Special relativity

- But according to Newtonian mechanics, velocity is always relative to the observer.
- Each observer has its own frame of reference.
- If two observers are at rest relative to each other, then they are in the same frame. The observers will agree on the speed of an object.
- If two observers are moving relative to each other, then they are in different frames. Each observer will see the same object move at a different speed.

Special relativity

- Consider for example Alice and Bob, who are moving at 10 m/s relative to each other. A star ★ will indicate which frame we're in.
- In Alice's frame, she is at rest, and Bob is moving away from her at a speed of 10 m/s.

Special relativity

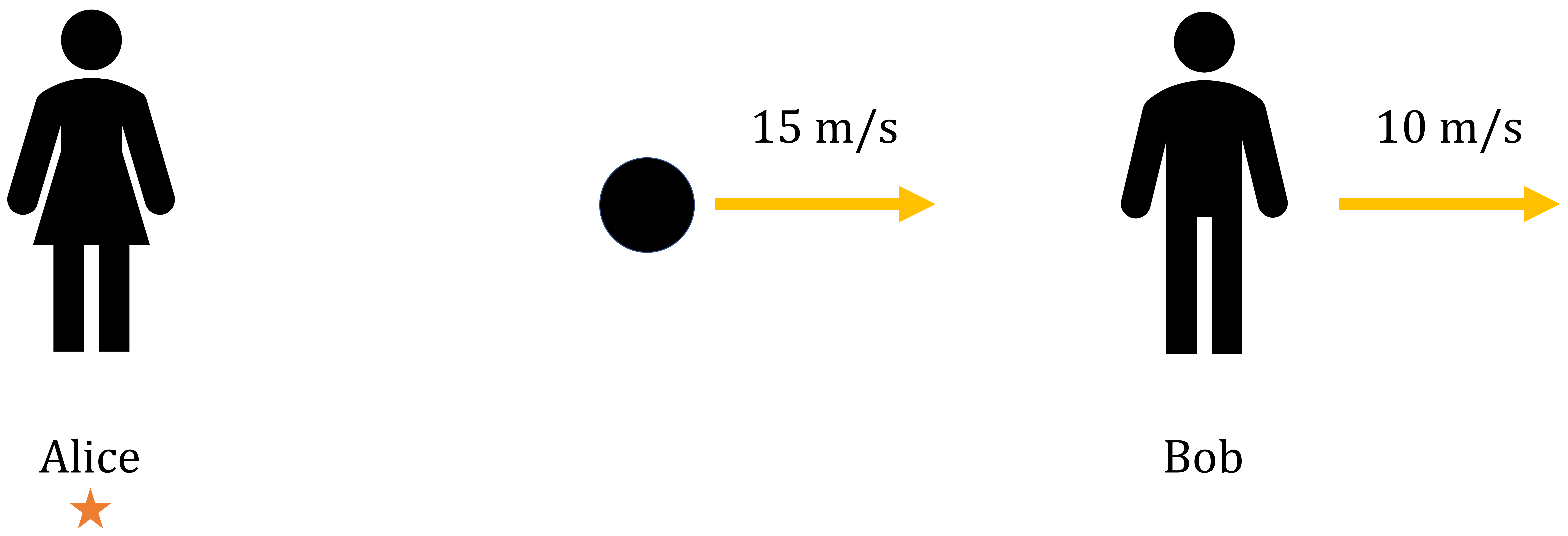
- But in Bob’s frame, he is at rest, and Alice is moving at 10 m/s away from him.

Special relativity

- Both Alice and Bob are correct! Speed is relative, so different observers see different speeds.
- There is no such thing as an “absolute speed”, only speed relative to a specific observer.

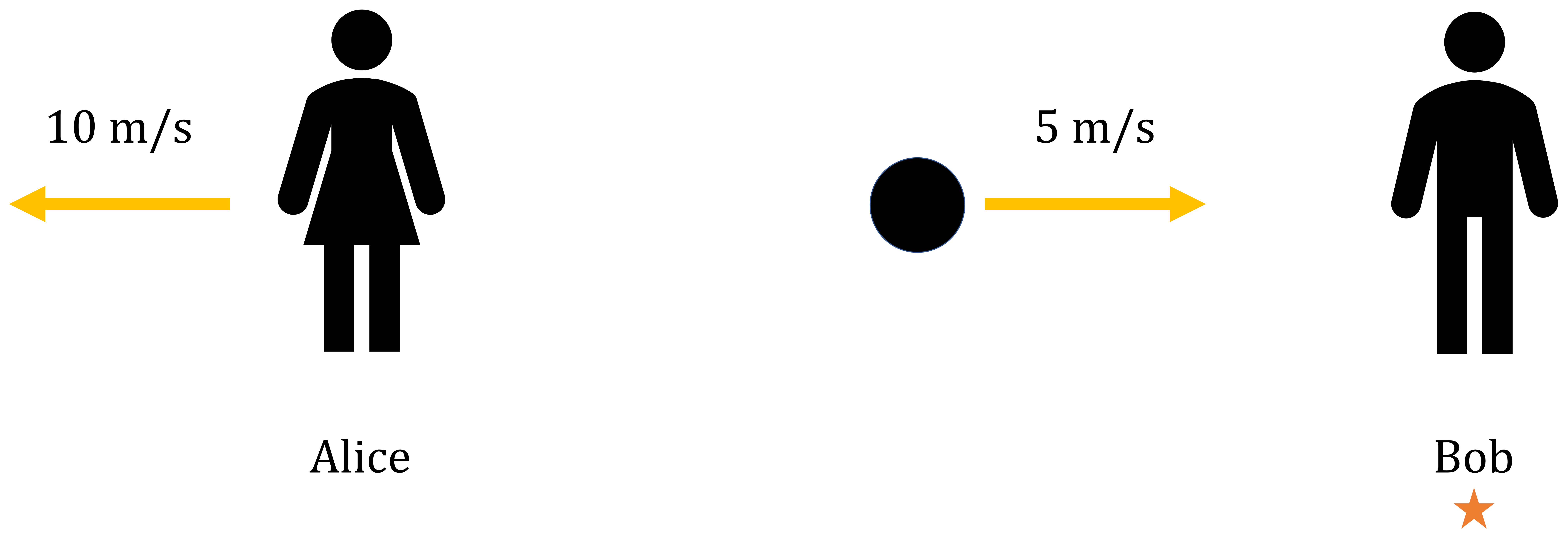
Special relativity

- Now consider what happens when Alice throws a ball toward Bob at 15 m/s.
- In Alice's frame, the ball moves at 15 m/s, because she threw the ball at that speed in her frame.



Special relativity

- In Bob's frame, all the velocities are shifted by 10 m/s to the left.
- $15 - 10 = 5$, so Bob sees the ball move at 5 m/s.

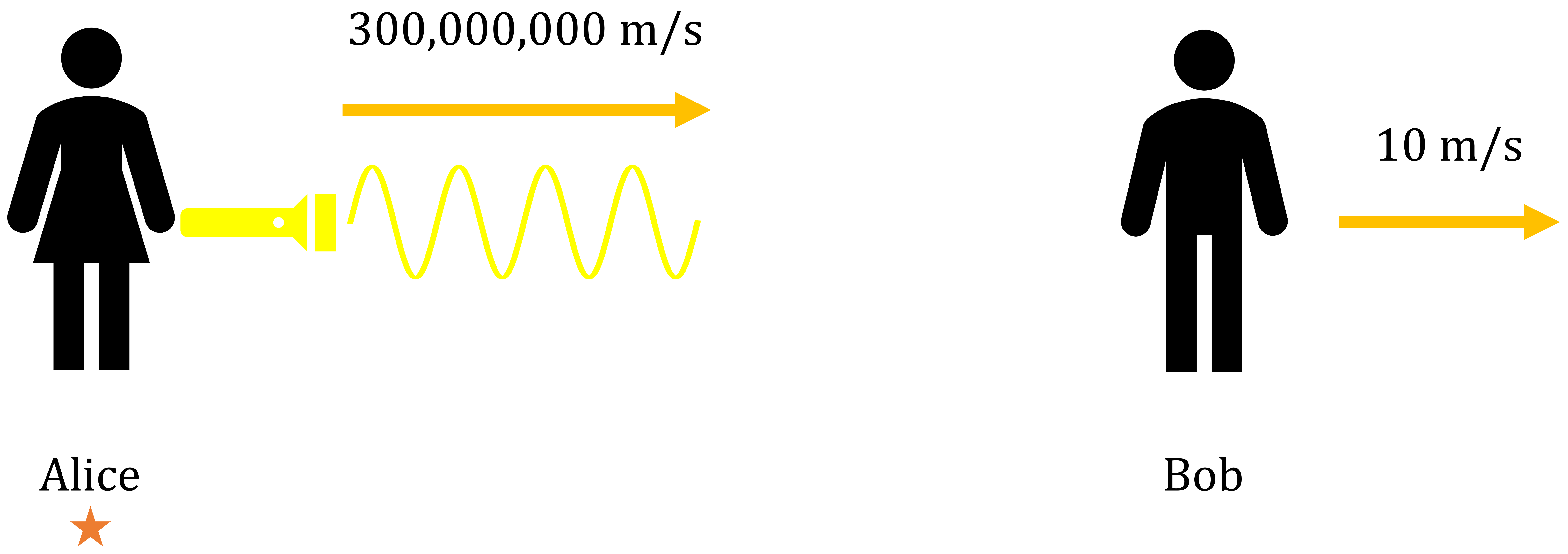


Special relativity

- Again, both Alice and Bob are correct.
- The ball can't have a "constant speed" that is the same for every observer.
- The ball's speed is 15 m/s relative to Alice and 5 m/s relative to Bob.
- But now, consider what happens when Alice shines a flashlight toward Bob.
    - Note: the speed of light is exactly 299,792,458 m/s, but for simplicity we will use the round value of 300,000,000 m/s in this example.

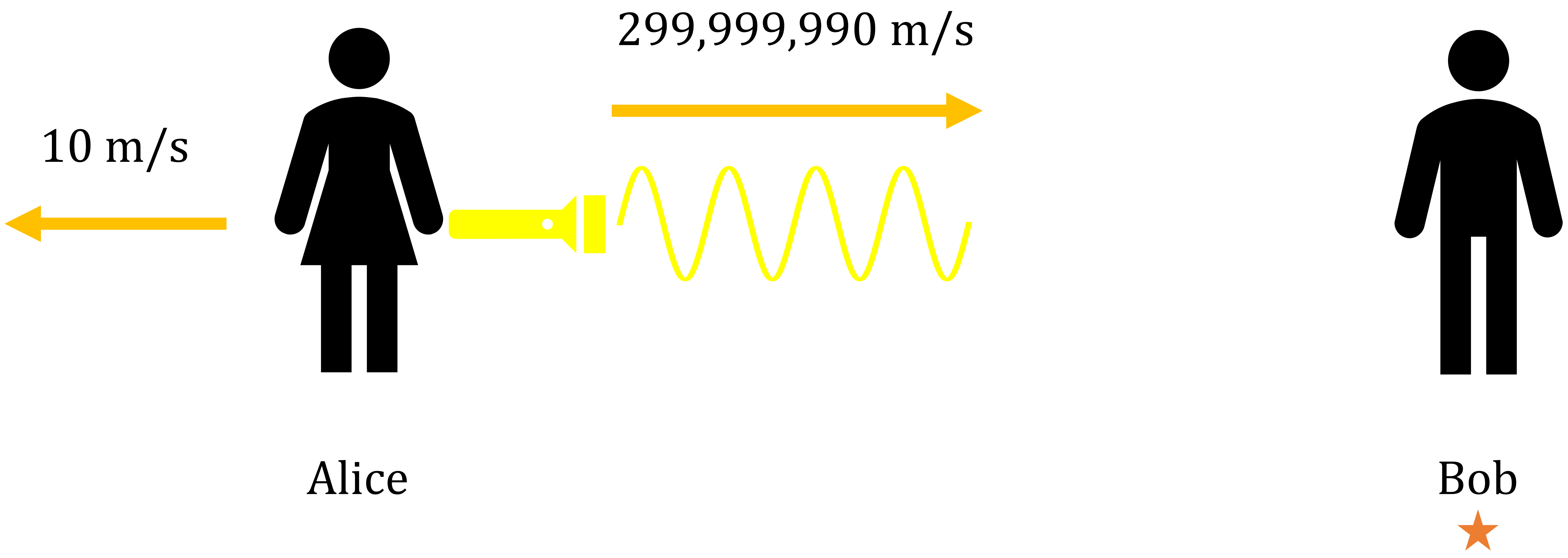
Special relativity

- In Alice's frame, the light moves at the speed of light, which is 300,000,000 m/s.



Special relativity

- But according to Newtonian mechanics, in Bob's frame, the light should move 10 m/s slower, at 299,999,990 m/s.



Special relativity

- This contradicts Maxwell's theory of electromagnetism, which says that light must travel at a constant speed of 300,000,000 m/s for all observers!
- Therefore, Newtonian mechanics is incompatible with electromagnetism.

Special relativity

- Albert Einstein published his special theory of relativity in 1905.
- Einstein's theory allows velocities to be relative, while also keeping the speed of light constant for all observers.
- This seems contradictory, but it is in fact how the universe works!
- As we will see later, this apparent contradiction leads to some weird consequences.

Special relativity

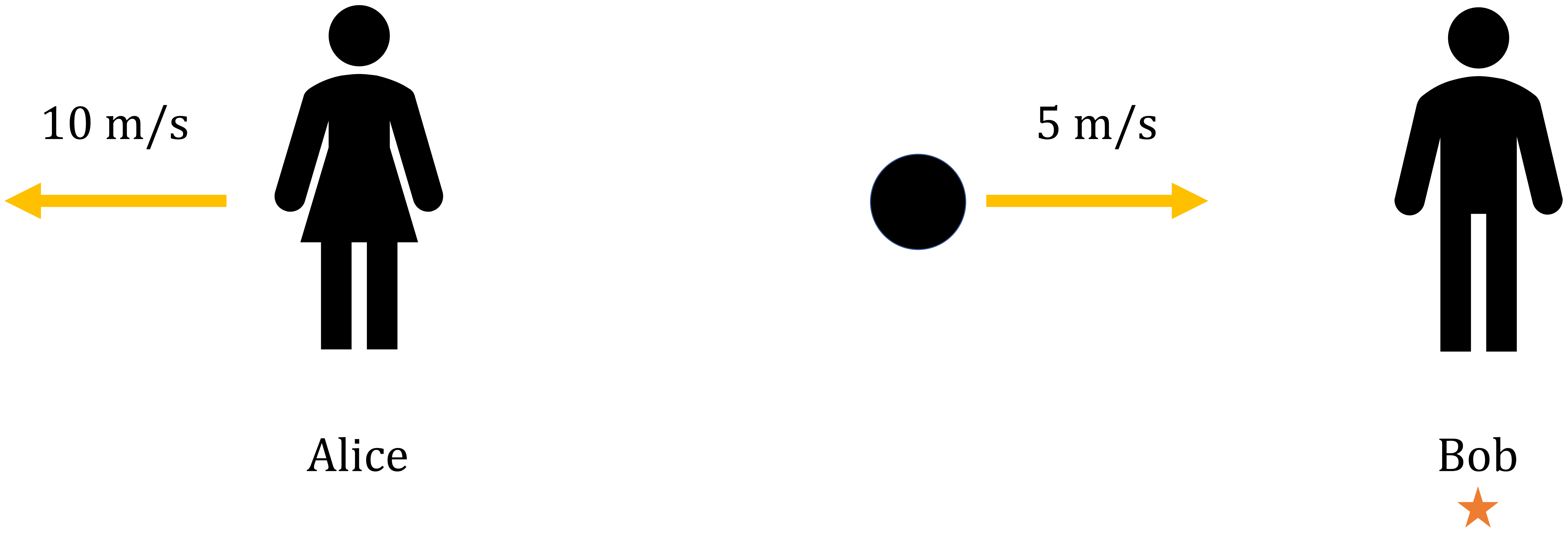
- As long as things are moving slow relative to the speed of light, relativity reproduces all the predictions of Newtonian mechanics.
- This is an example of an extremely important principle in science: new theories must always reproduce the predictions of older theories in cases where the older theories are valid.
- Newtonian mechanics was accepted for more than 200 years. Its predictions were verified in countless experiments, and it had many applications in science and engineering.
- So relativity must be consistent with Newtonian mechanics in all of these cases.

Special relativity

- We saw another example of this principle when we learned about Kepler's laws.
- Newtonian mechanics introduced new laws of motion that should apply to all objects in the universe.
- But Kepler's laws were already consistent with observations of the planets in the solar system.
- Therefore, Newtonian mechanics had to reproduce the predictions of Kepler's laws, which is indeed what happened.
- Newtonian mechanics generalizes (or expands) Kepler's laws, and similarly, relativity generalizes Newtonian mechanics.

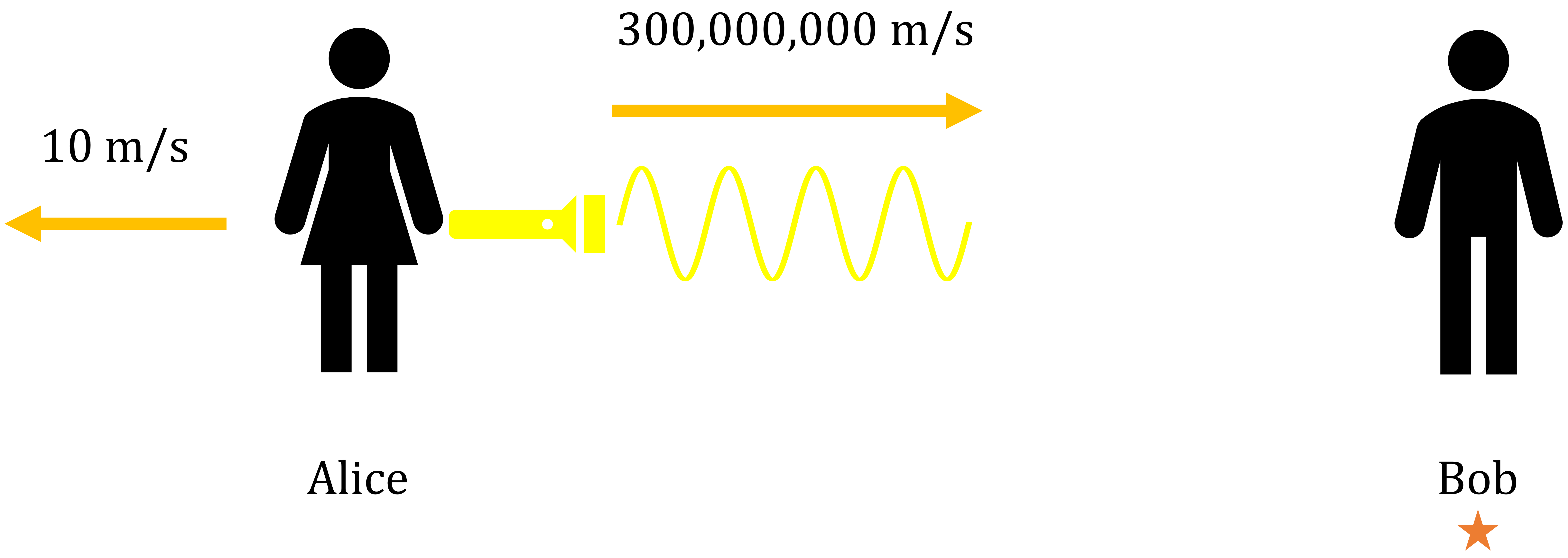
Special relativity

- When Alice throws a ball at 15 m/s toward Bob, relativity predicts that Bob will see the ball move at 5 m/s.
- This prediction is consistent with that of Newtonian mechanics, as we saw earlier.



Special relativity

- However, relativity also predicts that both Alice and Bob will see the light beam move at 300,000,000 m/s.



Special relativity

- In other words, relativity is consistent with Newtonian mechanics at speeds much slower than light, as in the case of the ball.
- At the same time, relativity is also consistent with Maxwell’s theory of electromagnetism at the speed of light.
- So relativity unifies Newtonian mechanics and Maxwell’s theory together into one theory that works at any speed.

Special relativity

- When speeds are slow compared to light, which is the case for most speeds we experience in daily life, Newtonian mechanics is correct, and we can use it.
- But when speeds are comparable to light, including light itself, Newtonian mechanics is no longer correct, and we must use relativity.

Special relativity

- We could in principle use relativity for slow speeds too, but we usually don't, because the math is more complicated, and the differences are negligible.
- For example, in the case of Alice throwing the ball, relativity predicts that Bob will see it move at 5.000000000000008 m/s.
- The difference between this and the Newtonian prediction of 5 m/s is 8×10^{-15} m/s, roughly the size of a single atomic nucleus per second.
- This difference is so small it cannot even be measured, so we might as well just use Newtonian mechanics.

Spacetime

- Special relativity is based on two fundamental postulates.
- The first postulate is the principle of relativity: the laws of physics are the same in all inertial frames of reference.
- Remember that an inertial frame is one that is not accelerating.
  - If a car is parked or moving at a constant speed, its frame is inertial.
  - But when the car accelerates (changes its speed), its frame is not inertial.
- The first postulate was already formulated by Galileo in slightly different terms 400 years earlier (see lecture 5).

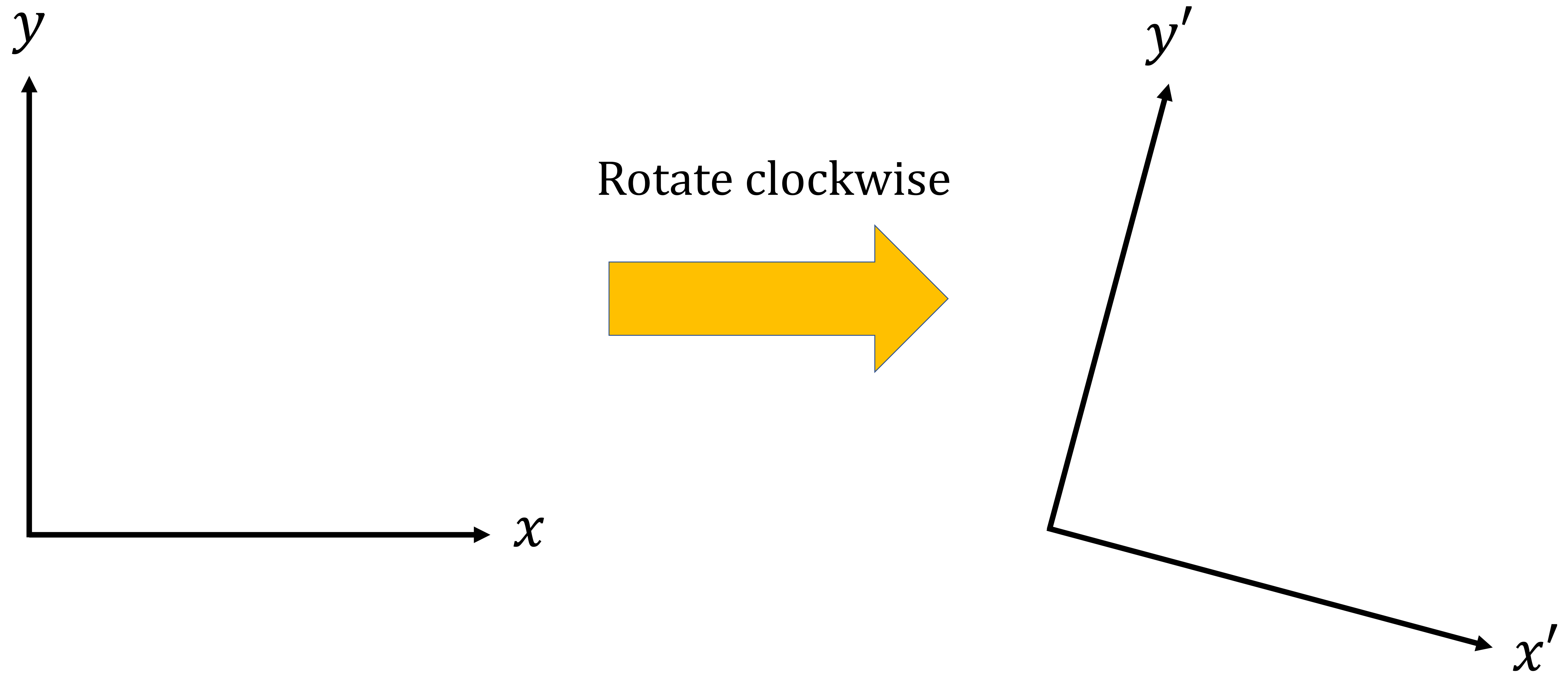
Spacetime

- The second postulate is the principle of invariant speed of light: the speed of light in vacuum is the same in all inertial frames, regardless of the motion of the light source.
- The second postulate is new, and it is the one responsible for all the weird consequences we will discuss in a bit.
- For this postulate to work, there must be a consistent way for Alice and Bob to both somehow see light move at the same speed.
- It turns out that this is only possible if space and time are combined into a single entity called spacetime.

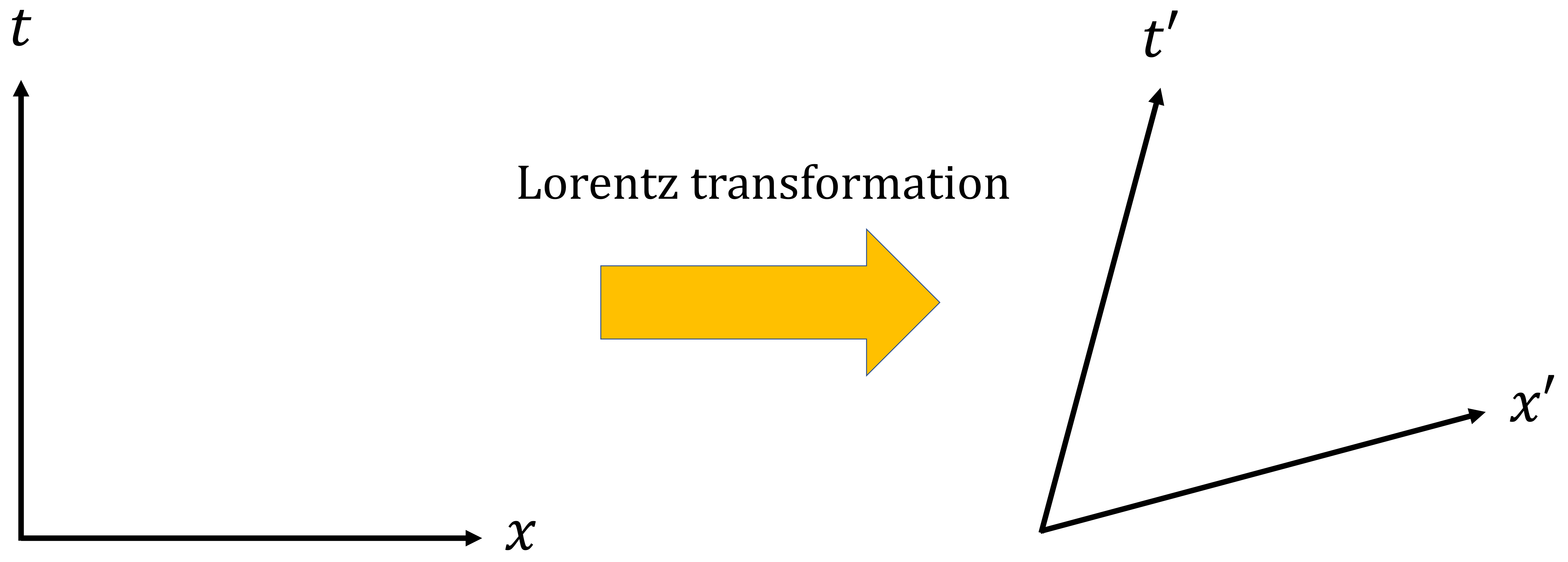
Spacetime

- There are 3 dimensions of space, and 1 dimension of time, so spacetime has a total of 4 dimensions.
- Combining space and time into spacetime is more than just adding dimensions, it also means that space and time can be mixed together.
- This mixing is a kind of “rotation” in spacetime, called a Lorentz transformation.
- The Lorentz transformation can be used to change frames of reference, and depends on the relative speed between the frames.

- Consider 2 space dimensions, with axes (or coordinates) x and y.
- When we rotate the axes, they rotate together in the same direction. We indicate the new axes x' and y'.



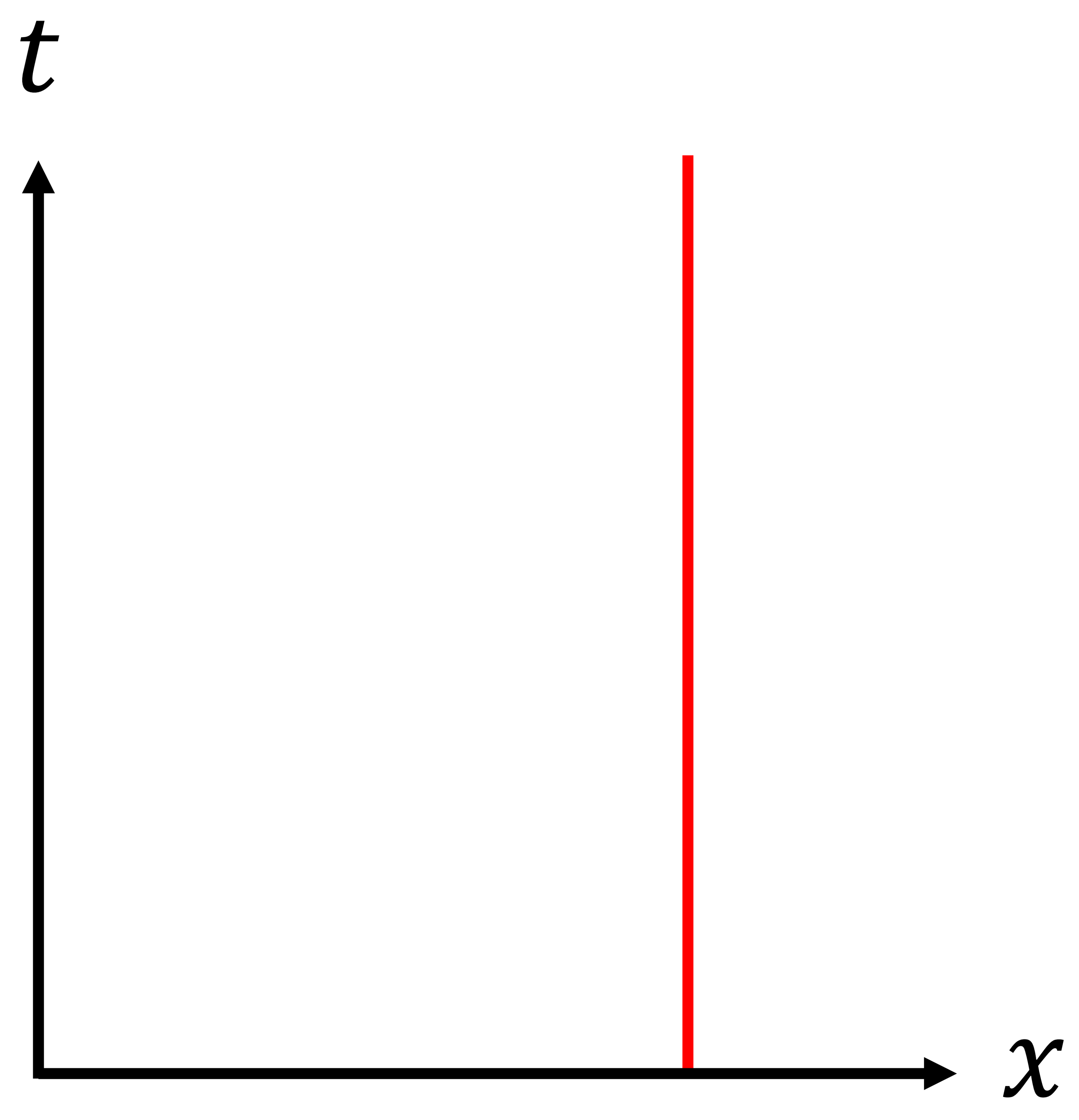
- Now consider a simplified spacetime with 1 space and 1 time dimension, with axes x (space) and t (time).
- When we apply a Lorentz transformation to the axes, they rotate in opposite directions! The new axes are x' and t'.



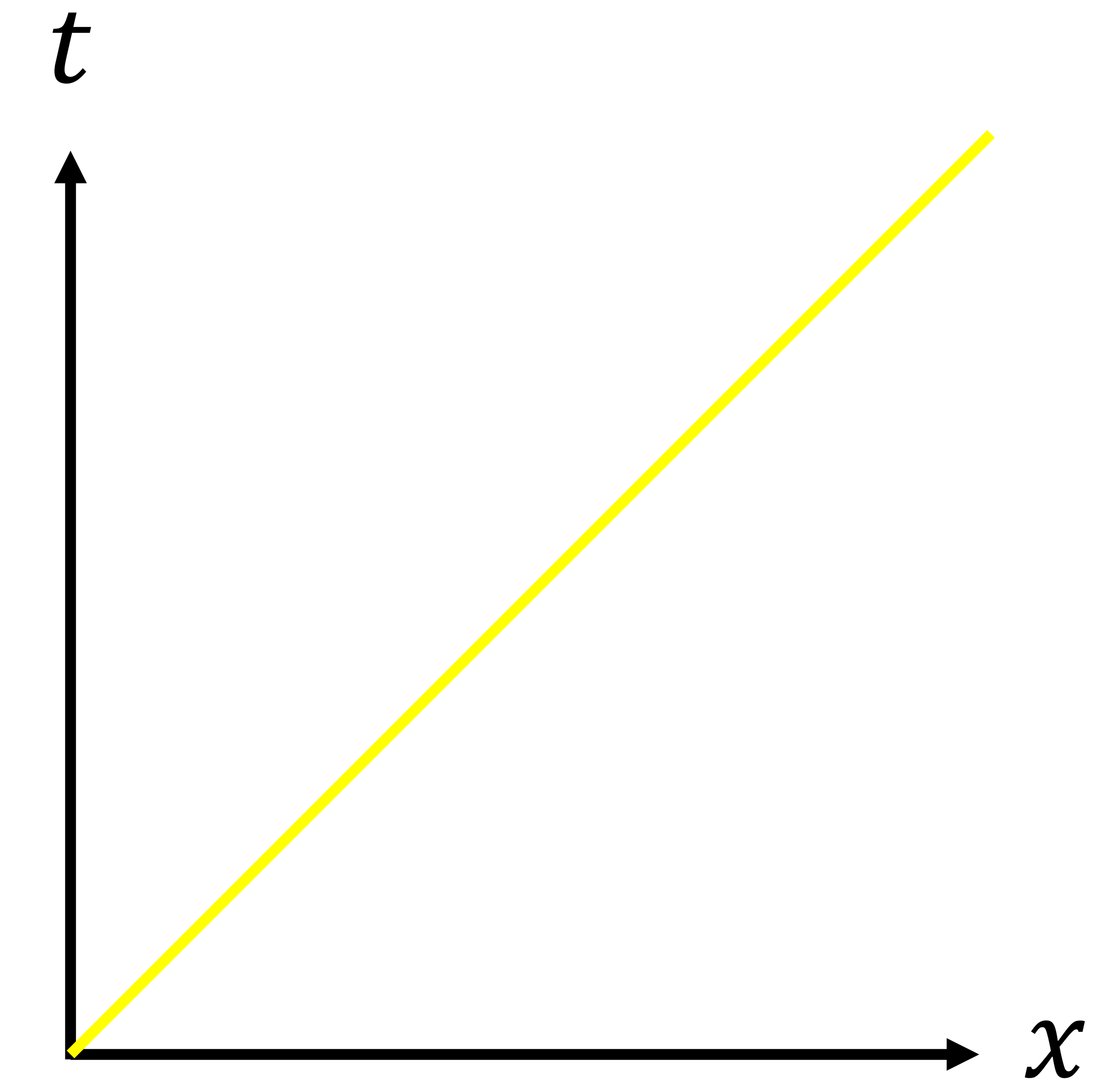
Spacetime

- To understand why this happens, we need to consider motion in spacetime.
- We will use units where time t is measured in seconds and distance x is measured in light-seconds.
  - A light-second is the distance light travels in 1 second, which is $\sim$300,000,000 meters.
- In these units, the speed of light is exactly 1 light-second per second.
- A diagram that shows the paths of objects in spacetime is called a spacetime diagram.

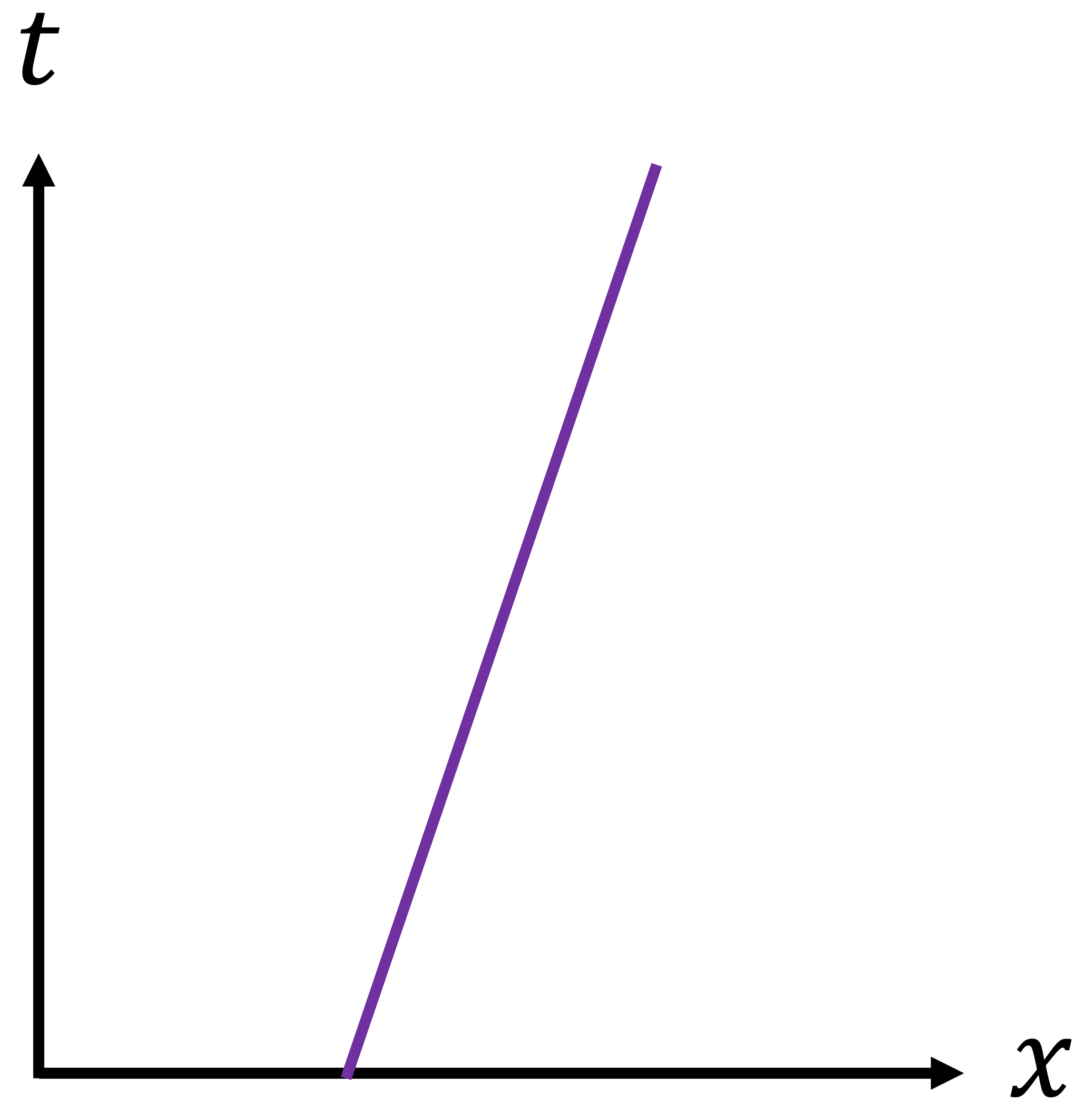
- An object at rest stays at the same position x in space at all times t. So its path in spacetime is just a vertical line.



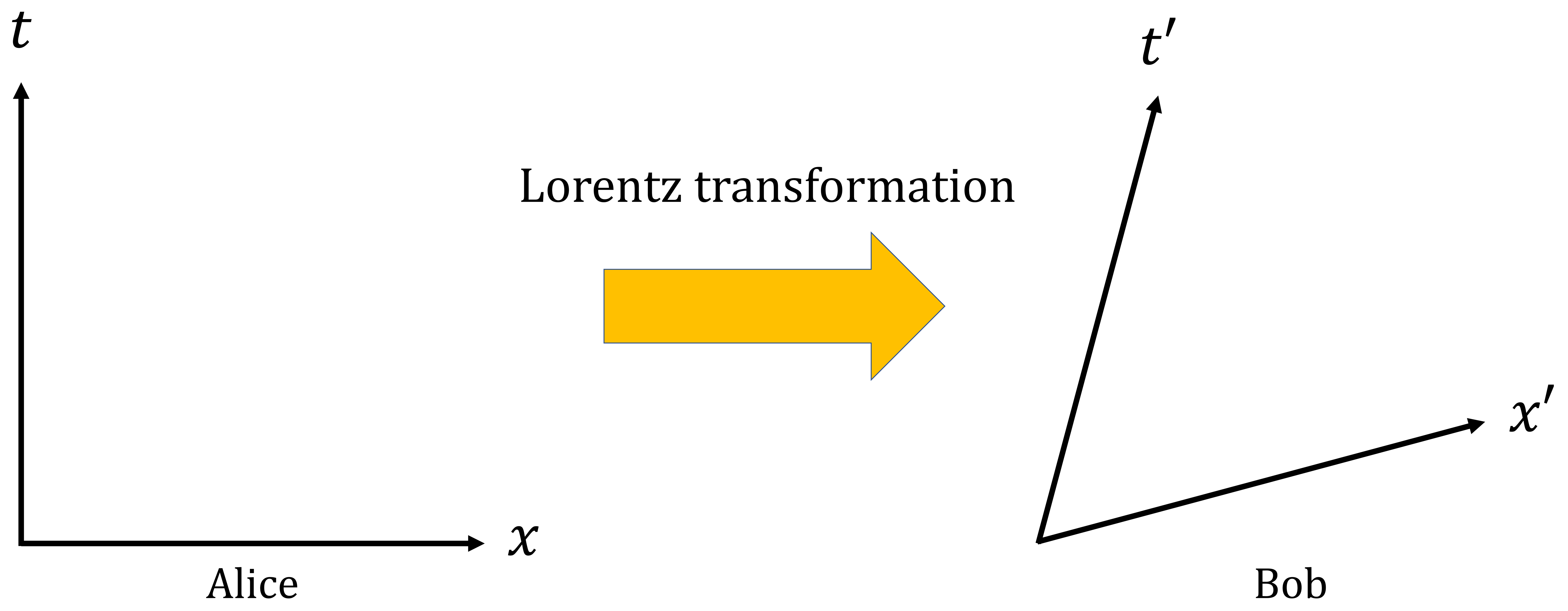
- Light moves at speed 1. This means that when t increases by 1 (so 1 second has passed), x also increases by 1 (so a distance of 1 light-second has been traveled).
- Therefore, light traces a diagonal path in spacetime. The path has a slope of 1, and is at an angle of exactly 45°.



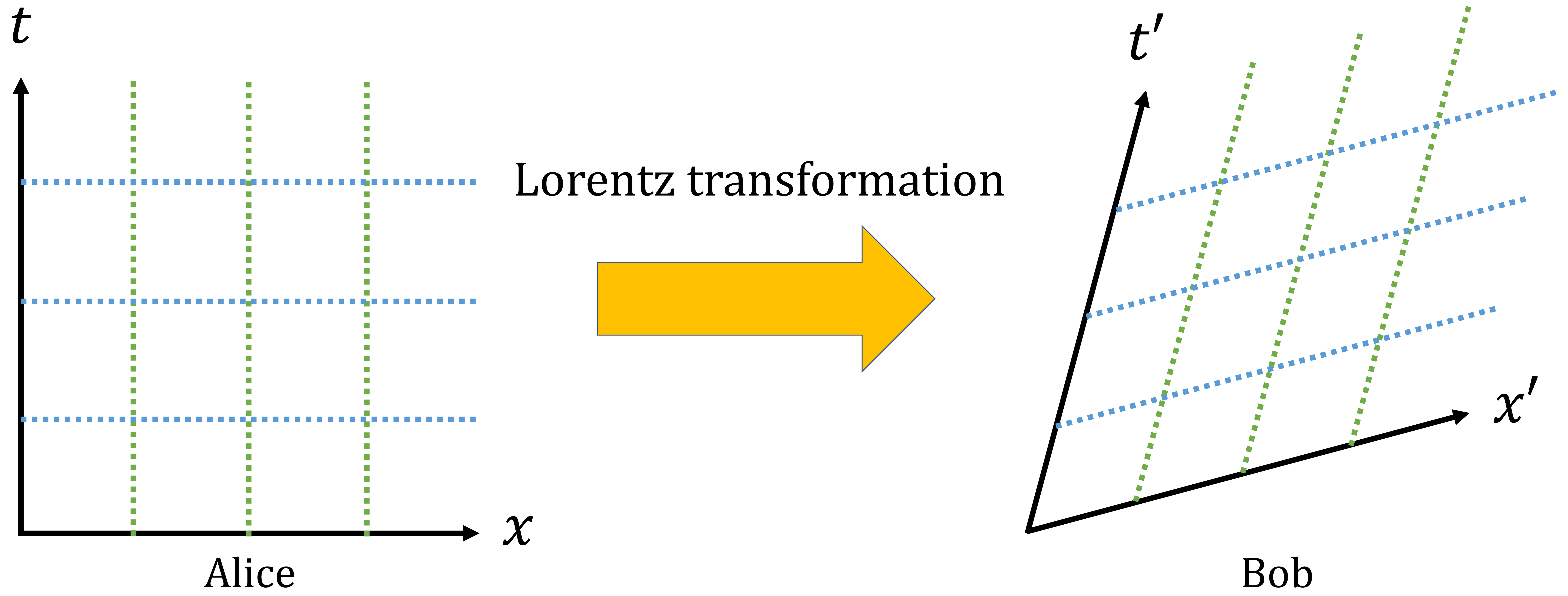
- An object moving slower then light (e.g. a ball) will be diagonal with a slope greater than 1.
- This means the object is moving at less than 1 light-second per second.



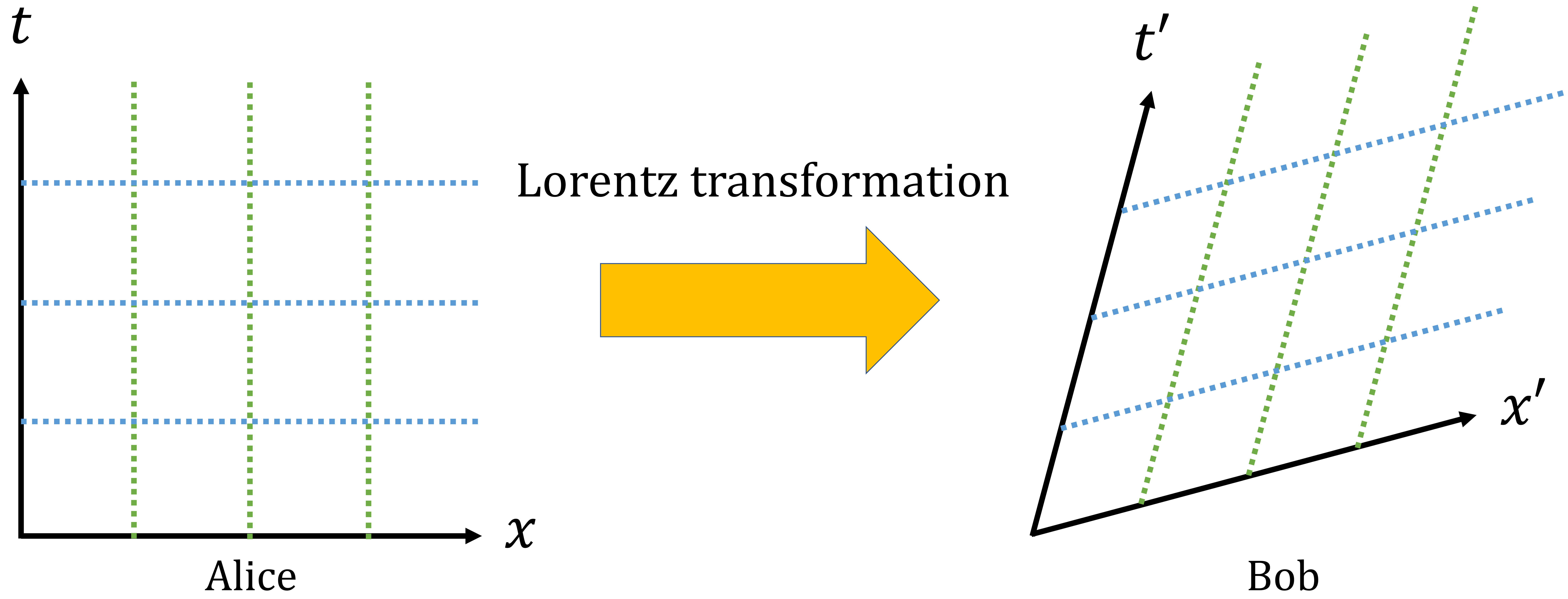
- Now let’s move from the previous frame (Alice) to the frame of an observer moving at a constant relative speed (Bob).
- We do this with a Lorentz transformation, which “rotates” the axes in opposite directions.
- The paths that objects make in spacetime don’t change; only the frame of reference changes.



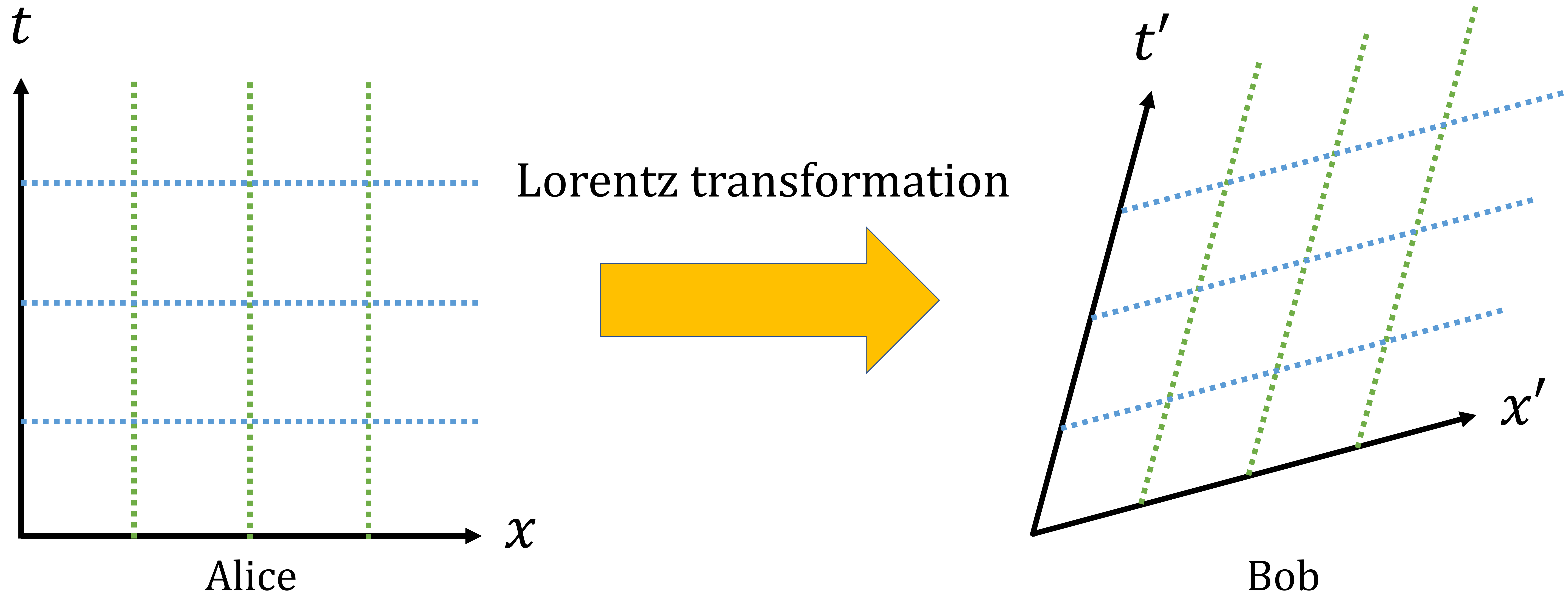
- Let's add a grid to understand what happens in this transformation.
- Note how the spacetime grid gets "squished" for Bob.
- This means that Bob has a different notion of space and time compared to Alice.



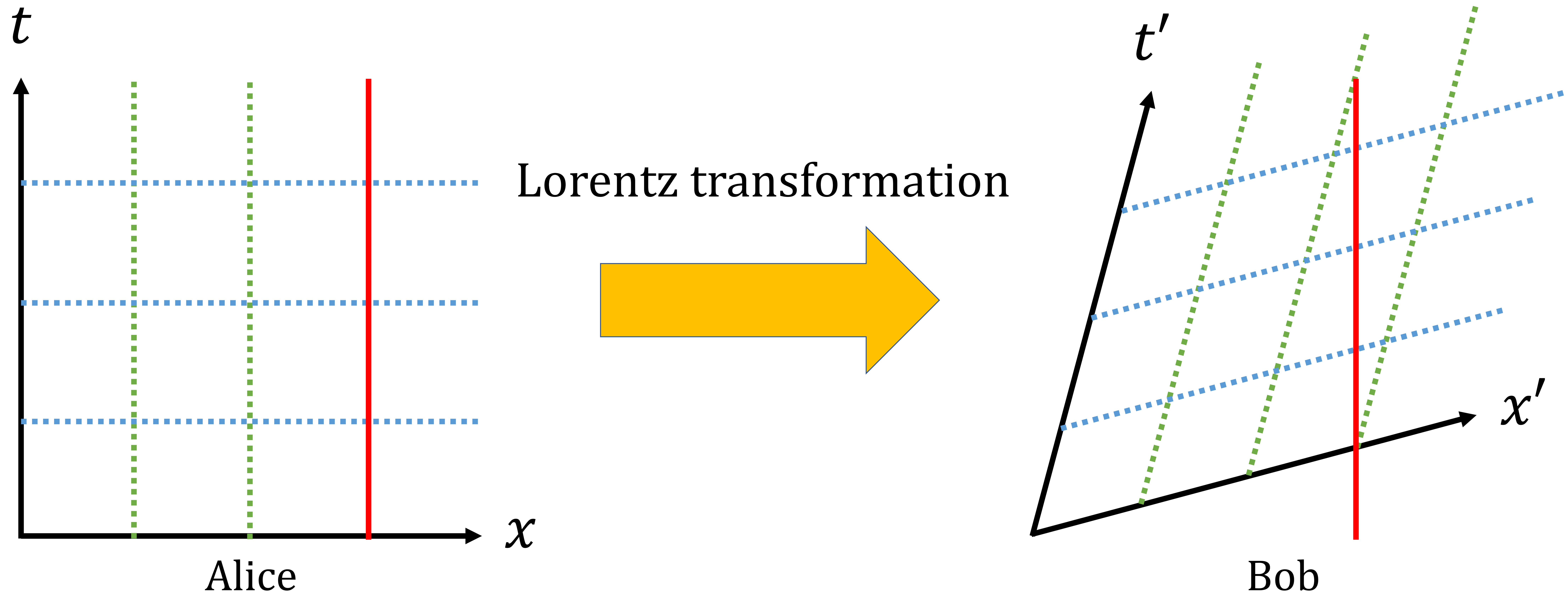
- The green lines represent constant values of x, i.e. fixed location.
- All the points along the same green line are considered to be in the same location in each frame.
- Alice and Bob each have a different notion of what it means to be in the "same location".



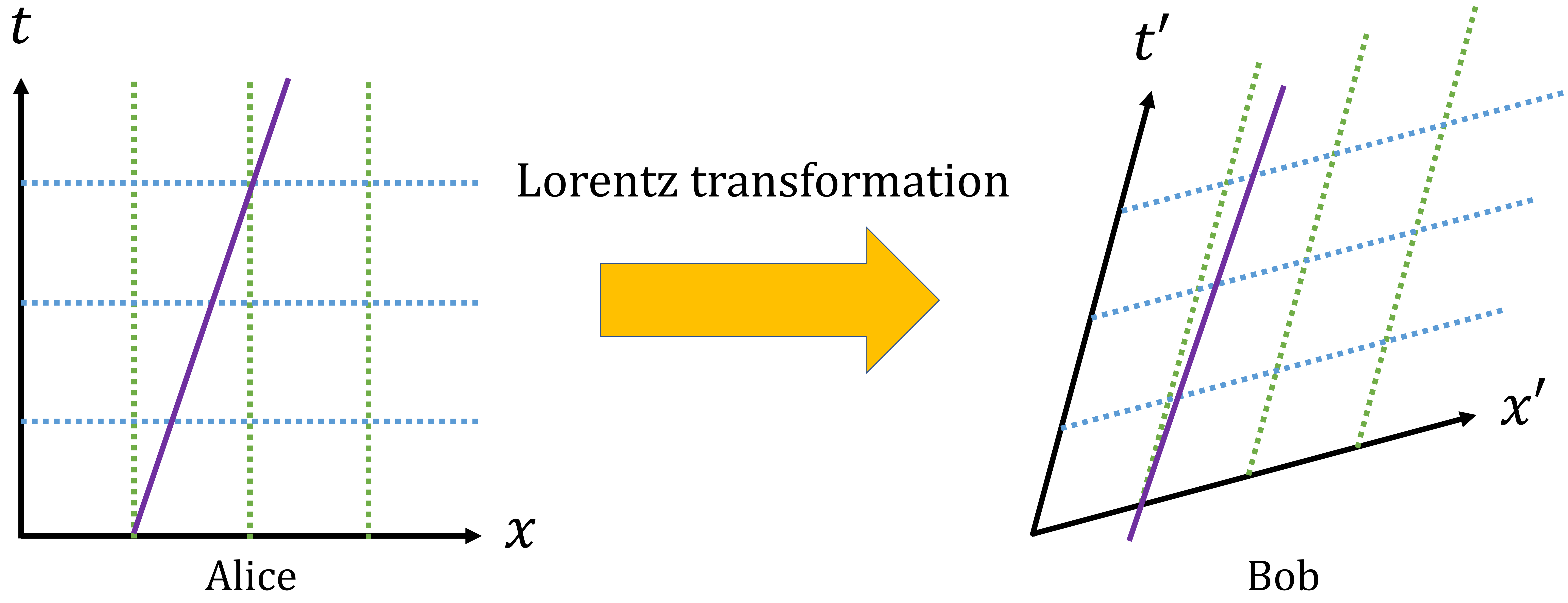
- The blue lines represent constant values of t, i.e. fixed times.
- All the points along the same blue line are considered to happen at the same time in each frame.
- Alice and Bob each have a different notion of what it means to happen at the "same time".



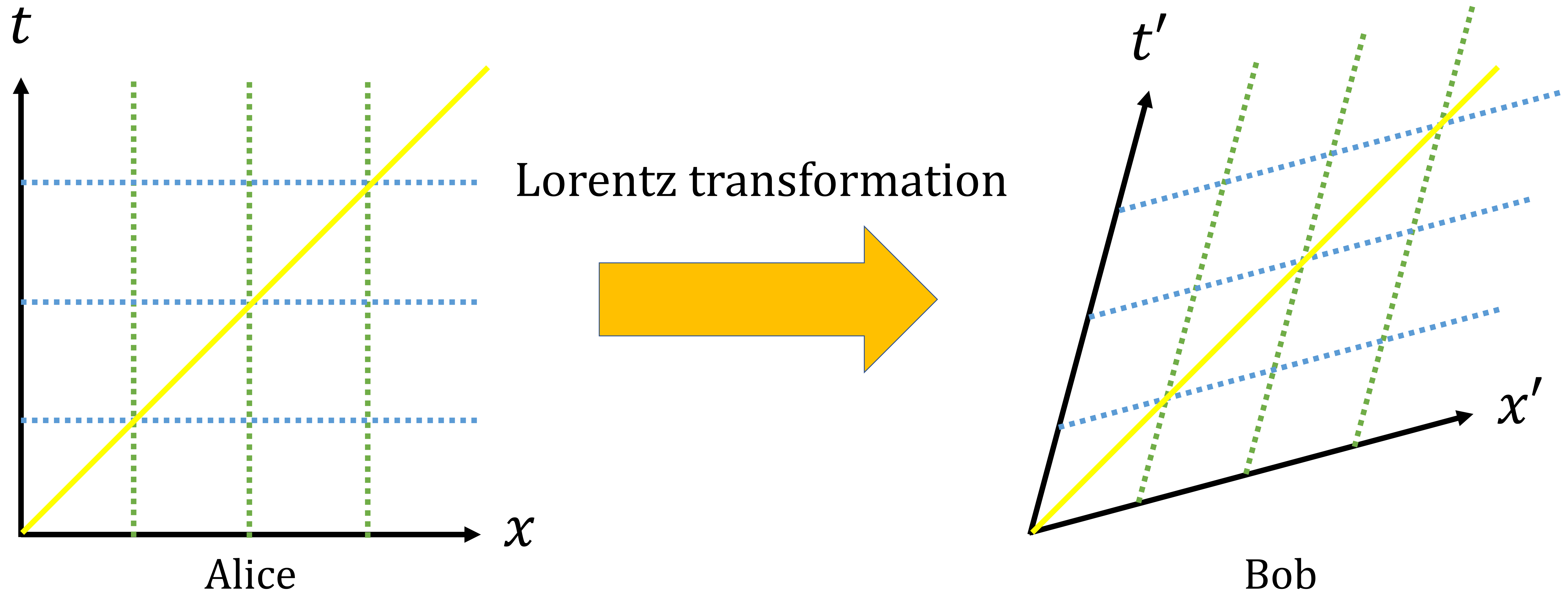
- The red line represents Alice. Alice sees herself at rest, but Bob sees her move to the left.
- This can be seen from the diagram. Note how the line stays at the same place in Alice’s frame, but moves between different green lines, i.e. different places, in Bob’s frame.



- The purple line represents the ball that Alice throws. From the diagram we see that the ball moves slower in Bob's frame compared to Alice's frame.
- In Alice's frame the ball moves 1 light-second in 3 seconds. In Bob's frame it moves a much shorter distance in the same time period.
  - Note: the speed of the ball in the diagram is 1/3 of the speed of light, which is much faster than 15 m/s, for illustration purposes.



- The yellow line represents light. It moves exactly 1 green line (1 light-second) for each blue line (1 second) in both diagrams.
- So both Alice and Bob see light moving at exactly the speed of light: 1 light-second per second.
- The Lorentz transformation resolves the contradiction between Newtonian mechanics and electromagnetism!



Consequences of special relativity

- To summarize: the inertial frames of two observers (e.g. Alice and Bob) who are moving at constant speed relative to each other are related by a Lorentz transformation.
- This transformation guarantees that the speed of light remains constant for both observers.
- But in the process, we find that each observer has a different notion of “space” and “time”.
- There is just one spacetime, but every observer sees this spacetime from a different perspective, and arranges their own personal space and time axes in different directions.

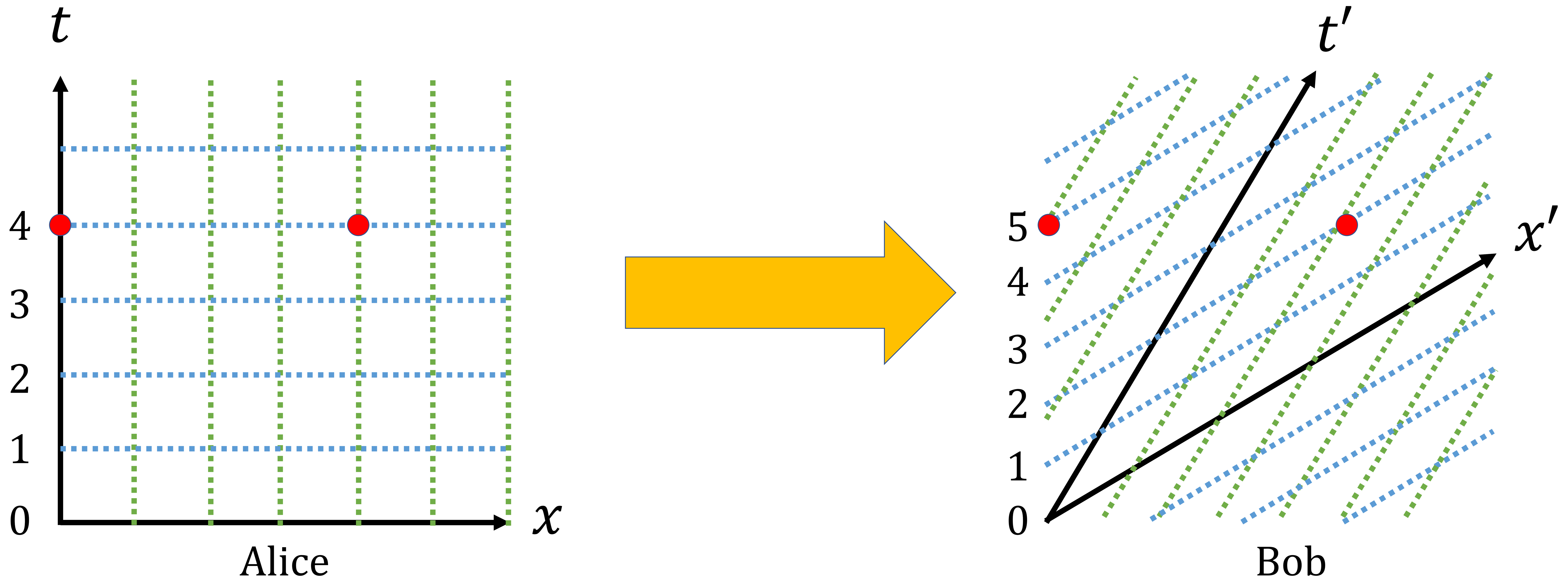
Consequences of special relativity

- This turns out to have some weird consequences, which we will now discuss.
- However, these weird consequences only become significant at relativistic speeds: when the relative speed between the two observers is close to the speed of light.
- This is why we don't notice them in our daily lives; we usually move at extremely slow speeds compared to light.
- For example, a car can move at $\sim$120 km/h ($\sim$33 m/s), but that's nothing compared to light, which moves at $\sim$1 billion km/h ($\sim$300,000,000 m/s)!

Consequences of special relativity

- The first consequence is relativity of simultaneity.
- Consider two events (for example, lights turning on) in two different places.
- It seems obvious to us that anyone who sees the events agrees on whether they happened at the same time (simultaneously) or not.
- However, when two observers are moving at relativistic speeds, that is not the case.

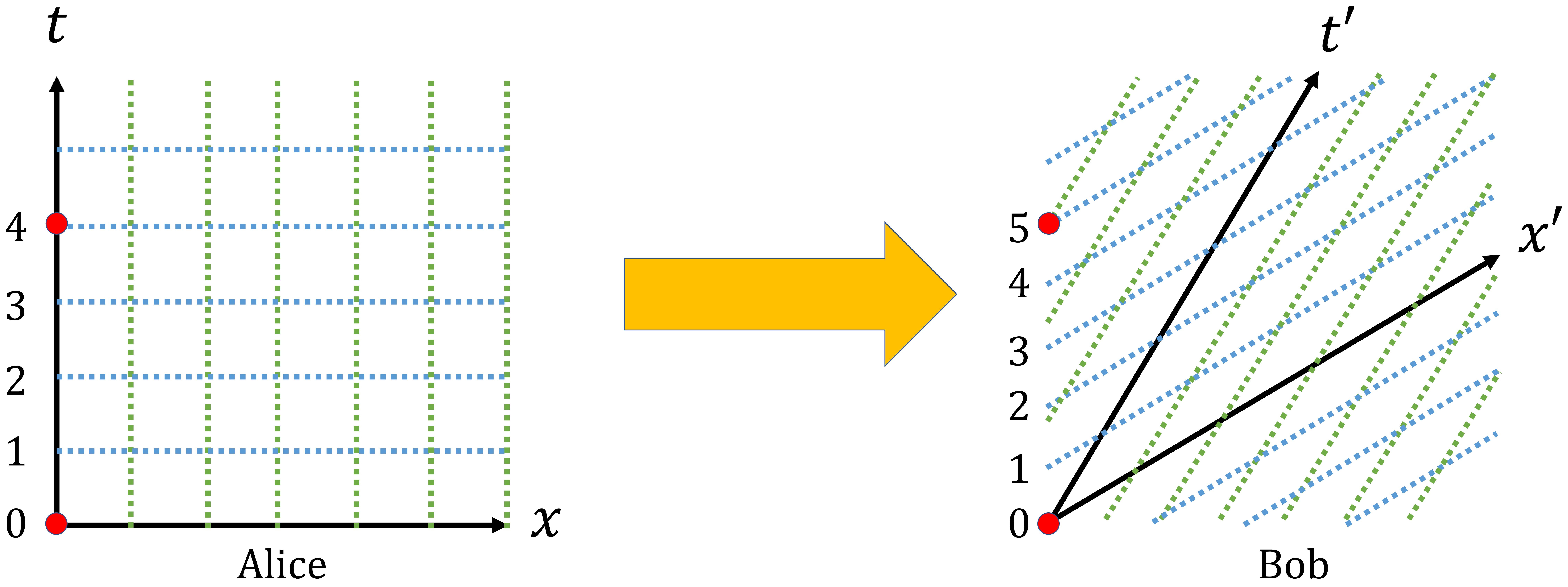
- The two events in red happen at the same time for Alice: $t = 4$ sec.
- However, for Bob, the event on the right happens first (at $t' = 2$), and the event on the left happens later (at $t = 5$)!
- Note that both observers see the same two events, in the same spacetime; only the perspective of each observer changed.



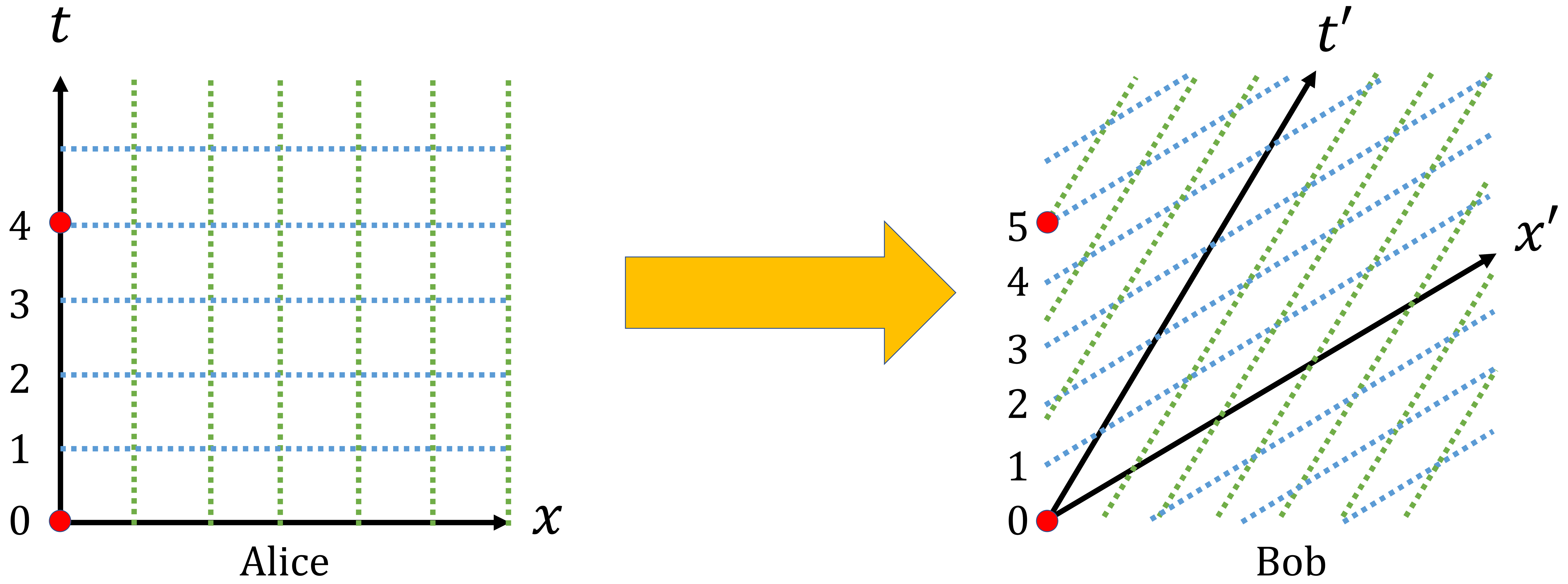
Consequences of special relativity

- The second consequence is time dilation.
- Consider two events that happen at two different times.
- Again, it seems obvious to us that anyone who sees the events agrees on how much time passed between them; but when two observers are moving at relativistic speeds, that is not the case.

- The two events in red happen 4 seconds apart for Alice. The first event happens at $t = 0$ and the second at $t = 4$.
- However, for Bob, the two events happen 5 seconds apart. The first event happens around $t = 0$ and the second at $t = 5$.
- In other words, Bob sees time dilated from 4 to 5 seconds.



- If both events are located at the same place, the time difference between them is called proper time. Proper time is always the shortest time between the events.
- Alice is measuring proper time, because both events are happening at the same place for her.
- Bob is not measuring proper time, because the events are happening at different places (different green lines) for him.



Consequences of special relativity

- The ratio of dilated to proper time is called the Lorentz factor and denoted by γ (the Greek letter gamma).
- In our example, Alice measures a proper time of 4 seconds and Bob measures a dilated time of 5 seconds, so the Lorentz factor is:

$$\gamma = 5/4 = 1.25$$

Consequences of special relativity

- The Lorentz factor increases as the relative speed increases.
- The precise relation between the Lorentz factor and the speed is:

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

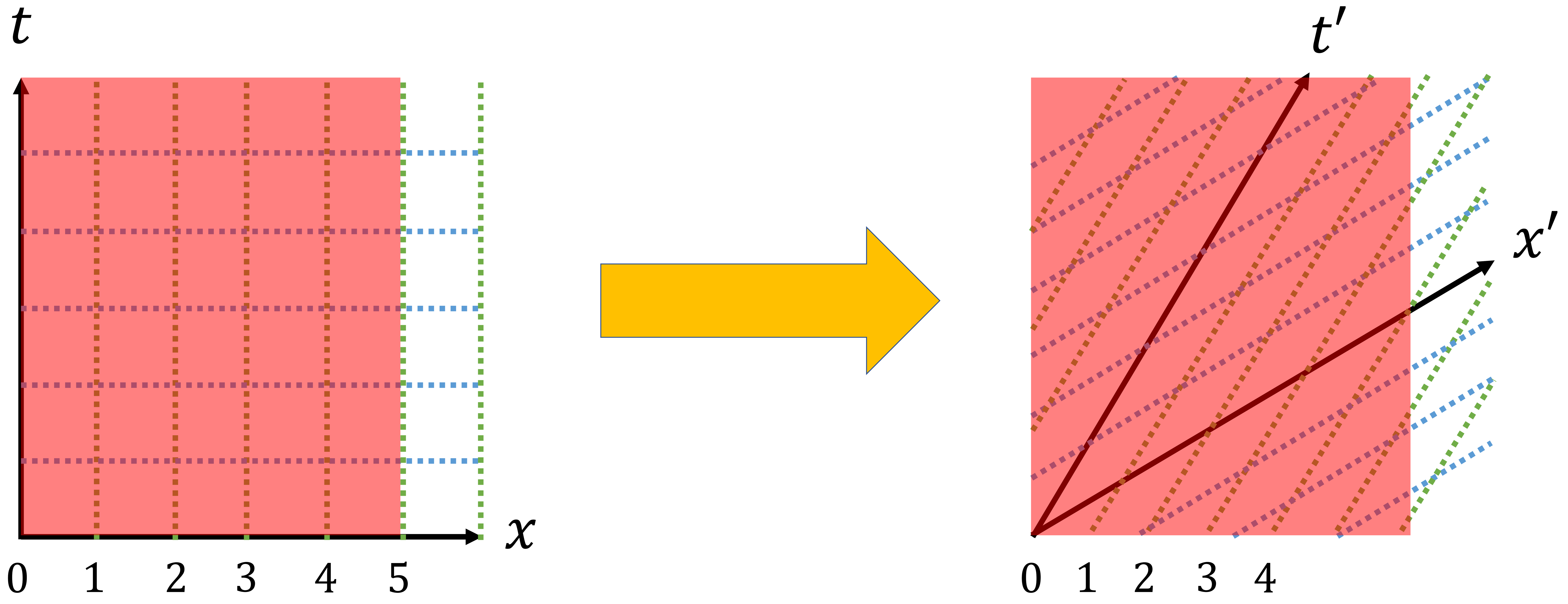
Where $\beta \equiv v/c$ is the ratio of the relative speed between the observers to the speed of light.

- In our example, $\gamma = 1.25$, which corresponds to $\beta = 0.6$, so the relative speed is 60% of the speed of light.
  - You won't need to know this precise relation for the test.

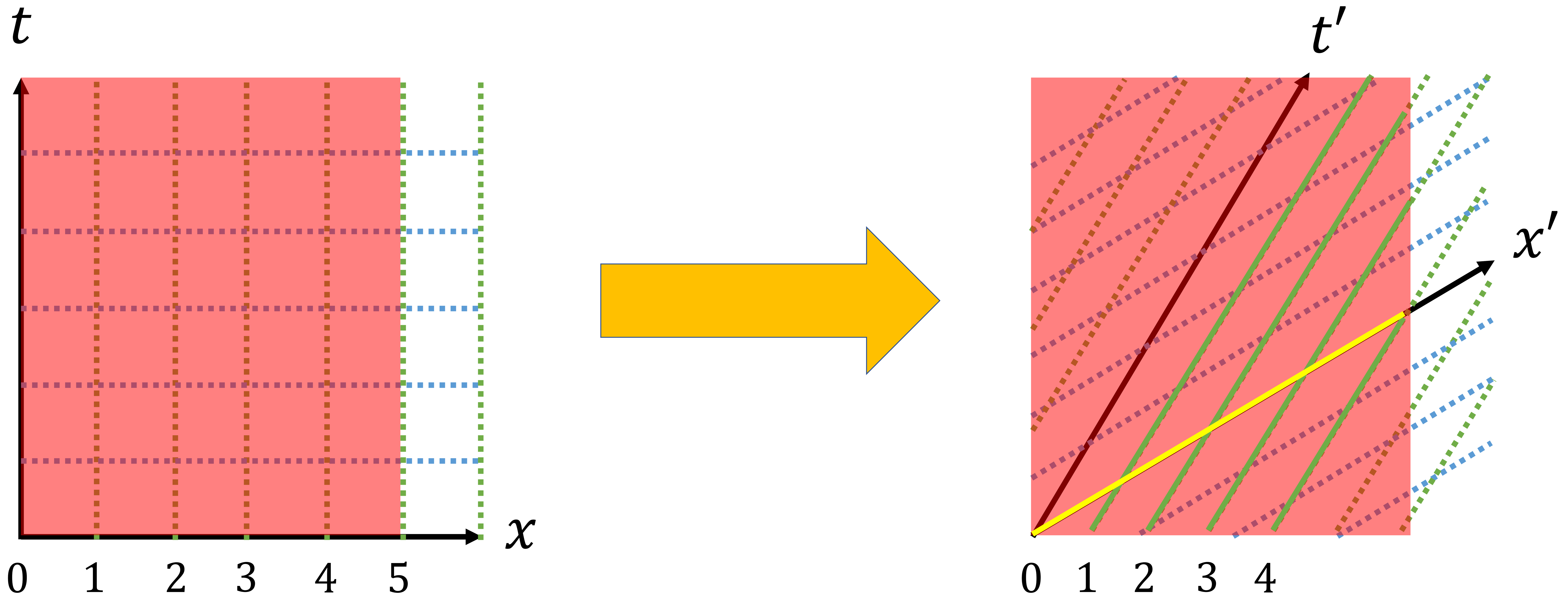
Consequences of special relativity

- The third consequence of special relativity is length contraction.
- Consider an object (e.g. a desk) at rest in Alice's frame.
- If the relative speed is relativistic, Alice and Bob will not agree on the length of the object.

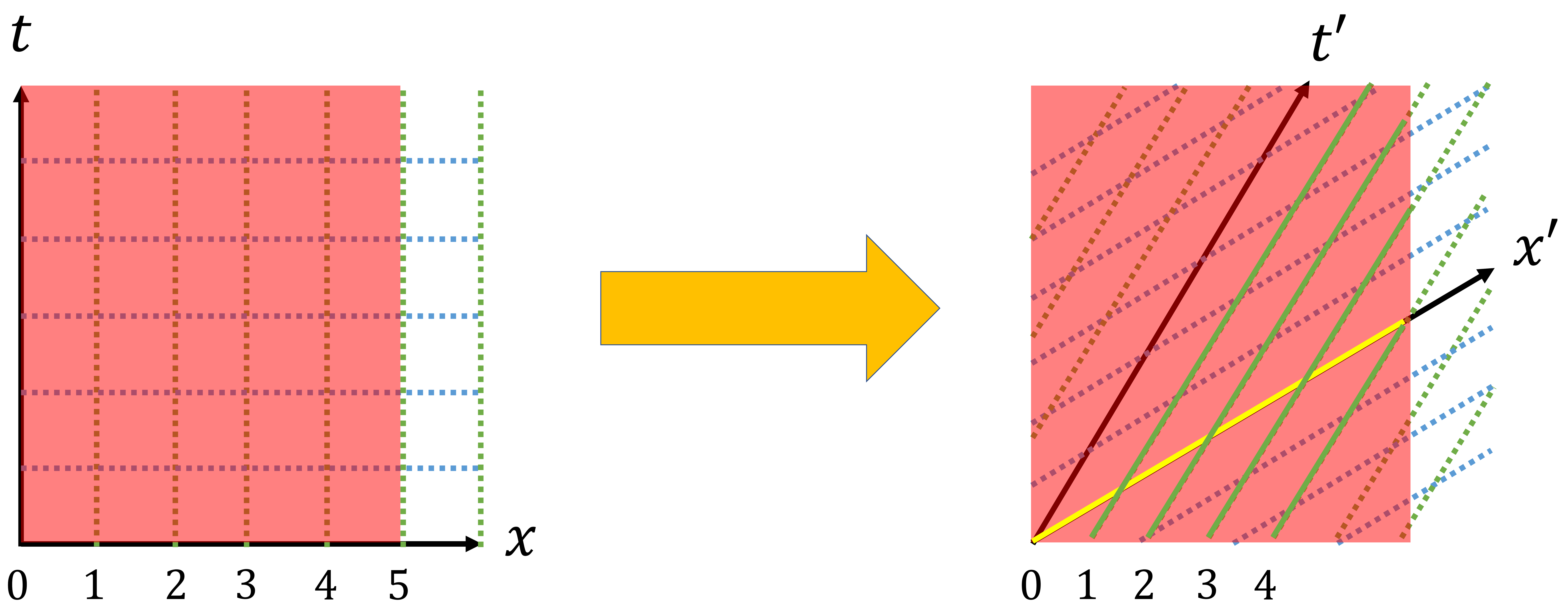
- In Alice's frame, the object is at rest, so it stays in the same place at all times. Therefore, we can represent the object as a rectangle.
- Alice measures the length of the object to be 5.
- In Bob's frame, the object is moving. Bob can only measure the length of the object if he freezes it in time, that is, along a blue line or the x' axis.



- Let's draw the object "frozen in time" in Bob's frame as a yellow line.
- If we count the number of green lines intersecting the yellow line, we find that Bob measures the length of the object as 4.
- Bob sees the length as being contracted, with the same Lorentz factor, $\gamma = 5/4 = 1.25$.



- The length of the object in the frame where it is at rest (Alice's frame) is called proper length.
- Proper length is always the longest length of an object.
- Here, Alice is measuring a proper length of 5, while Bob is measuring a contracted length of 4.



Consequences of special relativity

- Remember that there is no such thing as “absolute speed”. Both observers are moving relative to each other. It’s important to understand that these effects are relative as well.
- If Alice measures a proper time of 4 in her frame, then Bob measures a dilated time of 5 for the same two events in his frame.
- However, if Bob measures a proper time of 4 in his frame, Alice measures a dilated time of 5 for the same two events in her frame.
- No observer actually experiences time faster or slower than the other; that is a common misconception. They just don’t agree on proper vs. dilated time.

Consequences of special relativity

- We see that there is a perfect symmetry between the observers.
- However, this only applies as long as both frames are inertial.
- If one of the observers is accelerating, then they are not in an inertial frame, and the symmetry is broken, so each observer can actually experience different time durations.
- A famous example of this is the twin paradox.
- Alice and Bob are twins. Alice stays on Earth, while Bob travels at relativistic speeds on a spaceship to another planet and back. When they meet again, which twin will be older?

Consequences of special relativity

- Let's assume that the Lorentz factor is $\gamma = 1.25$ like before, corresponding to a speed of $0.6c$ (60% of the speed of light).
- Let's also assume that the total distance of Bob's trip is 6 light-years. Then the trip takes $6/0.6 = 10$ years according to Alice.
- Alice knows that Bob will experience time dilation. Correcting for that, she predicts that the trip will take only $10/1.25 = 8$ years according to Bob.
- Therefore, Alice predicts that when the twins meet again, she will be older than Bob.

Consequences of special relativity

- The paradox is that from Bob’s perspective, he is at rest in his spaceship, and Alice is the one that is moving, because planet Earth moves away from Bob and then back toward him.
- So if Bob does the exact same calculation, he expects himself to age 10 years and Alice to age only 8 years.
- Each twin expects that they will be older than the other twin when they meet again. Obviously, they cannot both be right!

Consequences of special relativity

- To resolve the paradox, we need to check if we made any wrong assumptions.
- Indeed, we secretly assumed that both Alice and Bob’s frames are inertial, but that is not actually the case.
- Alice stays on Earth, so her frame is inertial.
- Bob leaves Earth, travels to the other planet, and comes back to Earth. During each leg of the trip, his frame is also inertial.
- But when Bob turns around, he is changing his velocity from $0.6c$ toward the planet to $0.6c$ toward Earth.
- At that point in time, Bob is accelerating, so his frame is not inertial.

Consequences of special relativity

- Since Bob's frame is not inertial for the entire trip, the symmetry is broken.
- We cannot just assume that Bob is at rest and Alice is moving, because at the moment of turnaround, Bob is not in an inertial frame, so he cannot possibly be at rest.
- In conclusion, there is no paradox: we were right the first time, and Alice will indeed be the older twin when they meet again.

Video

- This video provides a visual way to understand the twin paradox.
- The video can be found at this URL:

https://youtu.be/h8GqaAp3cGs

Consequences of special relativity

- While special relativity has weird consequences, it is indeed correct, and its effects have been proven in many experiments.
- The reason we do not experience effects like time dilation in our daily lives is that we move much slower than light, so Newtonian mechanics is a good approximation to special relativity.
- However, particle accelerators (such as the Large Hadron Collider at CERN) accelerate particles to relativistic speeds.
- Special relativity correctly predicts the results of experiments in particle accelerators. If we tried to use Newtonian mechanics, we would get extremely incorrect predictions.

Consequences of special relativity

- In 1977, physicists used muons (unstable subatomic particles) to prove that a clock that moves away and back to its initial position will measure less time than a clock that stayed at rest.
- This both proved time dilation and demonstrated the twin paradox.
- Length contraction was also proven in many experiments.
- For example, heavy ions are spherical when at rest, but when they move they are length-contracted along the direction of motion, so they take the shape of a flat disk.
- This can be observed due to an increase in density.

Simulation

- I will use a simulation to visualize the effects we discussed on a spacetime diagram in real time.
- I will also demonstrate some other weird consequences of special relativity.
- It is recommended that the students play with this simulation on their own to fully understand the relativistic effects we discussed.
- The simulation can be found at this URL:

https://github.com/bshoshany/spacetime-diagrams

General relativity

- Einstein's special theory of relativity, published in 1905, was a huge success. It unified Newtonian mechanics and electromagnetism and provided an accurate description of motion at relativistic speeds.
- However, special relativity did not incorporate gravity.
- In 1915, Einstein published his general theory of relativity.
- As the name suggests, this theory generalized relativity to include gravity, and special relativity is a special case of general relativity, which applies when there is no gravity.
  - This is similar to how Newtonian mechanics is a special case of special relativity, which applies when speeds are slow compared to light.

General relativity

- An important principle in general relativity is the equivalence principle.
- We first learned about this principle in lecture 11.
- Remember that mass has two different meanings:
    1. Resistance to acceleration by any force (inertial mass).
    2. Strength of the gravitational force (gravitational mass).
- The equivalence principle says that these two types of mass are equivalent: the inertial mass and gravitational mass of any object are always the same, so we can just call it “mass”.

General relativity

- The equivalence of inertial and gravitational mass is also known as the universality of free fall.
- We already discussed this idea in lecture 5: Galileo showed in an experiment that falling objects accelerate uniformly.
- All objects dropped in a gravitational field fall at the same rate, no matter what their mass is.
- This is only possible if inertial and gravitational mass are the same, i.e., the equivalence principle is satisfied.

General relativity

- The formulation of the equivalence principle in terms of mass is called the weak equivalence principle (WEP).
- The issue with the WEP is that, unlike in Newtonian gravity, mass is actually not the only thing that affects gravity in general relativity.
- One consequence of special relativity, which we learned in previous lectures, is the relation $E = mc^2$, which tells us that mass is just a form of energy.
- Therefore, it's more precise to say that, in relativity, gravity is affected by energy, not mass.

- A better way to state the equivalence principle is the Einstein equivalence principle (EEP).
- The EEP says that acceleration is locally indistinguishable from gravity.
- Imagine that you drop a ball in a small sealed and soundproof room with no windows.
- Then you have no way to distinguish between the following two possibilities:

1. The room is on the ground on Earth, and the ball is falling due to gravity.
2. The room is inside a rocket, far from any sources of gravity, accelerating in space with the same acceleration as gravity on Earth ($\sim 9.8\ m/s^2$).

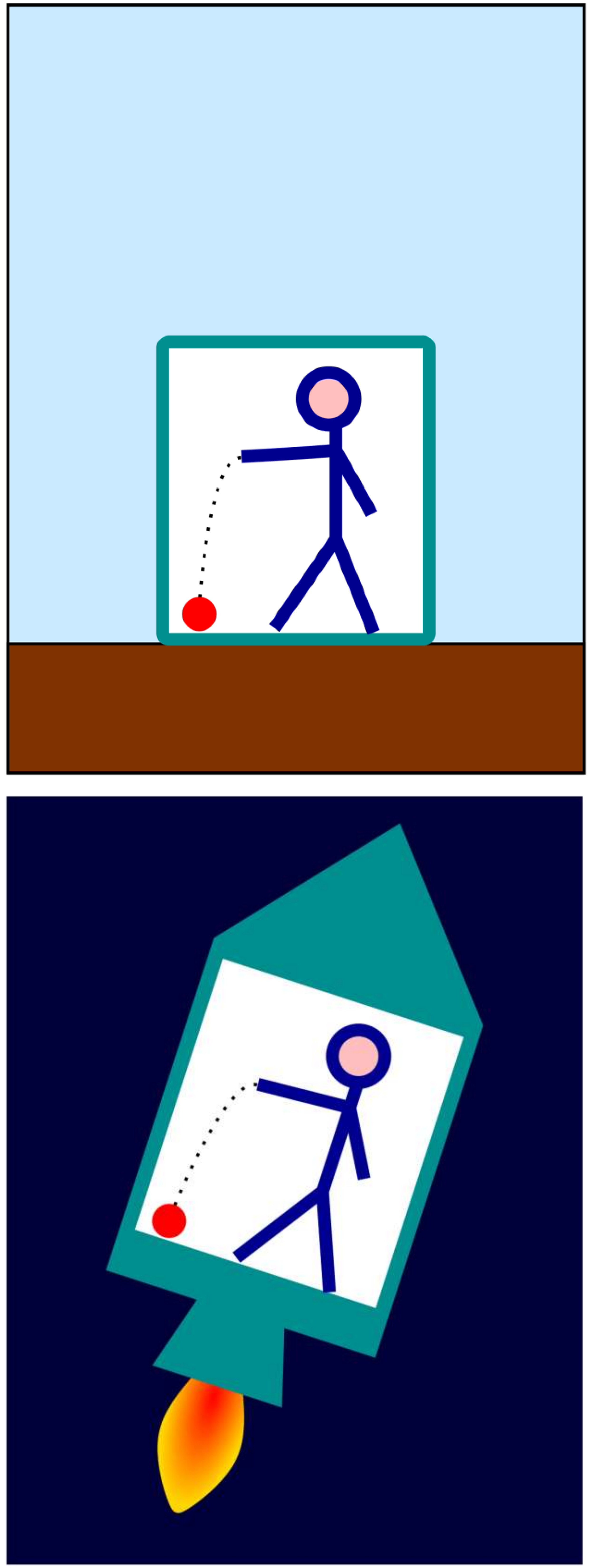


Illustration of the equivalence principle.
Credits: Mapos (Wikipedia)

- It’s important to note two things about this.
- First, the equivalence principle says that acceleration is locally indistinguishable from gravity.
- If the room was large enough, you could notice non-local gravitational effects such as tidal forces, or that gravity is weaker at higher altitudes.
- Second, the rocket needs to accelerate.
- If the rocket is just moving at a constant speed, then the ball will not drop, it will just float in the air.
- The same thing would happen if the room was in free fall toward Earth’s surface; the ball would float.
- There is no way to distinguish between free fall in gravity and constant velocity without gravity.

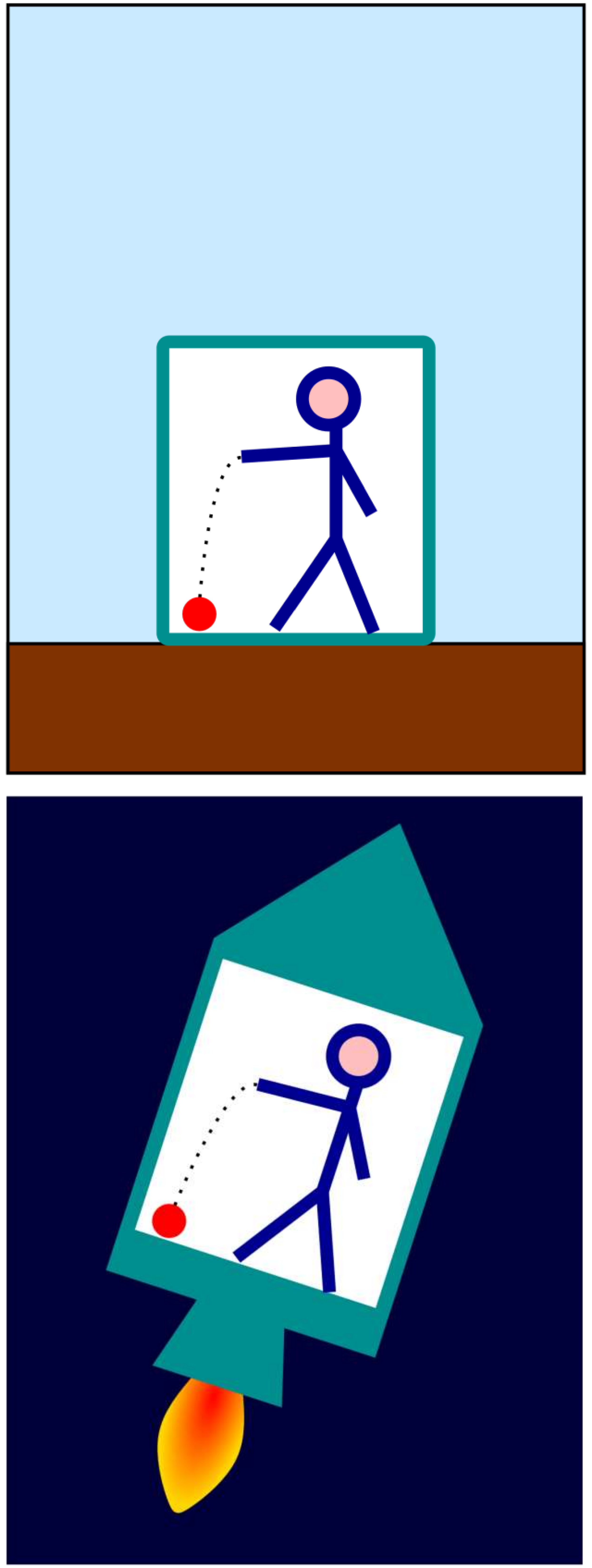


Illustration of the equivalence principle.
Credits: Mapos (Wikipedia)

Astronauts on the International Space Station are well within the gravitational field of Earth, but they are in free fall, so they float in the air instead of falling down.
Credits: NASA

General relativity

- According to relativity, there is no such thing as "absolute speed", because there is no way to detect what speed you're moving at.
- Any object moving at a constant speed (including zero speed, i.e. at rest) is in an inertial frame, and physics in all inertial frames is the same according to the principle of relativity.
- However, in relativity there is still absolute acceleration. This is because an accelerating object is in a non-inertial frame.
- You can detect that you're accelerating by, for example, dropping a ball. If it falls down, then you must be accelerating up.

General relativity

- Think back on the twin paradox. If Alice and Bob were both just moving at constant speed relative to each other, then both would have aged the same.
- In that scenario, there would have been a symmetry between Alice's and Bob's frames. Both are moving at constant speed relative to each other, and neither one is "special".
- However, Bob must accelerate to turn back, which breaks the symmetry. Acceleration is absolute, so we can't say that both Alice and Bob are accelerating "relative" to each other. Bob is special, because he is the only one accelerating.

General relativity

- From the equivalence principle, Einstein deduced that free fall is a type of inertial motion.
- In Newtonian mechanics:
  - When you're falling, the force of gravity accelerates you, so you're in a non-inertial frame.
  - When you're on the ground, the force of gravity cancels with the normal force of the ground pushing up on you, so you're in an inertial frame.
- But according to the equivalence principle, the opposite is true:
  - When you're falling, it's just like floating in space at constant speed, so you're in an inertial frame.
  - When you're on the ground, it's just like being accelerated by a rocket in space, so you're in a non-inertial frame.

General relativity

- What we learn from this is that gravity is not a force!
- From Newton's 2^{nd} law, force is proportional to acceleration.
- When you're falling, no force acts on you, so you're not being accelerated, and you are in an inertial frame.
  - It looks to you like you're accelerating down, but actually, everything else is accelerating up.
- When you're on the ground, the normal force pushes up on you, so you are being accelerated, and you are in a non-inertial frame.
  - It looks to you like you're not accelerating, but actually, you are accelerating up.

General relativity

- But how can everything on the ground be accelerating up all the time (relative to an inertial frame), and yet stay in the same place?
- The answer is that spacetime must be curved.
- When things are in free fall, they follow paths in spacetime called geodesics.
- In geometry, geodesics are the shortest paths. A geodesic on a flat surface will just be a straight line. But a geodesic on a curved surface, like a sphere, will be a curved line.
- The geometry of spacetime is more complicated. A geodesic is not the shortest path, but it is the “easiest” path to take in a curved spacetime.

General relativity

- Due to the curvature of the Earth, your geodesic wants to take you down all the way toward the center of the planet.
- However, you are clearly not falling to the center of the Earth. That’s because the ground (or chair, etc.) is in your way.
- The ground is applying a normal force that is pushing you up. This is the only force acting on you, since gravity is not a force. Force equals acceleration, so you are being accelerated upward.
- This means you are currently not following a geodesic. Therefore, you are not in free fall, since if you were, you would have been following a geodesic. You are in a non-inertial frame.
- In a curved spacetime, you must move to stay in place!

Bonus video

- In this video, Derek Muller (Veritasium) explains the equivalence principle, why gravity is not a force, and the role of acceleration and geodesics in general relativity.
- The video is available at this URL:

https://youtu.be/XRr1kaXKBsU

Gravitational redshift and time dilation

- In 1907, based on the equivalence principle, Einstein predicted an effect called gravitational redshift.
- The wavelength of photons redshifts (shifts towards longer wavelengths) as they travel away from a source of gravity.
- Conversely, the wavelength of photons blueshifts (shifts towards shorter wavelengths) as they travel toward a source of gravity.
- While this is usually presented as a consequence of general relativity, only the equivalence principle itself is needed to derive it.
  - Einstein predicted gravitational redshift 8 years before publishing general relativity in 1915.

Gravitational redshift and time dilation

- To see how gravitational redshift arises from the equivalence principle, consider a lab inside an accelerating rocket.
- Light is emitted upwards from the floor of the lab toward a detector on the ceiling.
- But according to a free-falling (inertial) observer, by the time the light reaches the ceiling, the ceiling has accelerated away from it.
- According to the Doppler effect (see lecture 12), since the detector is moving away from the source of light, it will detect the light as being redshifted.

Gravitational redshift and time dilation

- By the equivalence principle, physics inside an accelerating rocket is indistinguishable from physics in a gravitational field.
- Therefore, if we repeat this experiment in a lab located on the surface of the Earth, we will observe the same redshift.
- In other words, when light moves away from the Earth (or any other source of gravity), it will be redshifted.
- Gravitational redshift has been demonstrated experimentally many times, in the form of shifts in the spectral lines of the Sun, white dwarfs, and even stars passing near supermassive black holes.

Gravitational redshift and time dilation

- A related effect is gravitational time dilation.
- This is a separate effect from special-relativistic time dilation, a.k.a. kinematic time dilation, which we discussed earlier.
- Gravitational time dilation predicts that clocks run slower if they are closer to a source of gravity.
- For example, a clock on Earth at sea level will run slower than a clock on Mount Everest.

Gravitational redshift and time dilation

- To see how this follows from gravitational redshift, recall that the wavelength of light is inversely proportional to its frequency.
- Imagine a clock on Earth emitting light at a constant frequency, as measured by the clock itself (e.g. 1 Hz = 1 wavelength per second). An observer at a fixed altitude in space detects the light.
- The closer the clock is to the source of gravity, the more redshifted it will be according to the observer.
- But when light is redshifted, its wavelength gets longer. This means that its frequency becomes slower.
- So the observer in space will see the clock tick at slower and slower frequencies the closer the clock is to the surface of the Earth.

Gravitational redshift and time dilation

- Gravitational time dilation has been proven in many experiments.
- For example, in the Hafele-Keating experiment in 1971, atomic clocks were flown on planes and compared to clocks that stayed on the ground.
- The readings on the clocks differed in exactly the way relativity predicts.
- This included both kinematic time dilation (due to differences in velocity) and gravitational time dilation (due to differences in height).

Gravitational redshift and time dilation

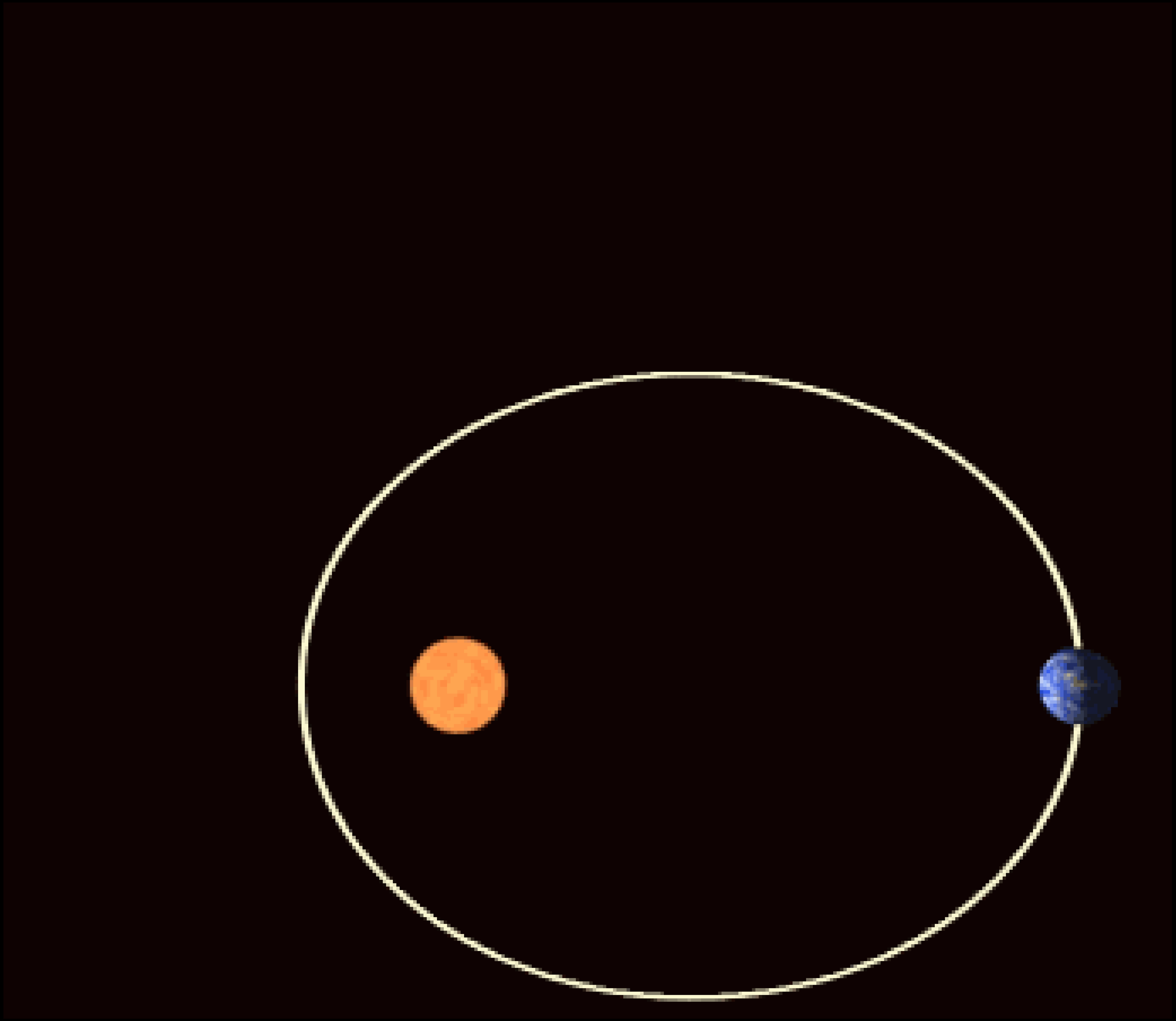
- In this experiment, the clocks differed by only a few nanoseconds.
- You could theoretically age slower if you lived at a lower altitude on Earth, but not by any significant amount.
- However, the difference can hypothetically become significant in an extremely strong gravitational field, such as near a black hole.
- This was demonstrated in the movie Interstellar (2014), where the characters stayed for 1 hour on a planet near a supermassive black hole, and found out that 23 years have passed for everyone else.
  - Note: The first half of this movie is scientifically accurate, but the second half is complete nonsense.

Gravitational redshift and time dilation

- Interestingly, the Global Positioning System (GPS) we use on a daily basis also serves as concrete proof of gravitational time dilation.
- This system consists of many satellites, carrying atomic clocks, which broadcast their position and time continuously.
- A receiver on the ground can compare the data from several satellites to calculate its position accurately.
- However, the GPS satellites are at a higher altitude, so their clocks run faster than clocks on Earth, due to gravitational time dilation.
- Without correcting for this effect, the GPS would not have worked.

Perihelion precession of Mercury

- According to Kepler's 1st law, a planet orbits the Sun in an ellipse, with the Sun at one of the foci of the ellipse.
- This law can be derived from Newtonian physics, which also predicts that the elliptical orbit itself remains fixed.
- The perihelion is the point where the planet is closest to the Sun.
- If there is more than one planet, Newtonian physics predicts that the elliptical orbit will undergo perihelion precession, meaning that the perihelion point, and thus the orbit itself, will rotate around the Sun.



Perihelion precession.
Credits: WillowW (Wikipedia), animation available at this URL: https://en.wikipedia.org/wiki/File:Precessing_Kepler_orbit_280frames_e0.6_smaller.gif

Perihelion precession of Mercury

- In our solar system, the only planet that undergoes significant perihelion precession is Mercury.
- In 1859, it was shown that the precession of Mercury doesn’t match the Newtonian prediction.
- At first, astronomers thought this might be due to an undiscovered planet between the Sun and Mercury, which was named Vulcan.
    - This was motivated by the fact that Neptune was previously found based on anomalies in the orbit of Uranus (see lecture 10).
- However, despite extensive searches, this planet was not found.

Perihelion precession of Mercury

- The true reason for the anomalous precession of Mercury was only found in 1915, when Einstein published his general theory of relativity.
- Einstein showed that his theory correctly predicted the perihelion precession of Mercury.
- In addition to the Newtonian sources of precession, the curvature of spacetime introduces additional precession that Newton's theory doesn't account for.

Perihelion precession of Mercury

- This was a very significant discovery, solving a long-standing problem in astronomy.
- Therefore, it helped motivate physicists and astronomers to adopt general relativity as a more precise theory of gravity.
- Today, we know that general relativity provides the most precise description of gravity, and Newtonian gravity only applies in the Newtonian limit, when:
  - Particles move slowly compared to light,
  - Gravity is weak,
  - And the gravitational field is static (does not change with time).

Deflection of light by the Sun

- It was already known in 1801 that Newtonian gravity predicts that light will bend around a massive object.
- This means that light from stars will be deflected by the Sun.
- General relativity also predicts that light will be bent by the Sun. The Sun curves spacetime; light follows geodesics, and in a curved spacetime, geodesics are not straight lines.
- Einstein calculated that light will be deflected by 1.75 arcseconds as it passes near the Sun. This was double the value predicted by Newtonian gravity.

Deflection of light by the Sun

- This prediction was tested in 1919 in the Eddington experiment, organized by Arthur Eddington and Frank Dyson.
- Normally, we cannot see light from stars being deflected by the Sun, because when the Sun is in the sky, we cannot see the stars.
- However, on May 29, 1919, there was a total solar eclipse. At that time, the Sun’s light was momentarily blocked, and stars near the Sun could be observed.
- Observations were performed in two places on the path of the eclipse: a town in Brazil and an island near the west coast of Africa.

Deflection of light by the Sun

- Photos were taken of the stars near the Sun, and compared with photos of the same stars taken earlier, during nighttime.
- The Eddington experiment showed that the deflection of the stars indeed matches Einstein’s predictions, and not the predictions of Newtonian gravity.
- This was such a significant scientific discovery that it made the front page of most major newspapers, and Einstein became famous around the world.

Deflection of light by the Sun

- The results of the Eddington experiment were not very precise.
- However, similar experiments have been performed several times since then, with better equipment, and the results closely matched Einstein's prediction.
- In 2017, the deflection of light was shown for the first time using a star other than the Sun: the nearby white dwarf Stein 2051 B, located $\sim$18 light-years from Earth.
- When the white dwarf passed in front of a more distant star, light from that star was deflected. This was used to measure the white dwarf's mass, since the amount of deflection depends on the mass.

Video

- This video summarizes the Eddington experiment.
- The video can be found at this URL:

https://youtu.be/HLxvq_M4218

Gravitational lens

- The Sun is not the only object that can deflect light. Any massive object curves spacetime and therefore affects the path of light.
- This effect is known as gravitational lensing and the object causing the lensing is called a gravitational lens.
- The effect is more significant for objects with more mass.
- A collection of massive objects, such as a galaxy or even a cluster of galaxies, can also serve as a gravitational lens.

Gravitational lens

- Although the words “lens” is used, gravitational lens do not behave like optical lens (as used in glasses, cameras, etc.).
- In the ideal case:
    1. The lens has a circular shape,
    2. The light source, the gravitational lens, and the observer are all perfectly aligned along a straight line.
- In that case, the light source will appear as a ring around the lens, called an Einstein ring.
- In the more common case, where the alignment is not perfect and/or the shape isn’t circular, the light source might appear as an arc segment instead.

A red galaxy acting as a gravitational lens and distorting the light from a much more distant blue galaxy behind it. Here the alignment is almost perfect, resulting in an almost full circle.
Credits: ESA/Hubble & NASA

Video

- In this video, we see a simulation of a gravitational lens moving against a background field of galaxies.
- The video can be found at this URL:

https://youtu.be/k6JHryhlNVk

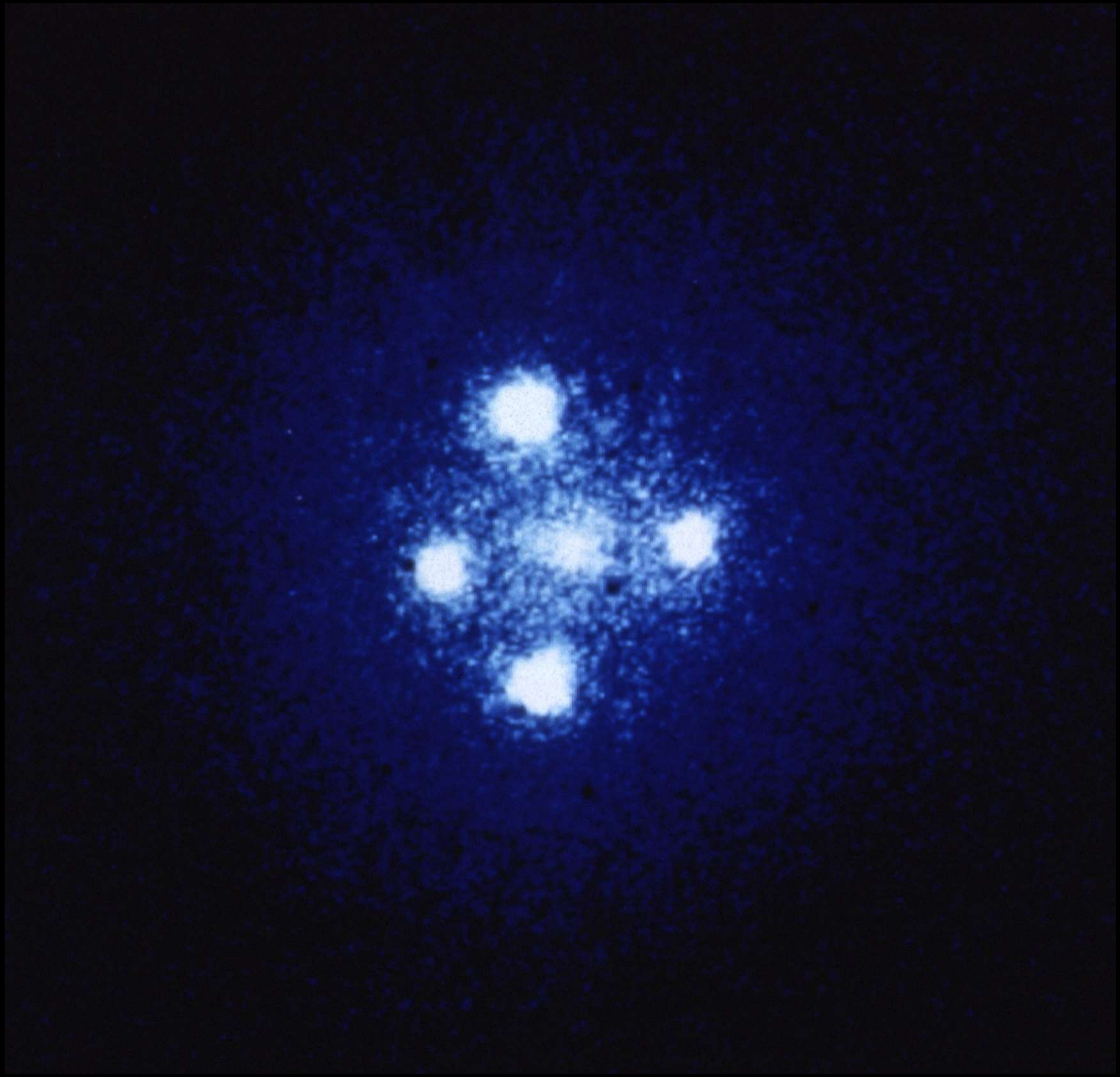
Video

- In this video, we see an animation illustrating how gravitational lensing works.
- The video can be found at this URL:

https://www.eso.org/public/videos/eso1522a/

Gravitational lens

- One particularly interesting instance of gravitational lensing is the Einstein cross.
- An active galactic nucleus (AGN) is a region at the center of a galaxy that has very high luminosity, thought to be caused by accretion of matter by a supermassive black hole.
- The most luminous AGNs are called quasars (quasi-stellar radio sources).
- The Einstein cross is a quasar that is being gravitationally lensed by a galaxy into 4 distinct copies of itself. It does not look like a ring or arc because of the unusual alignment and shape of the lens.



The Einstein Cross.
Credits: NASA, ESA, and STScI

Video

- In this video, we see an animation illustrating how gravitational lensing results in an Einstein cross.
- The video can be found at this URL:

https://esahubble.org/videos/heic1702a/

Gravitational lens

- Gravitational lens aren't just cool to see, they are also useful, because they magnify the images of very distant objects.
- This allows us to see farther in space (and therefore also in time, since light travels at a finite speed) than we could have otherwise.
- Our telescopes can only see up to a certain distance (or time).
- The James Webb Space Telescope (JWST) is currently our most powerful telescope, enabling us to see farther than ever before.
- However, even the JWST has its limits, and gravitational lens can allow it to see even farther than it could have otherwise.

Gravitational waves

- In lecture 12 we learned that a wave is a regular disturbance propagating in space.
- The disturbance can be, for example, in the electromagnetic field (for light), in the air (for a sound wave), and so on.
- In general relativity, spacetime is curved. This allows for the possibility of regular propagating disturbances in the curvature of spacetime itself: gravitational waves.
- Like electromagnetic waves, gravitational waves always propagate at the speed of light.

Gravitational waves

- Gravitational waves are radiated by massive objects whenever they are accelerating.
- If a massive object is moving at constant speed (or at rest), it will not radiate gravitational waves.
  - Remember: in relativity speed is relative, but acceleration is absolute.
- Also, even if the object is accelerating, it will not radiate gravitational waves if the motion is:
  - Spherically symmetric (e.g. an expanding or contracting sphere)
  - Rotationally symmetric (e.g. a spinning disk or sphere).

Gravitational waves

- A common source of gravitational waves is any two massive objects orbiting each other.
- However, the strength of the waves is only significant when the objects are extremely massive, such as neutron stars and/or black holes.
- A supernova will also generate gravitational waves.

Gravitational waves

- Unlike electromagnetic waves, gravitational waves are very hard to detect.
- You can't just make a camera (or antenna) and absorb the incoming wave, you need to detect small changes in space and time themselves.
- Although gravitational waves were predicted by Einstein in 1916, the first indirect evidence for them came in 1974, and the first direct evidence only in 2015.

Gravitational waves

- Like all waves, gravitational waves carry energy. This means that when a source emits gravitational waves, it gradually loses energy.
- If the source is two massive bodies orbiting each other, this loss of energy will cause the orbit to decay (decrease in radius).
- For example, when the Earth orbits around the Sun, it releases gravitational waves and loses ~ 200 joules of energy every second.
- However, this is negligible compared to the total rotational energy of $\sim 10^{36}$ joules. The Earth's orbit only decays by $\sim 10^{-15}$ m (the diameter of a proton) per day.

Gravitational waves

- The first indirect detection of gravitational waves used the Hulse-Taylor pulsar, a binary system of two neutron stars, one of which is a pulsar.
- The pulsar has a pulse period of $\sim$59 milliseconds, and orbits the neutron star every $\sim$7.75 hours.
- Both neutron stars have a mass of $\sim 1.4\ M_{\odot}$ and orbit with a semi-major axis of $\sim$2,000,000 km.
- Since the binary system's discovery in 1974, its orbit has decayed, in precise agreement with loss of energy due to gravitational waves.

Gravitational waves

- The energy carried by the gravitational waves from the Hulse-Taylor pulsar is $\sim 7.35 \times 10^{24}$ W.
  - Compare this to ~ 200 W of gravitational waves from the Earth-Sun system (as mentioned before) and $\sim 5{,}000$ W from the entire solar system.
  - This huge amount of energy is due to the large masses and small separation of the neutron stars compared to the solar system.
- This huge loss of energy results in an observable decrease of the orbital period by ~ 76.5 microseconds per year and orbital radius by ~ 3.5 meters per year.
- It is expected that due to this decay, the two neutron stars will merge in ~ 300 million years.

Video

- This video shows an animation of the Hulse–Taylor pulsar along with a short explanation.
- The video can be found at this URL:

https://youtu.be/zT3TkA_u0Ws

Gravitational waves

- This discovery was indirect because we have observed the expected orbital decay due to gravitational waves, but not the waves themselves.
- The first direct discovery of gravitational waves was made by the Laser Interferometer Gravitational-Wave Observatory (LIGO) in collaboration with the Virgo interferometer in 2015.
- LIGO consists of two observatories located in the US, at the Hanford Site in Washington and in Livingston, Louisiana.

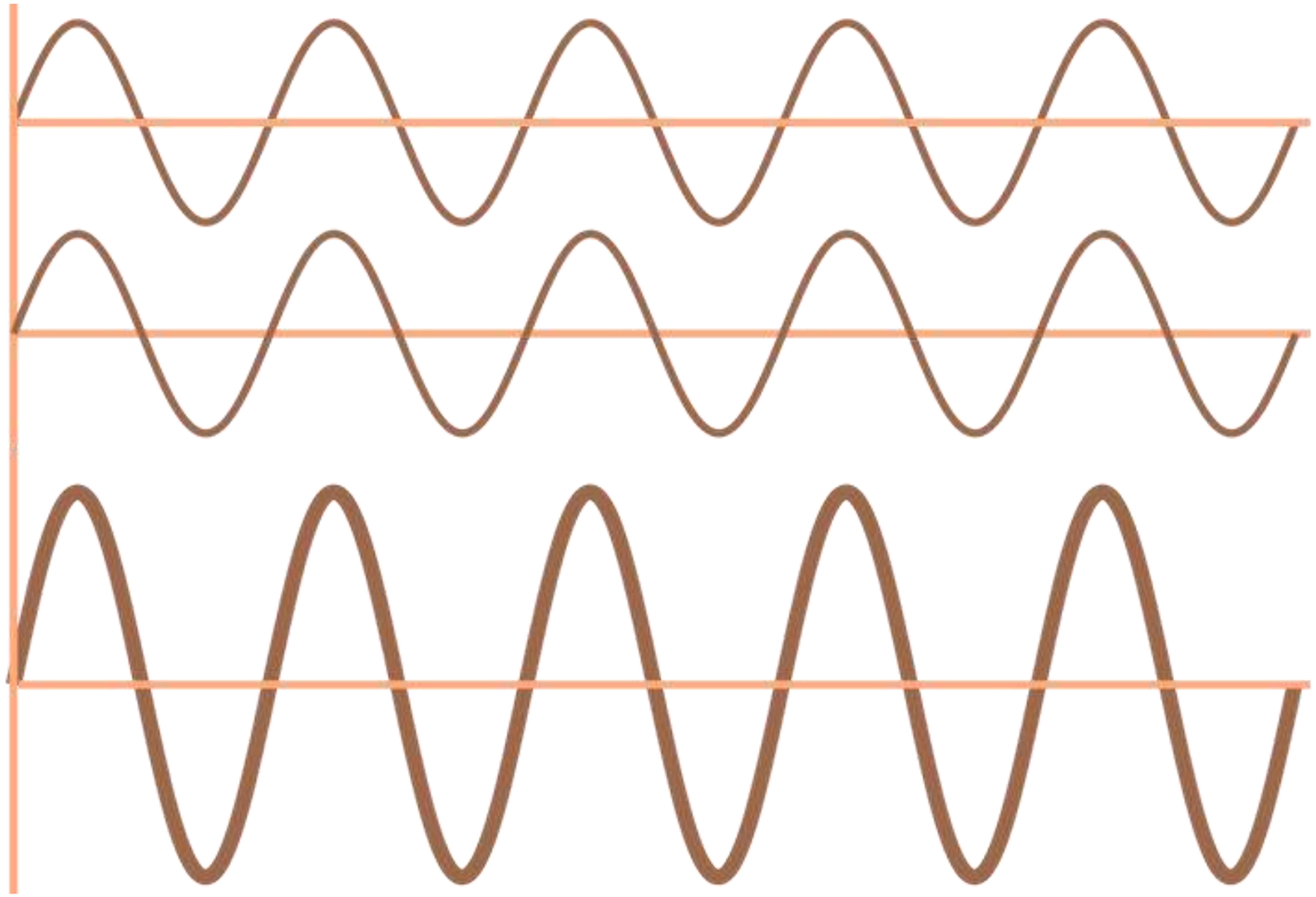
Gravitational waves

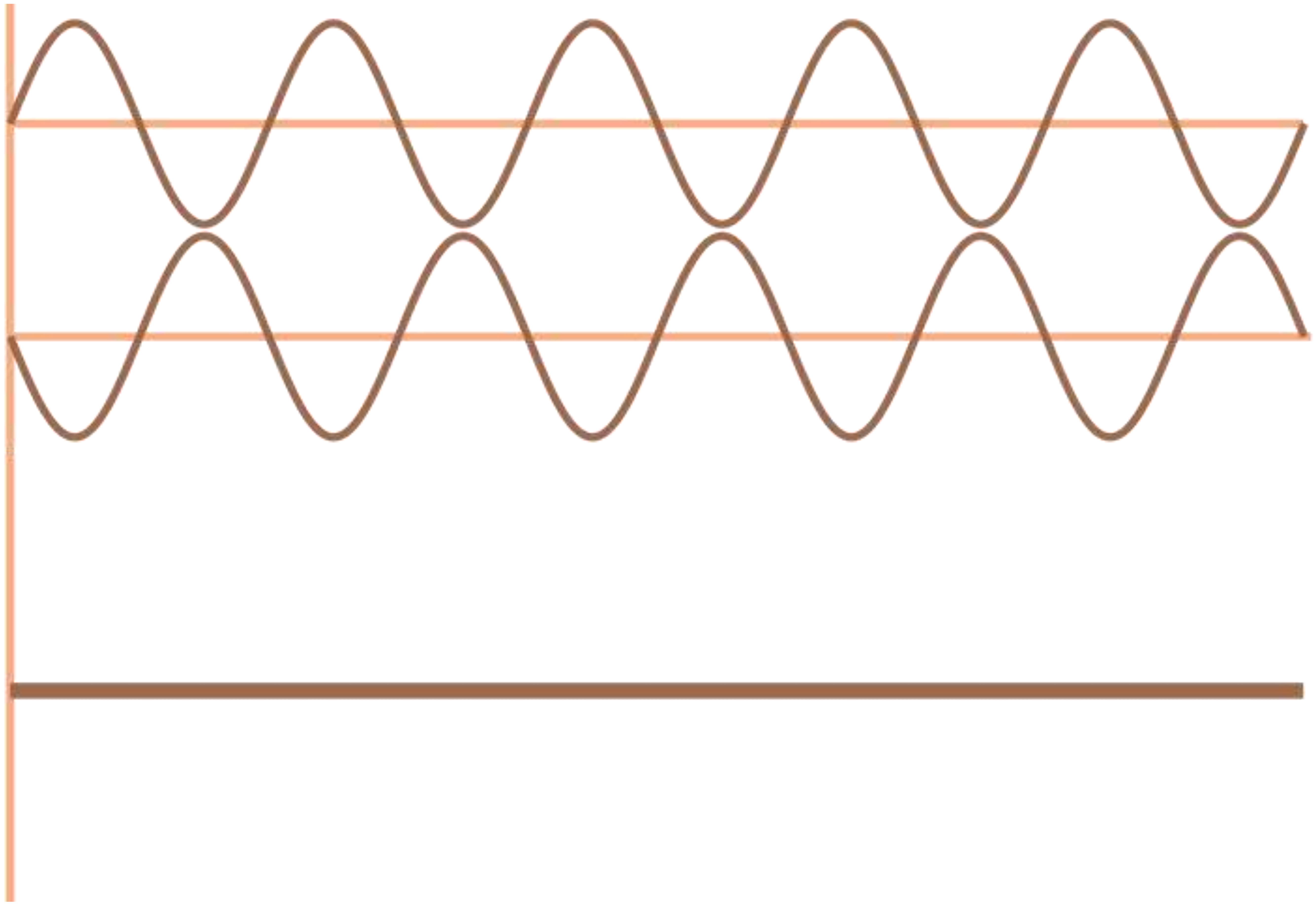
- Each of the two LIGO observatories is equipped with an interferometer, which is an experiment that measures interference between two laser beams.
- The interferometer is shaped like the letter “L”. A laser beam is split into two parts and sent along both arms, each ~4 km long.
- When both arms have the same length, the two light waves exactly cancel each other.
- But when a gravitational wave passes through it, spacetime will be distorted, and one arm will be at a different length than the other. The beams will no longer cancel each other.

The locations of the two LIGO observatories.
Credits: Wikipedia

The LIGO Hanford Observatory. Note the “L” shape of the interferometer.
Credits: LIGO Laboratory

The LIGO Livingston Observatory.
Credits: Caltech/MIT/LIGO Lab





Constructive interference
(waves add up)

Destructive interference
(waves cancel)

Example of interference. The two waves on the top sum up into the wave on the bottom. Left: identical waves sum into a larger wave. Right: opposite waves cancel each other.
Credits: Wikipedia

The LIGO interferometer (simplified). Laser is emitted from the bottom, split into two beams using a beam splitter, reflected off mirrors (red), and the beams then interfere with each other.
Credits: Cmglee (Wikipedia)

Gravitational waves

- The LIGO interferometers are so precise they can detect a change in arm length of $\sim 10^{-18}$ m, 1,000 times smaller than a single proton!
- The two observatories are separated in space by $\sim$3,002 km.
  - The separation is $\sim$3,030 km along the surface of the Earth, but gravitational waves will pass through the Earth.
- Gravitational waves travel at the speed of light, $\sim$300,000 km/s, so it can take them up to $\sim$10 ms to travel between the observatories.
- Measuring the time difference between detections can be used to determine where the wave originated from in space.

Gravitational waves

- The Virgo interferometer is another gravitational wave observatory, located in a village near Pisa, Italy.
- It is also "L"-shaped, with ~3 km arms, and works similarly to LIGO.
- The interferometer is operated by the Virgo Collaboration, with ~650 members, representing 119 institutions in 14 countries.
- LIGO and Virgo work together, sharing data and jointly publishing their results.
- This allows them to increase their confidence in the validity of the signal and locate the source of the waves more accurately.

Aerial view of the Virgo detector.
Credits: The Virgo collaboration

Gravitational waves

- The first detection of gravitational waves was made on September 14, 2015 by LIGO.
- The signal lasted over $\sim$0.2 seconds, increasing in frequency from $\sim$35 Hz to $\sim$250 Hz.
- Interestingly, this signal is in the human audible range ($\sim$20-20,000 Hz), so it can be converted to sound, and sounds like a bird “chirp”.
    - This sound was very popular in the media, but it is important to understand that the wave was a gravitational wave, not a sound wave!
- This event was named GW150914 (GW = gravitational wave, followed by the date of detection).

Gravitational waves

- By analyzing the GW150914 signal, we know that it was produced by a merger of two black holes $\sim$1.4 billion light-years away.
- The two black holes had masses of $\sim 30\ M_{\odot}$ and $\sim 35\ M_{\odot}$, and merged into a final black hole with $\sim 62\ M_{\odot}$.
- This means that a total of $\sim 3\ M_{\odot}$ of rest energy was radiated away in the form of gravitational waves.
- During the final 20 milliseconds of the merger, the power of the radiated gravitational waves peaked at $\sim 3.6 \times 10^{49}$ watts.
    - This is 50 times greater than the combined power of all light radiated by all the stars in the observable universe!

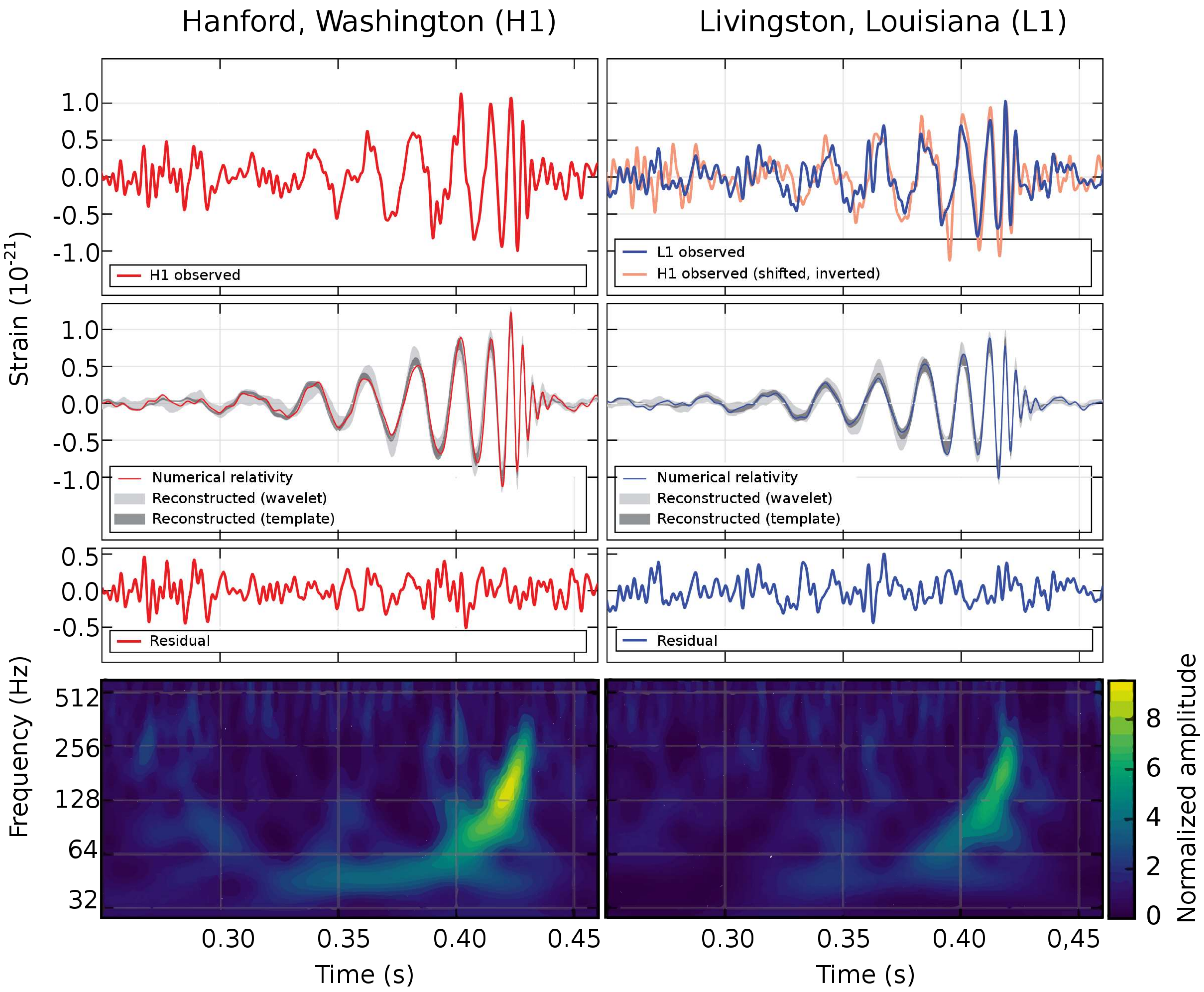
Gravitational waves

- During the 0.2-second signal, the relative orbiting velocity of the black holes increased from ~30% to ~60% of the speed of light.
- The black holes were orbiting each other at only ~350 km by the time they merged.
- There was a time delay of ~6.9 ms between the detections at the two LIGO observatories.
- By analyzing that, and other differences between the two detections, LIGO was able to locate the source somewhere in the direction of the Magellanic Clouds (but much farther).

Video

- This video shows a computer simulation of the two merging black holes.
- Note the strong gravitational lensing created by the black holes.
- The video is available at this URL:

https://www.ligo.caltech.edu/video/ligo20160211v3



The gravitational wave signal GW150914, compared with the prediction of general relativity.
Credits: B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)

Video

- In this video we can "hear" the gravitational waves GW150914 when their data is converted into sound waves.
- The first two runs are at the original frequencies. The next two runs are at higher frequencies that are easier to hear.
- The video is available at this URL:

https://youtu.be/QyDcTbR-kEA

Gravitational waves

- Until 2015, astronomical observations have always been made using electromagnetic waves.
- In ancient times, observations were made only in visible light, with the naked eye.
- In 1609, Galileo made astronomical observations with a telescope for the first time, but still only in the visible spectrum.
- More recently, we began to observe the universe in other regions of the electromagnetic spectrum, from radio waves to gamma rays.
- Each new type of observation made it possible to detect new astronomical objects and phenomena, or understand existing ones better.

Gravitational waves

- The first detection of gravitational waves marked the beginning of a brand-new type of observation: gravitational-wave astronomy.
- This has several advantages over observation using electromagnetic waves.
- Gravitational waves can be generated even in cases where no electromagnetic waves will be generated, for example when two uncharged black holes merge without any nearby matter.
- Also, gravitational waves pass through any kind of matter without significant scattering. This means they cannot be blocked by interstellar dust or other objects, as light does.

Video

- This video from LIGO describes the first observation of gravitational waves.
- The video is available at this URL:

https://youtu.be/qI22gbZmYkQ

Gravitational waves

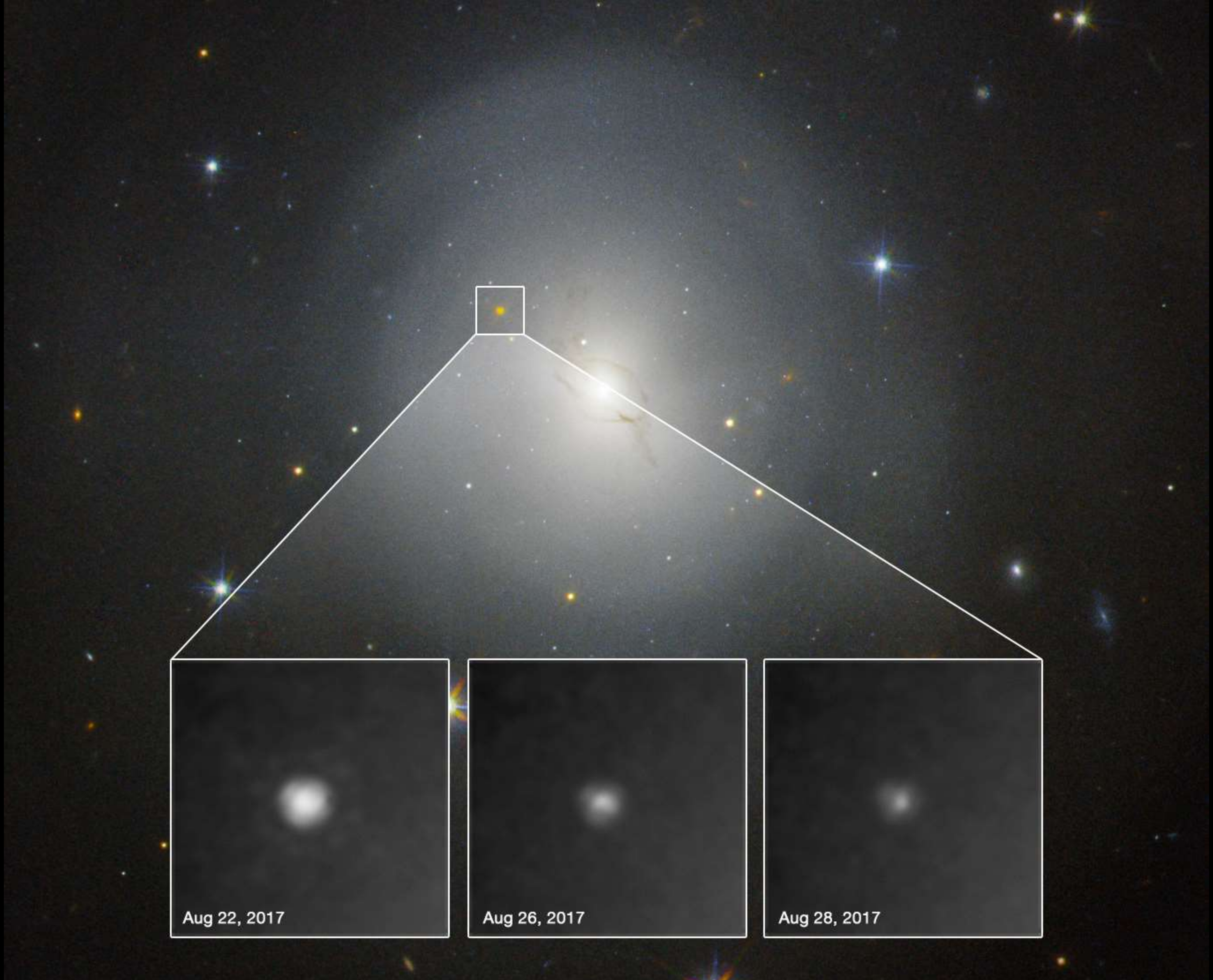
- Since 2015, both LIGO and Virgo have detected dozens of additional gravitational wave events.
- All the events come from the merger of two compact objects: two black holes, two neutron stars, or a black hole and a neutron star.
- In 2017, the first gravitational wave event from the merger of two neutron stars, GW170817, was detected.
- This event was particularly important since, unlike black hole merges, neutron star mergers also emit a detectable electromagnetic signal.

Gravitational waves

- The electromagnetic signal emitted from the GW170817 merger was seen by 70 observatories on all 7 continents and in space.
- This marked a significant breakthrough for multi-messenger astronomy, observation of the same event based on different types of signals.
- In this case, the same neutron star merger was observed using both gravitational waves and electromagnetic waves.
- Other possible types of “messengers” in multi-messenger astronomy are cosmic rays (very high energy charged particles) and neutrinos (neutral elementary particles with very small mass).

Gravitational waves

- The source of the gravitational wave event GW170817 was identified within 12 hours in the galaxy NGC 4993, located ~140 million light-years away.
- The neutron star merger created a kilonova, an event 1,000 times brighter than a nova.
- Recall that a nova happens when a white dwarf explodes after accreting matter from a binary companion.
- A kilonova can happen due to the merger of either two neutron stars, or a neutron star and a black hole.



Hubble Space Telescope image of the kilonova associated with GW170817, in the galaxy NGC 4993.

Credits: NASA and ESA. Acknowledgment: A.J. Levan (U. Warwick), N.R. Tanvir (U. Leicester), and A. Fruchter and O. Fox (STScI)

Gravitational waves

- Gamma-ray bursts (GRBs) are bursts of extremely high energy gamma rays, releasing as much energy in a few seconds as the Sun will in its entire lifetime!
- A gamma-ray burst designated GRB 170817A was detected ~1.7 seconds after GW170817.
- Gamma-ray bursts are thought to be released during supernovae, but thanks to GW170817, we now know they can also be released during neutron star mergers.

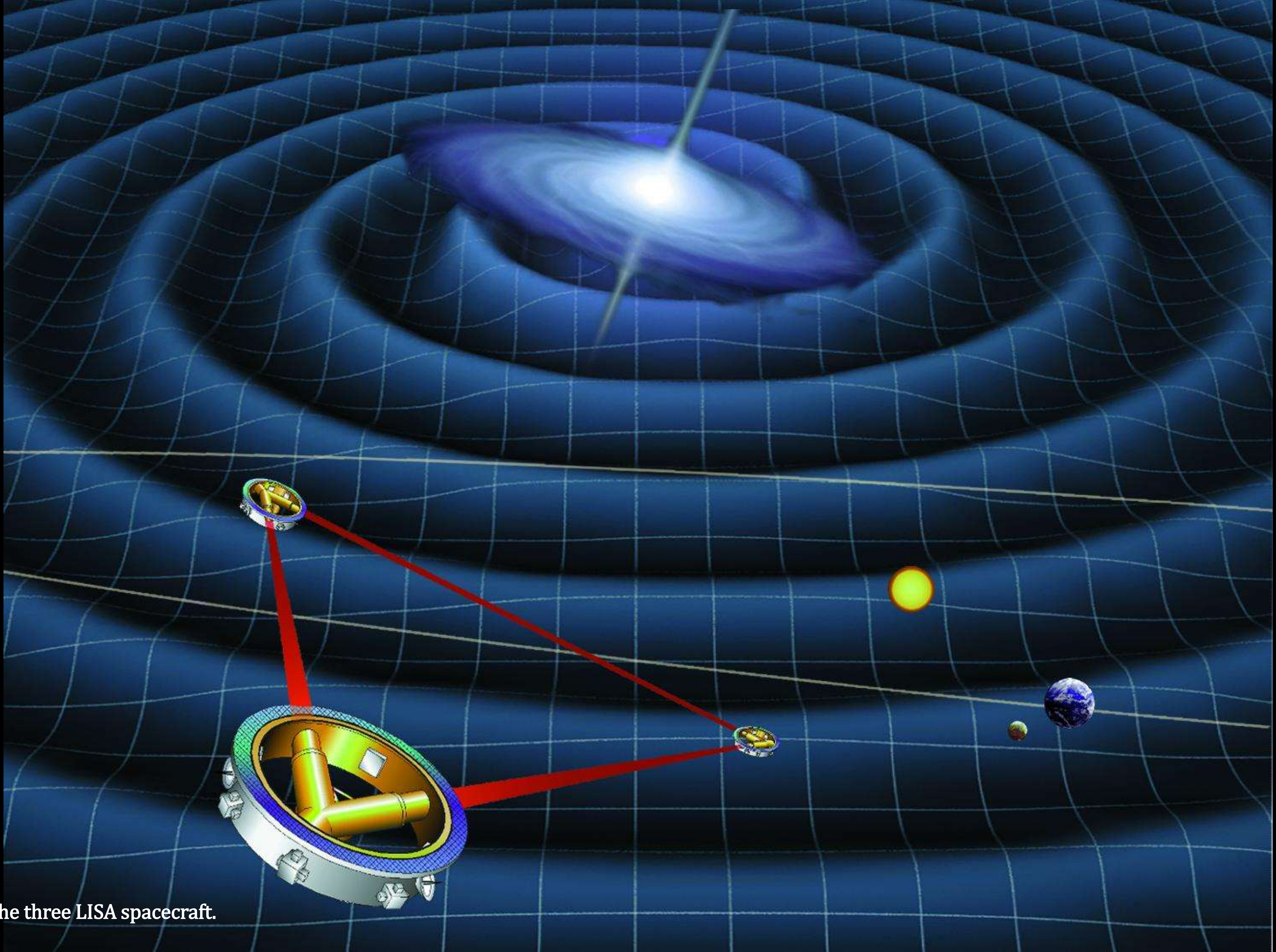
Gravitational-Wave Transient Catalog-1

Gravitational waves

- LIGO and Virgo are Earth-based gravitational wave detectors. However, space-based detectors are also being planned.
- One proposed detector is the Laser Interferometer Space Antenna (LISA), which will consist of 3 spacecraft orbiting the Sun, arranged in an equilateral triangle with sides ~2.5 million km long.
- When a gravitational wave passes by, it can be detected because the lengths of the triangle's sides will be modified.

Gravitational waves

- Due to the much longer side length, LISA will be much more sensitive to gravitational waves with longer wavelengths (and thus lower frequencies) compared to LIGO and Virgo.
- While LIGO and Virgo are sensitive to signals from black hole and/or neutron star mergers, LISA will be sensitive to signals from:
    - Binary stars within our galaxy,
    - Binary supermassive black holes in other galaxies,
    - Extreme mass ratio inspirals (EMRI), produced by stellar black holes or neutron stars orbiting supermassive black holes.
        - “Extreme mass ratio” refers to the supermassive black hole being much more massive.
        - “Inspiral” refers to the gradually shrinking orbit as the smaller body spirals inward.



Artist's conception of the three LISA spacecraft.
Credits: NASA

Space travel

- Now that we know relativity, we can discuss the possibility of space travel. We refer to travel to other stars as interstellar travel.
- The nearest star, Proxima Centauri, is $\sim$4.2 ly (light years) away. It has 3 exoplanets, which may or may not be habitable.
    - An exoplanet is any planet outside our solar system.
- Other star systems in the Milky Way with potentially habitable exoplanets are much farther away, up to thousands of light years.
- The diameter of the Milky Way galaxy is $\sim$175,000 ly. The Sun is $\sim$27,000 ly from the galactic center.

Space travel

- Intergalactic travel is travel to other galaxies.
- There are several dozen small satellite galaxies orbiting the Milky Way, at distances from ~65,000 ly (Sagittarius Dwarf) to ~1,365,000 ly (Leo T).
- The nearest large galaxy, Andromeda, is ~2,500,000 ly away. The Local Group consists of the Milky Way, Andromeda, and their satellite galaxies.
- There are ~100 billion galaxies in the observable universe. The most distant known galaxy is HD1, ~33.4 billion ly away.

Space travel

- The fastest human-made spacecraft today is the Parker Solar Probe, which will reach a maximum speed of $\sim$690,000 km/h or $\sim$190 km/s, which is 0.064% of the speed of light.
- At this speed, a trip to Proxima Centauri will take $\sim$6,600 years.
- A trip to Andromeda will take $\sim$3.9 billion years!
- Obviously, such long travel times are unfeasible.

Space travel

- According to relativity, it is impossible for any massive object to accelerate to the speed of light.
- As the object approaches the speed of light, accelerating it requires more and more energy.
- Accelerating to the speed of light itself would require an infinite amount of energy, which is impossible.
- Since we can never accelerate to the speed of light itself, we can never move faster than light either.
  - However, it could be possible to take advantage of the curvature of spacetime to effectively move faster than light. We will discuss that soon.

Space travel

- The best we can hope to achieve using conventional spacecraft propulsion is to get as close as possible to the speed of light.
- However, this requires a huge amount of energy.
- Consider a spacecraft weighing 50,000 tons (approximately the weight of the Titanic).
- Accelerating this spacecraft to 99% of the speed of light would require a total energy of $\sim 3 \times 10^{25}$ J.
- For comparison, this is $\sim$50,000 times larger than the total world energy consumption in 2021, which was $\sim 6 \times 10^{20}$ J!

Space travel

- The spacecraft would need to be accelerated as it leaves Earth, and then decelerated as it approaches the destination.
- Deceleration would require the same energy as acceleration. So the total energy consumption would be doubled.
- Also, this calculation assumes a perfect propulsion system, which doesn't waste any energy, but this is impossible – there is always some waste.
- In conclusion, accelerating a spacecraft to 99% of the speed of light is physically possible, but requires energy many orders of magnitude beyond what our current technology allows.

Space travel

- Even if we manage to accelerate a ship to 99% of the speed of light, it will still take years to get to even the nearest star systems.
- The ship will take ~4.2 years to get to Proxima Centauri, ~27,300 years to get to the center of the Milky Way, and ~2,525,000 years to get to the Andromeda galaxy.
- However, relativistic time dilation will make the trip be subjectively shorter for the passengers on the ship.
- At 99% of the speed of light, the Lorentz factor is ~7, so the ship can get to Proxima Centauri in only 7 months.

Space travel

- If we could accelerate the ship even closer to the speed of light, the Lorentz factor can be as high as we want it to be.
- For example, at 99.99999999999% of the speed of light, the Lorentz factor is 2,500,000, which means a trip to the Andromeda galaxy will last just 1 year for passengers on the ship!
  - However, this neglects the time needed to accelerate to that speed, which can be considerable (but we won't get into that here).
  - Also, accelerating our 50,000-ton spaceship to that speed would require 10^{31} J of energy, or $\sim 10^{10}$ times the world energy consumption.
  - Furthermore, the trip will still take ~2,525,000 years as measured by the people who stayed on Earth.

Space travel

- It is extremely unlikely that we will be able to create spaceships that can accelerate to a significant fraction of the speed of light any time soon.
- It may take hundreds or thousands of years to develop that technology, or possibly even longer.
- Therefore, a scenario where a spaceship travels at much slower speeds is much more likely.
- In this case, even travel to the closest star, Proxima Centauri, could take many hundreds of years; and intergalactic travel is completely out of the question.

Space travel

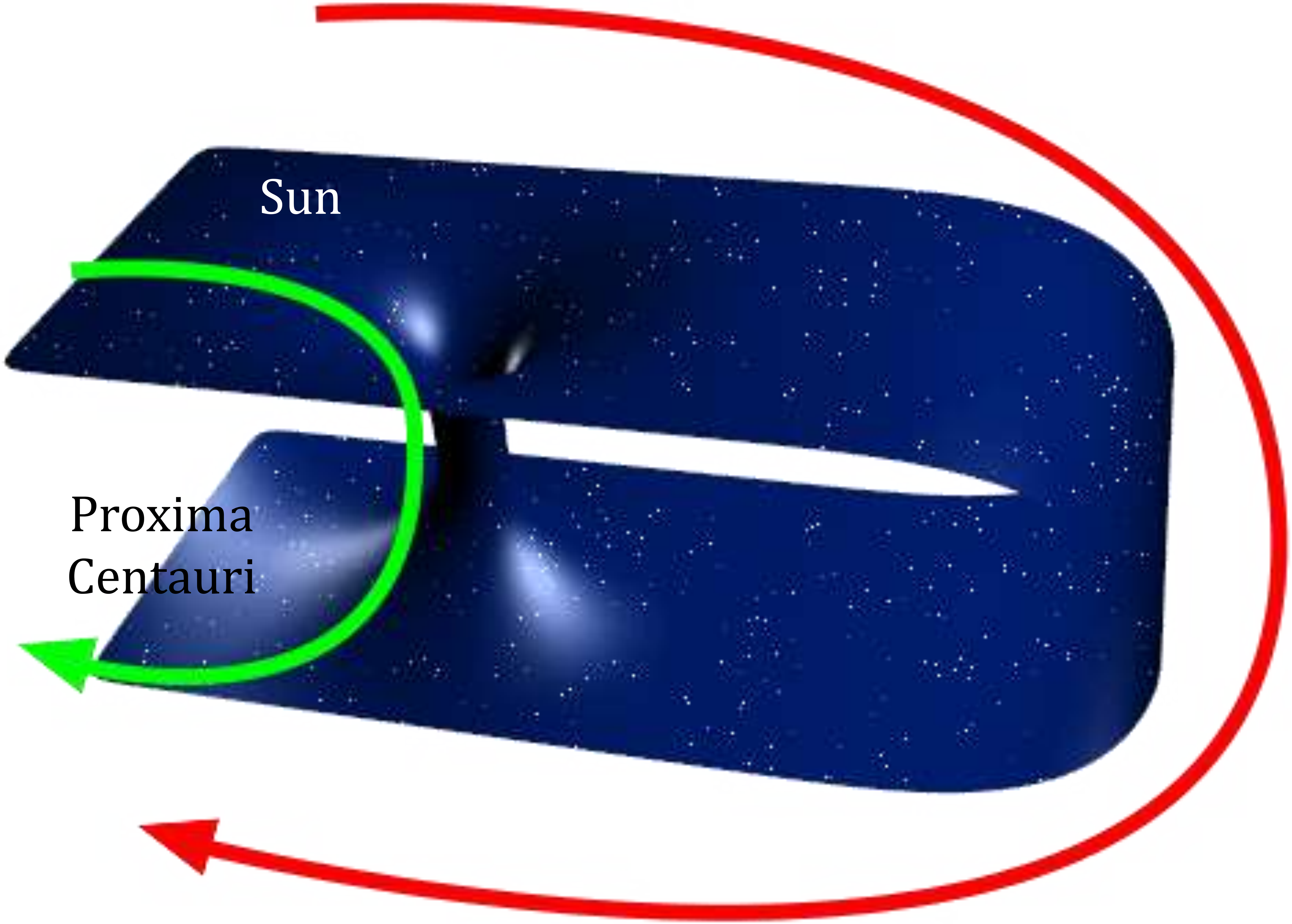
- There are several suggestions to make such a long trip possible, some of which may be familiar to fans of science fiction.
- A generation ship functions as a completely independent "world", where people live, reproduce, and die, and the ship arrives at its destination many generations later.
- In a sleeper ship, human passengers are in a state of suspended animation, to be woken up only at the end of the journey.
- It is also possible that, in the future, humans will achieve very long lifetimes (or even immortality), making journeys of hundreds of years tolerable.

Space travel

- A form of interstellar travel that is often invoked in science fiction is faster-than-light (FTL) travel, which refers to travel at speeds faster than the speed of light.
- As mentioned before, according to relativity, an extremely precise and well-tested theory, it is impossible to accelerate a massive object like a spaceship to the speed of light or beyond.
- However, there are some hypothetical “loopholes” that might allow bypassing this restriction.
- These “loopholes” are made possible due to the curvature of spacetime.

Space travel

- One commonly discussed FTL “loophole” is a wormhole, which is a “shortcut” in spacetime.
- Hypothetically, there could be a wormhole such that instead of traveling $\sim$4.2 ly, a spaceship could enter the wormhole near the Sun and immediately emerge near Proxima Centauri.
- The loophole is that the spaceship has effectively traveled much faster than light, since it crossed $\sim$4.2 ly in a very short time, but it actually traveled slower than light over a shorter distance.



Instead of traveling the long way (red arrow), the spaceship can take a shortcut through the wormhole and travel the short way (green arrow) to the same destination.
Credits: Panzi (Wikipedia)

Space travel

- Another hypothetical “loophole” is called a warp drive.
- While relativity forbids traveling faster than light within space, it still allows space itself to travel at any speed, and this loophole can be exploited.
- A warp drive consists of a warp bubble, a portion of space that moves on its own at FTL speeds due to its curvature.
- A spaceship inside the warp bubble will actually be at rest in space, but will move along with the bubble.

Space travel

- While both wormholes and warp drives seem to be mathematically consistent with general relativity, it is currently unknown if they can really exist in our universe.
- One major issue is that it appears that constructing them would require "exotic matter", which is matter with negative energy.
- The matter we encounter in our daily lives has positive energy. It is possible to create exotic matter in a lab, but only in extremely small quantities and for a very short amount of time.
- Another major issue is that we don't know how to construct them in the first place. All existing theoretical research into them simply assumes that they have already been constructed somehow.

Time travel

- Hypothetically, it could be possible to curve spacetime in a way that enables time travel. This might be possible using a wormhole, a warp drive, or even more exotic types of spacetime curvature.
  - “Time travel” refers specifically to travel to the past. Travel to the future is easy (you’re doing it right now), can be accelerated using time dilation, and doesn’t lead to any weird consequences.
- Time travel is defined as travel along a closed timelike curve.
  - “Curve”: A geodesic that follows the curvature of spacetime.
  - “Timelike”: The kind of geodesic that massive objects follow.
  - “Closed”: The curve forms a closed loop, so you end up at the same place in spacetime (so in particular, the same time) you started at.

Time travel

- It is currently unknown whether time travel is possible in our universe.
- According to Stephen Hawking's chronology protection conjecture, the laws of physics should prevent time travel from being possible.
- For example, even if wormholes could be used for FTL travel, quantum effects will still destroy any wormhole if we attempt to convert it into a time machine.
- However, this conjecture has not yet been proven, except in some special cases.

Time travel

- Another issue with time travel is that it seems to lead to time travel paradoxes.
- For example, consider the famous grandfather paradox:
  - A time traveler goes to the past and kills their grandfather before he met their grandmother.
  - This means the traveler will never be born.
  - But if they are never born, then they cannot travel to the past and kill their grandfather.
  - Therefore, the traveler will be born.
  - In other words, the traveler is born if and only if they are not born. This is a paradox!

Time travel

- However, time travel paradoxes do not necessarily forbid time travel, because they can hypothetically be resolved.
- The Novikov self-consistency conjecture says that it is impossible to change the past. Any attempt to do so will necessarily fail.
- For example, while trying to kill their grandfather, the traveler's gun jams, no matter how many times they try.
- Perhaps the traveler even accidentally causes their grandfather to meet their grandmother as a result of their failed attempts!

Time travel

- Another possible resolution of time travel paradoxes is parallel timelines.
- When the traveler arrives in the past, they actually cause the timeline to branch into two: the old timeline and a new timeline.
- The traveler successfully kills their grandfather in the new timeline. However, the old timeline remains unchanged.
- The traveler will never be born in the new timeline, but that’s perfectly consistent, since they came from the old timeline. Therefore, there is no paradox.

Time travel

- In the last few years, one of my main research topics has been time travel paradoxes and their resolutions.
- You can find more information about my research by reading my papers and listening to my talks and interviews, all of which are available on my website!

https://baraksh.com/

Dark matter

- Let us consider the way that galaxies rotate.
- According to Kepler's second law, stars farther away from the center of the galaxy should rotate slower.
  - This is similar to how, in our solar system, planets farther away from the Sun rotate slower.
- However, in reality, this does not seem to be the case. In most galaxies, stars rotate at roughly the same speed regardless of their distance from the center.
- This can be seen by looking at galaxy rotation curves: plots of rotation speed as a function of distance from the center.

An example of a galaxy rotation curve. A (dashed line): expected rotation velocity as a function of distance. B (solid line): actual observed velocity.
Credits: PhilHibbs (Wikipedia)

Video

- In this video, we see two different galaxy rotation curves.
- On the left is the one predicted from Kepler's laws, and on the right is the one we actually observe.
- The video can be found at this URL:

https://en.wikipedia.org/wiki/File:Galaxy_rotation_under_the_influence_of_dark_matter.ogv

Dark matter

- There are two ways to resolve this discrepancy.
- First, if general relativity is correct, then there must be some additional mass in the galaxy that we just can't see, and this mass affects the rotation curves.
- This additional mass is called dark matter, because we can't see it, so it's "dark". More precisely, this matter does not interact with light: it does not absorb, emit, or reflect it.
- Dark matter does not interact with electromagnetism, but it does interact with gravity, and possibly also with the weak interaction.

Dark matter

- However, another possible explanation is that general relativity is not correct, and should be modified in some way to account for the discrepancy.
- This approach is called modified gravity, but most physicists and astronomers don’t like it, because general relativity has been successfully tested in thousands of different experiments, so it’s hard to modify it and still be compatible with all these experiments.
- On the other hand, dark matter has never been detected directly, despite numerous experiments attempting to do so.

Cosmology

- Our final topic for this lecture (and the entire course) is cosmology: the study of the universe as a whole.
- One of the most important contributions of Einstein’s theory of relativity is that it allowed us to understand how our universe evolved over time.
- I will only go over the basics of cosmology in this course. Students who are interested in learning more should consider taking my course ASTR 2P42, where we will learn about cosmology in depth.

Cosmology

- Our universe is thought to have began ~13.8 billion years ago in an event called the Big Bang.
- Initially, the universe was in a state of extremely high density and temperature.
- In fact, general relativity predicts that the initial state was a singularity – a state of infinite density and temperature.
- However, as in the case of singularities inside black holes, most theoretical physicists think this singularity is just a place where the theory breaks down, not an actual physical phenomenon.

Cosmology

- The Big Bang was not an "explosion". Instead, ~13.8 billion years ago, the universe began to expand.
- As it expanded, the density and temperature decreased, until eventually it was possible for atoms to form, and later, galaxies, stars, planets, and everything else.
- It's important to understand that the universe isn't expanding "into" anything!
- When we say that the universe expands, what we actually mean is that the curvature of spacetime increases, so that the distances between galaxies become longer with time.

Cosmology

- This expansion can be observed in the form of Hubble's law: the observation that galaxies are moving away from us at speeds proportional to the distance from us.
- In equation form:

$$v = H_0 D$$

  - v is the recessional velocity of the galaxy (the speed at which it is moving away from us)
  - H_0 is called the Hubble parameter, and is a proportionality constant determined experimentally.
  - D is the distance to the galaxy.

Cosmology

- Hubble's law is observed from the fact that the redshift of a galaxy is larger for galaxies that are farther away.
- Remember that according to the Doppler effect, when a source of light moves away from us, its spectrum undergoes redshift.
- Since we see all galaxies being redshifted, we conclude that they are all moving away from us. The only possible explanation for this is that space between us and these galaxies is expanding.
- This also means that aliens on any other galaxy see the exact same thing: all other galaxies (including ours) are receding away from them. We are not special!

Cosmology

- Since the 1990s, we know that the universe is not expanding at a constant rate: the expansion is actually accelerating.
- While expansion at a constant rate does not require energy, accelerating the expansion does require energy, called dark energy.
- This is most likely just vacuum energy, that is, energy that the vacuum of space has even with no matter in it.
- The amount of vacuum energy is given by the cosmological constant, a fundamental constant in general relativity, first introduced by Einstein.

Cosmology

- The cosmic microwave background (CMB) is an almost uniform microwave radiation that exists everywhere in space.
- It was discovered in 1965 by accident, but provides important evidence for the Big Bang.
- In the beginning, the universe was so hot and dense that it was opaque, and no light could travel through it.
- However, as the universe cooled and became less dense, protons and electrons combined to form atoms, and light was finally able to pass through – the universe became transparent.

Cosmology

- This happened ~380,000 years after the Big Bang, and the CMB was released at that time.
- This is known as the time of last scattering, or the period of recombination (combination of protons and electrons into atoms) or decoupling.
- Due to the CMB, the universe is not at absolute zero; its temperature is actually ~2.7 K.

A timeline of the universe from the Big Bang until today.
Credits: NASA / WMAP Science Team

Conclusions

- In this lecture we learned about Einstein's theory of relativity and its many interesting and counter-intuitive consequences.
- We also learned how this theory revolutionized astronomy in many different ways.

- <u>Bonus Reading:</u> OpenStax Astronomy, chapters 24-30. (However, not all the material is those chapters was covered in this lecture, and much of the material covered is not in the textbook!)
- <u>Exercises:</u> Practice questions are available in the textbook and on the course website.

What’s next?

- If you enjoyed this course, you may also be interested in my 2nd-year astronomy course, ASTR 2P42: Astrophysics & Cosmology.
- This is a more advanced course, which delves much deeper into the material, including relevant math and physics.
- It therefore requires 1st-year physics and calculus as mandatory prerequisites.
- Please see the course website for more information:

  https://baraksh.com/2P42/

- I hope to see you there! 🥹

Thanks for taking
ASTR 1P01 and 1P02!

- Prof. Shoshany