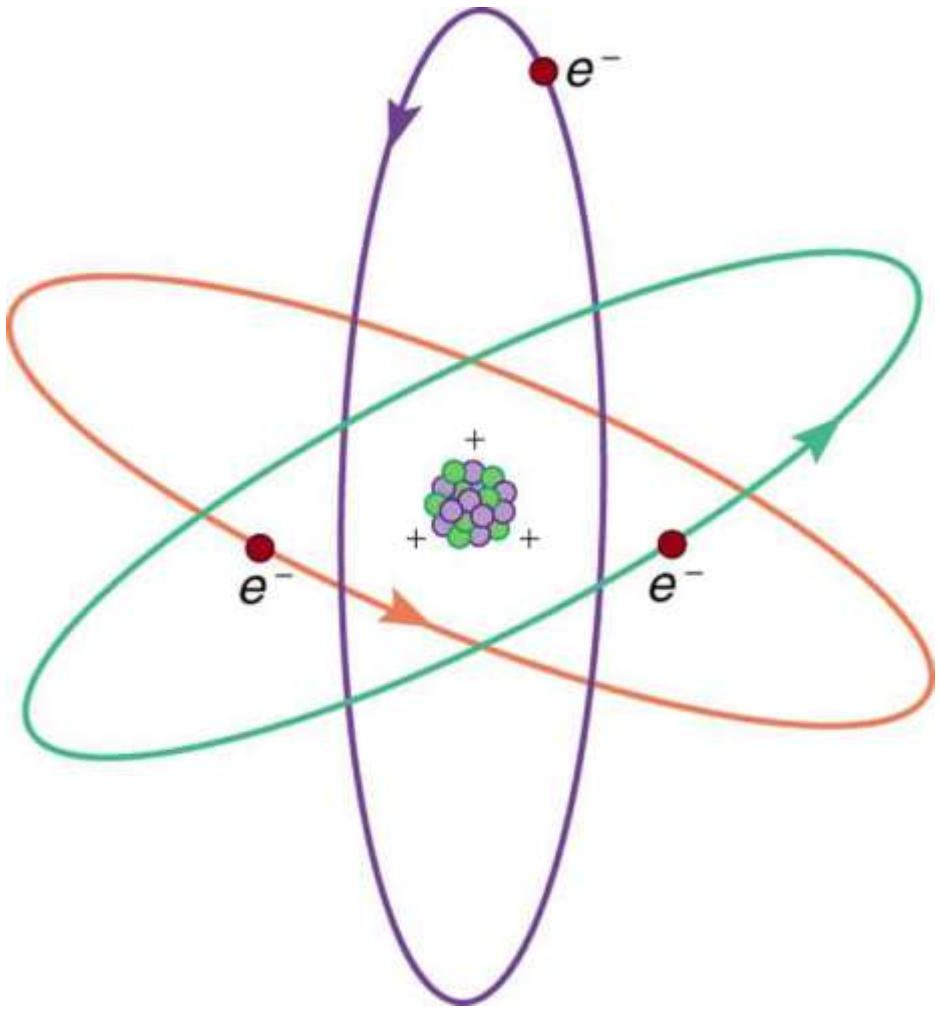
PHYS 1P22/92

Prof. Barak Shoshany

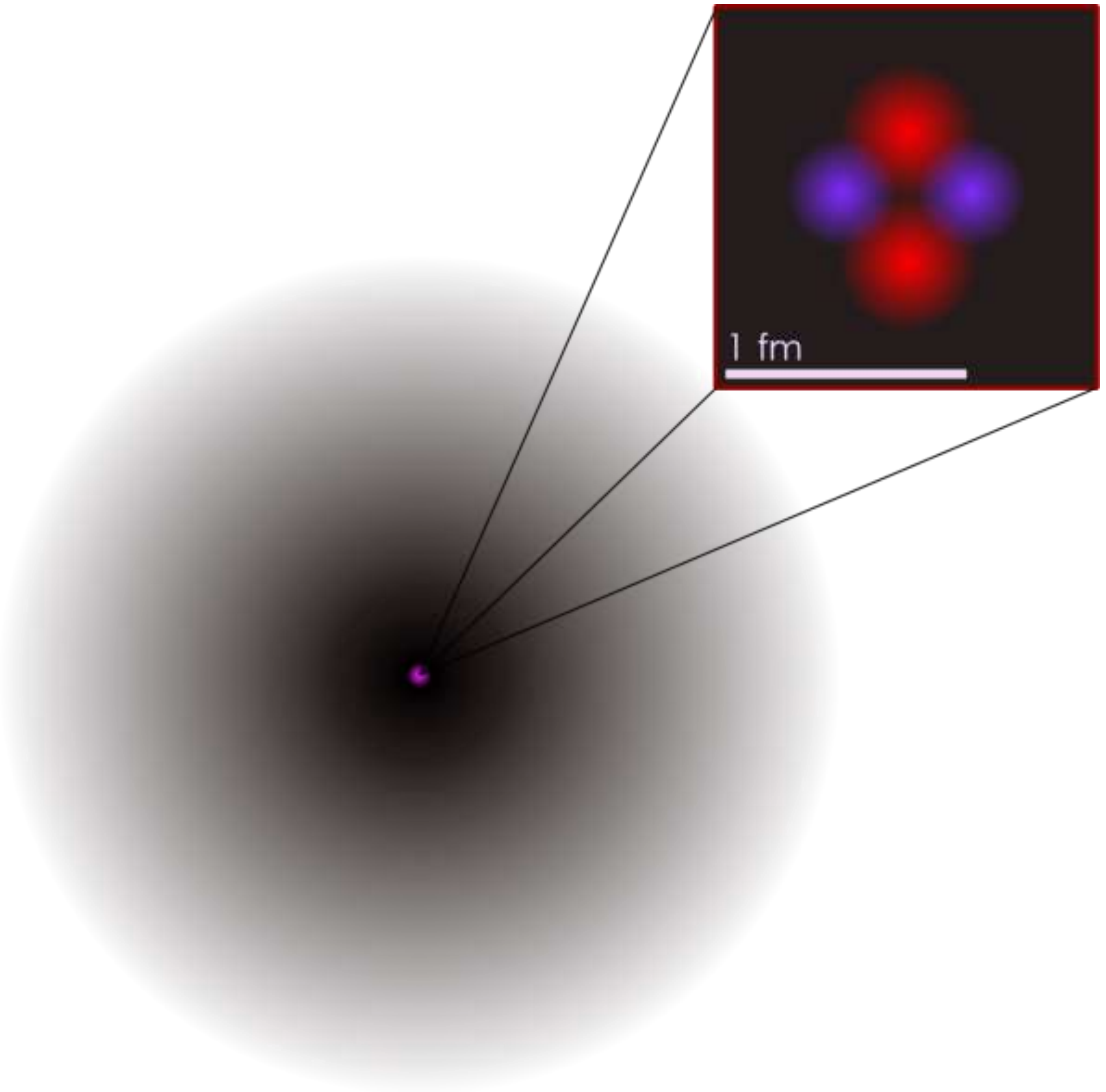
Spring 2024

18. Electric Charge and Electric Field

18.1 Static Electricity and Charge: Conservation of Charge



Planetary model of the atom (incorrect)



Orbital model of the atom (correct)

Electric charge

- SI derived unit: coulomb (C)
  - In SI base units: $\mathrm{C} \equiv \mathrm{A} \cdot \mathrm{s}$ (A = ampere, see ch. 20)
- Elementary charge (defined exactly):

$$e \equiv 1.602\,176\,634 \times 10^{-19}\ \mathrm{C}$$

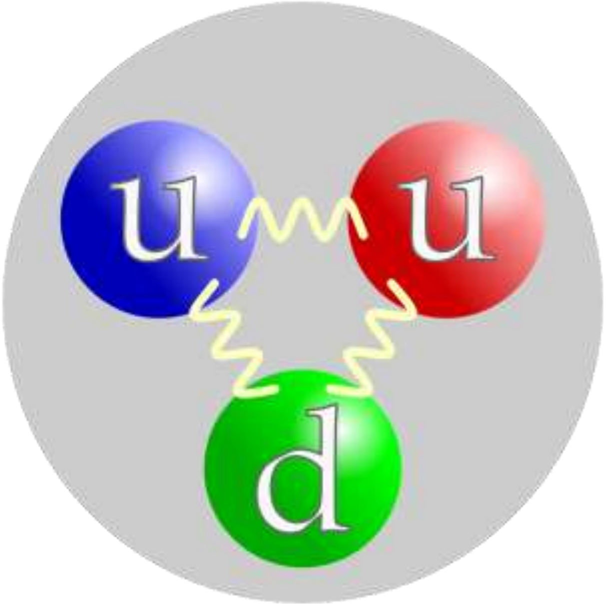
$$1\ \mathrm{C} \equiv \frac{1}{1.602\,176\,634 \times 10^{-19}}\ e \approx 6.24 \times 10^{18}\ e$$

- Electric charges of subatomic particles:
  - Proton: $+e$
  - Electron: $-e$
  - Neutron: 0

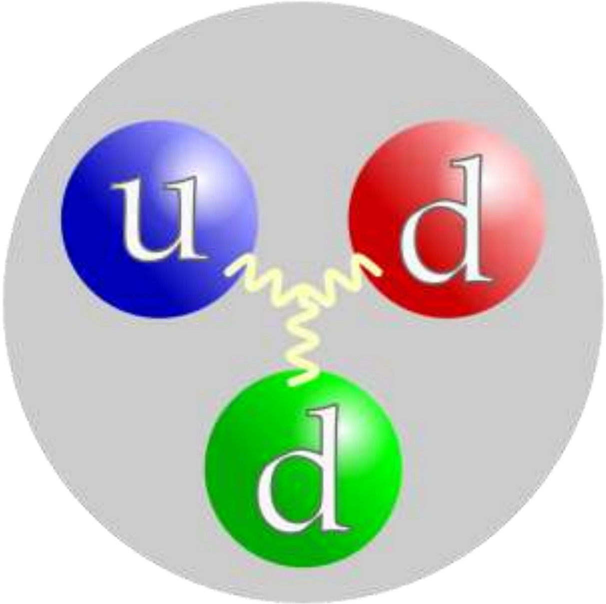
Up quark: $+2/3\ e$

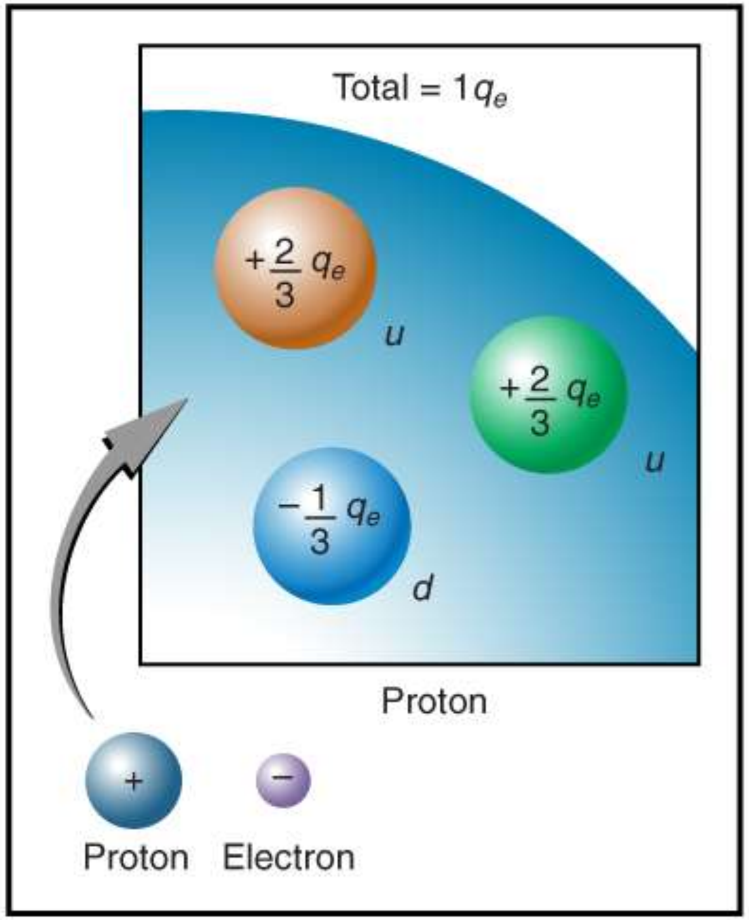
Down quark: $-1/3\ e$

Proton



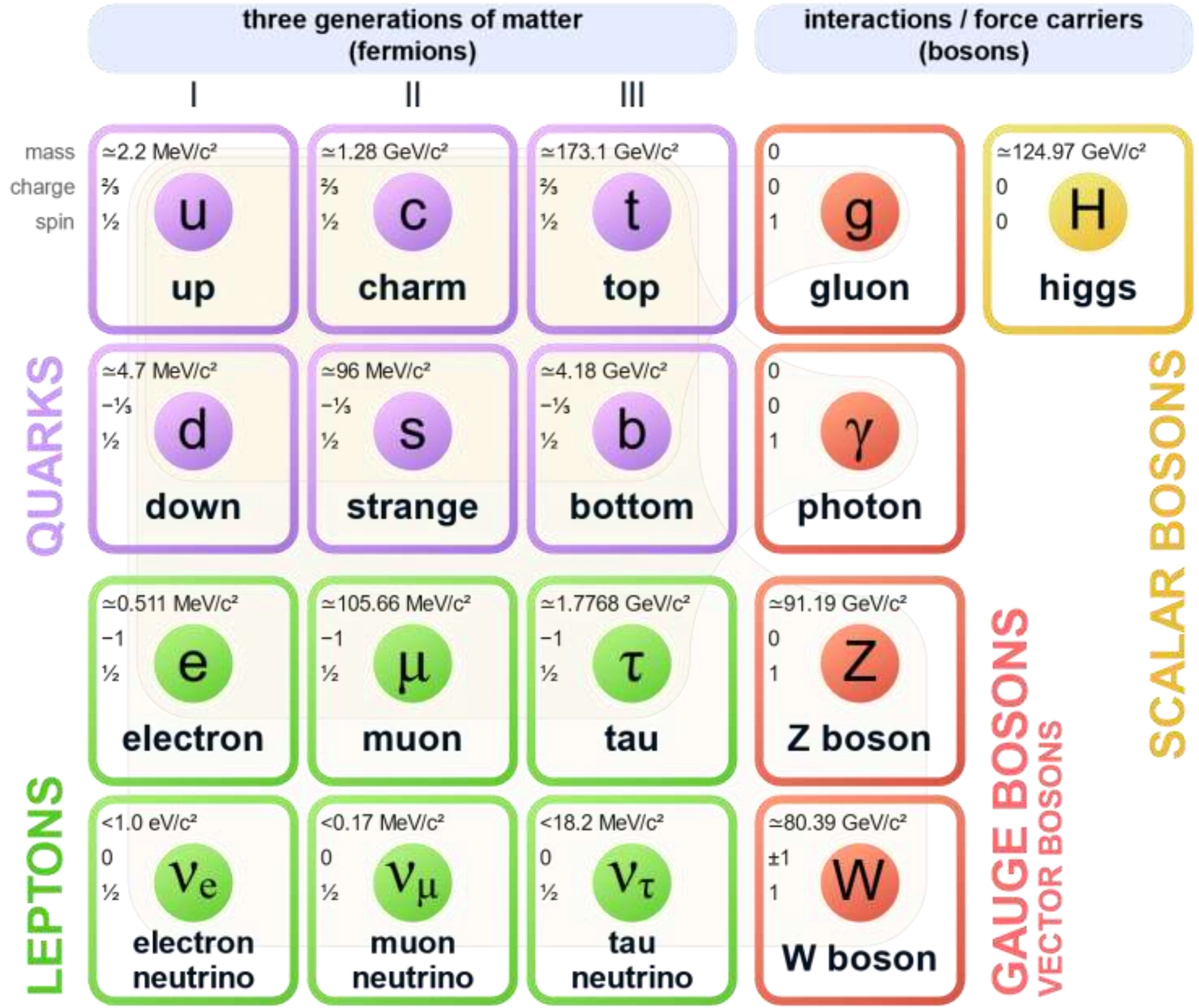
Neutron



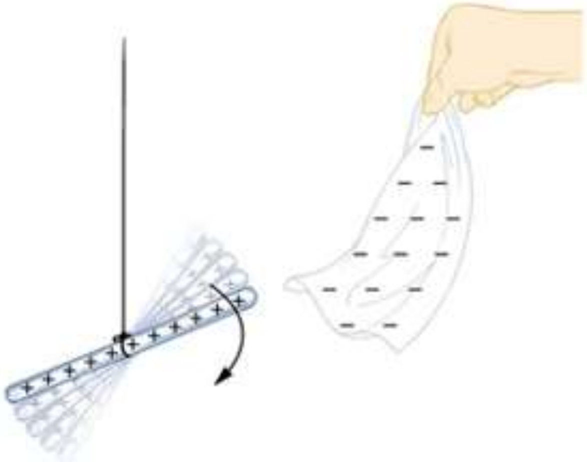


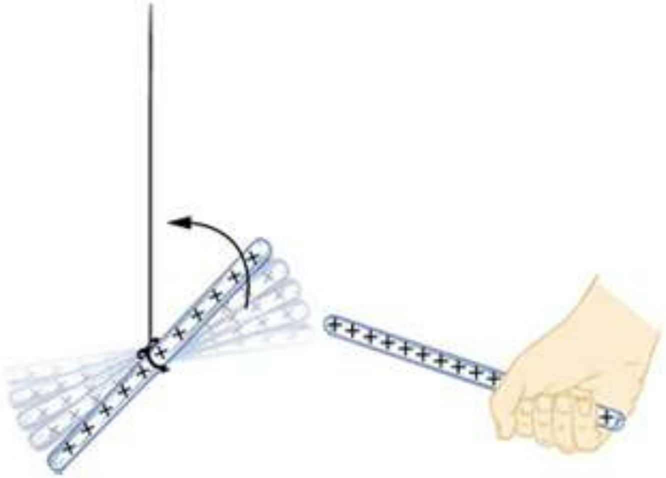
(In the book: $q_e \equiv e$)

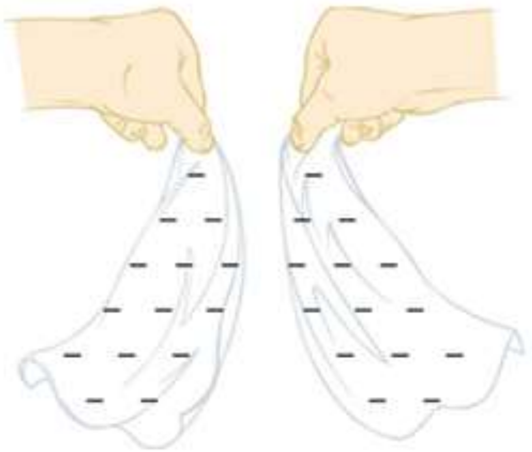
Standard Model of Elementary Particles



Static electricity





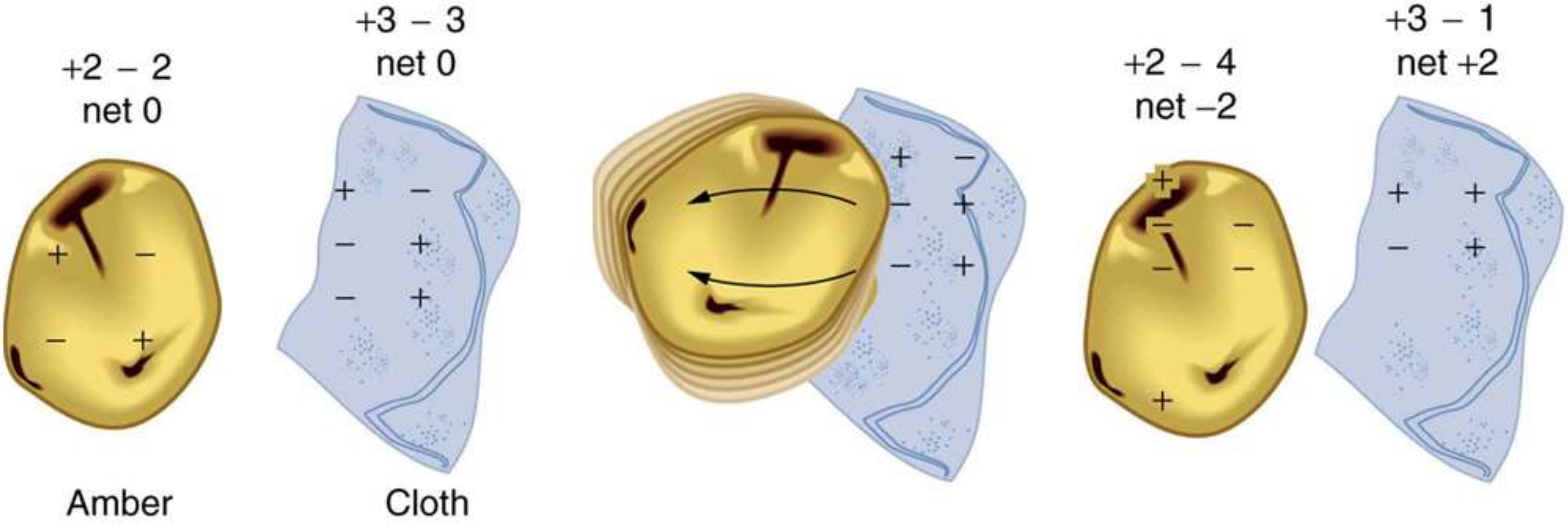


After rubbing
Glass rod: +
Silk: −

Like charges repel

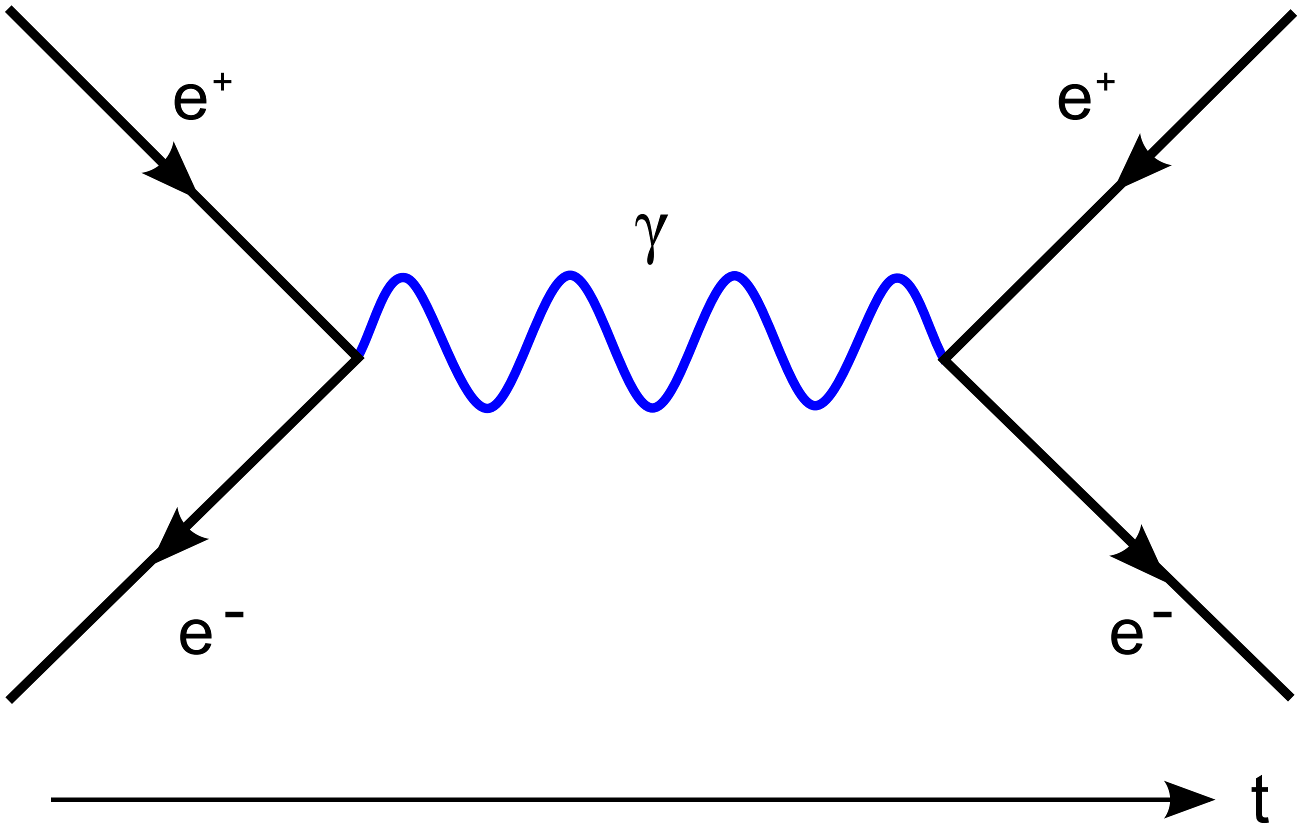

HAGEN

Conservation of charge



Relativity: $E = mc^2$

Mass is **not** conserved, but energy and charge are.



Pop quiz: what happened to the child's hair?

Pop quiz: what happened to the child's hair?

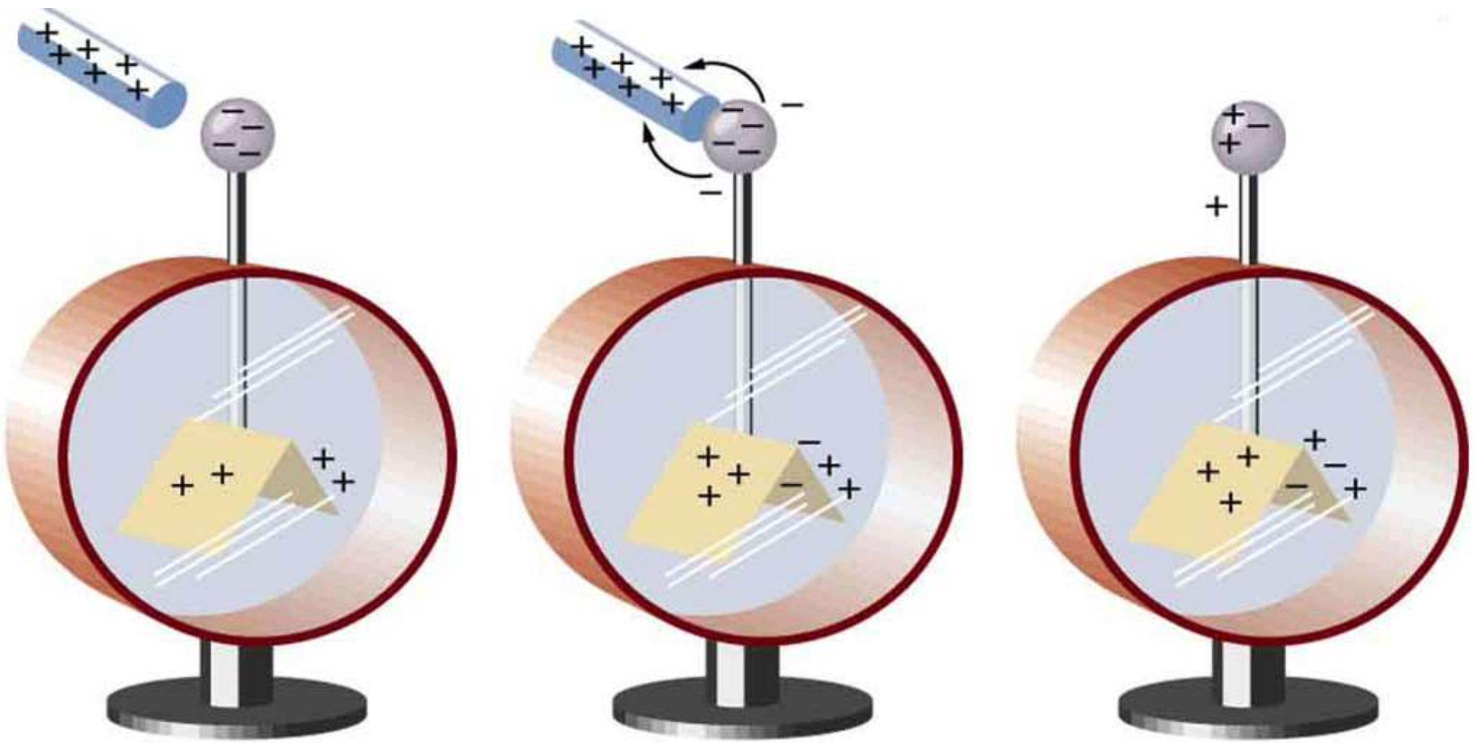
Answer: Going down the slide caused static charges to accumulate. The excess charges in each hair repel the excess charges in other hairs.

18.2 Conductors and Insulators

- Conductor = charges can move freely within the material.
- Usually electrons that are not bound to atoms.
  - Example: metals (silver, copper, aluminum).
- Ion = atom or molecule with # of electrons different from # of protons, resulting in non-zero charge.
- Moving charges can also be ions instead of electrons.
  - Example: salt water.
- Insulator: electrons and ions are bound and cannot move (easily).
  - Examples: rubber, plastic, pure water, dry salt.

Charging by contact

- No contact:
  - Electrons move to top
  - Net positive charge on gold leaves
  - Like charges repel, leaves separate
  - When rod removed, leaves return to normal. No charging took place.
- Contact:
  - Electrons transferred. Now leaf is charged.
- Glass rod removed:
  - Charge remains. Charges evenly distribute.



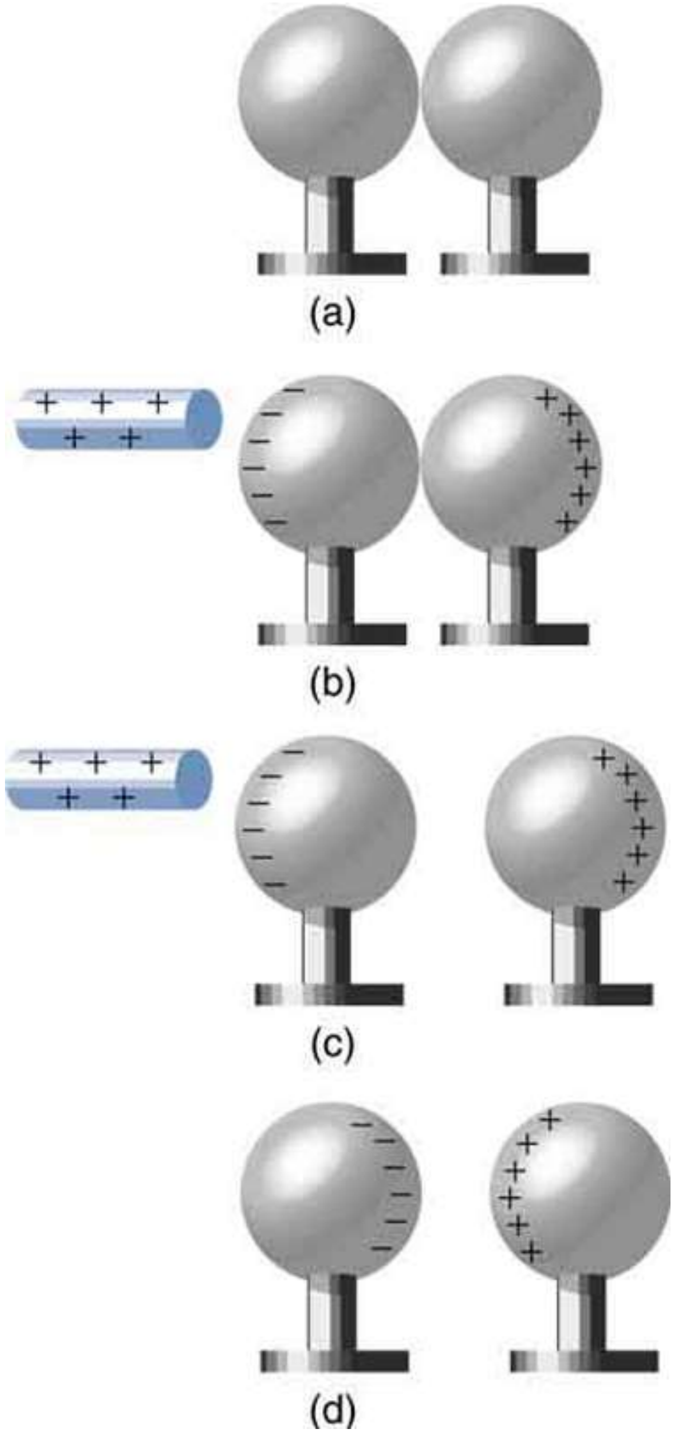
Charging by induction

(a) Two uncharged metal spheres in contact.

(b) Positively charged glass rod brought near (not touching).

(c) Spheres separated.

(d) Rod removed, spheres retain charge.



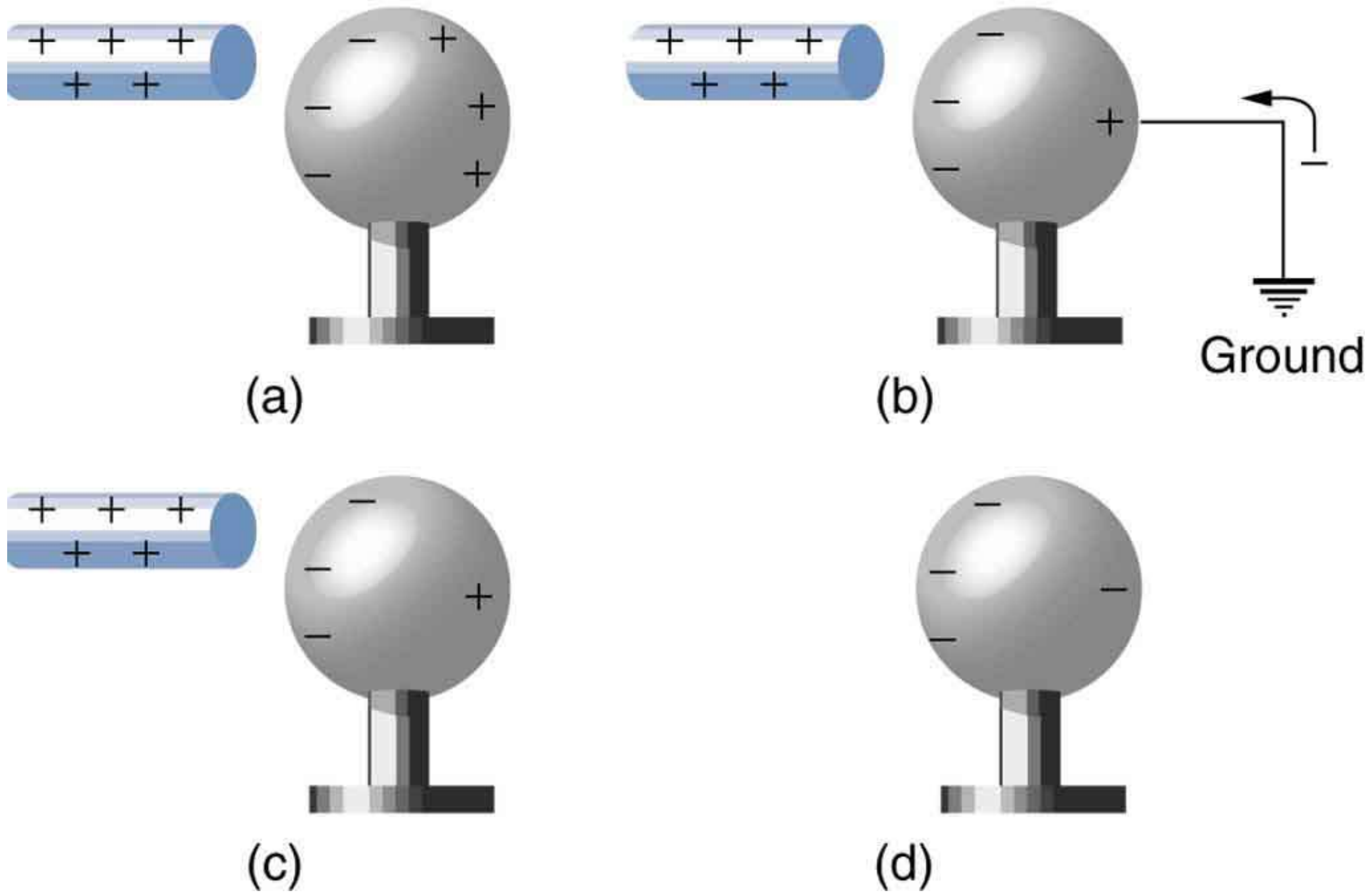
Charging by induction

(a) One uncharged sphere. Positive rod brought near. Charges polarize.

(b) Sphere grounded, electrons attracted from the ground.

(c) Ground connection broken.

(d) Rod removed, sphere retains charge.

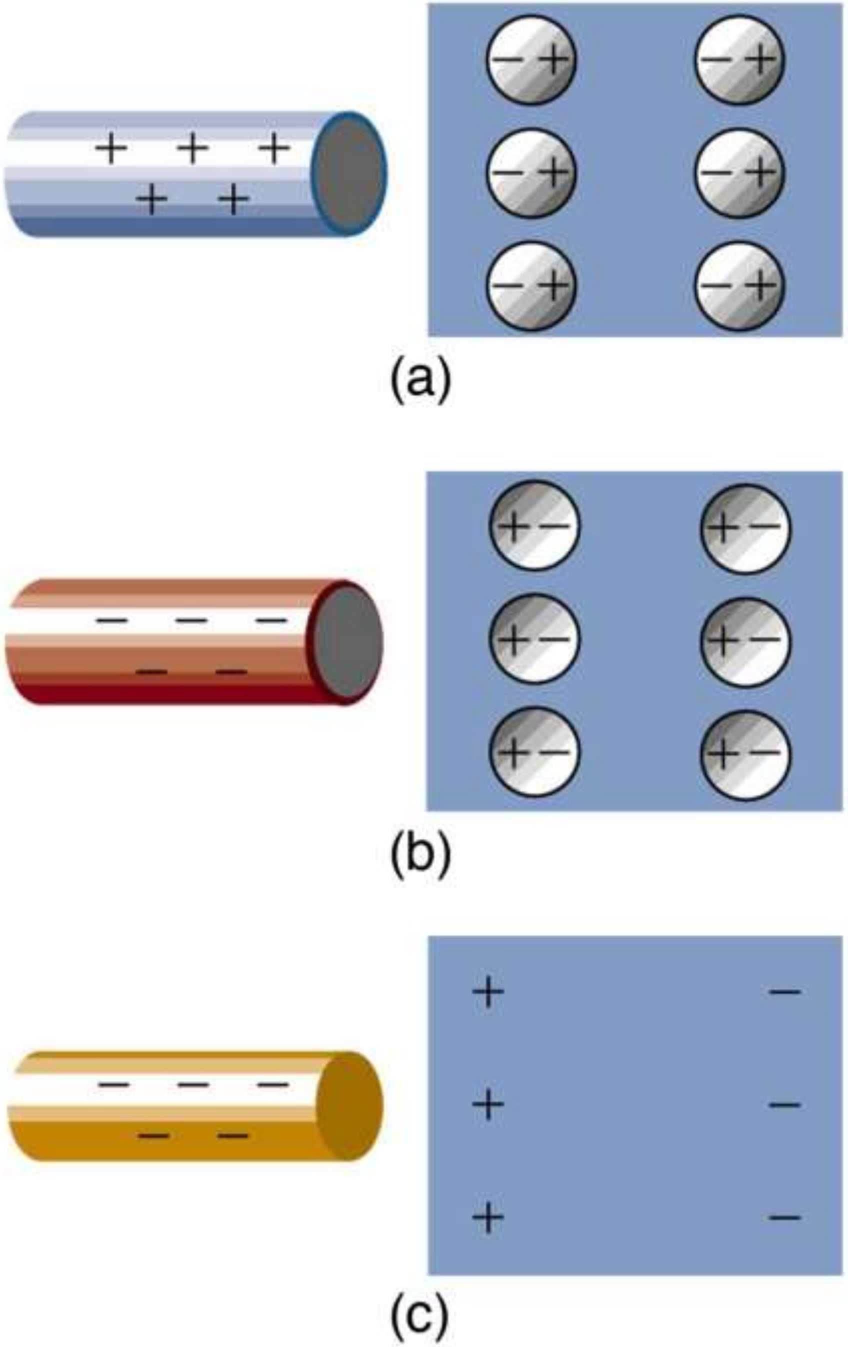


Polarization

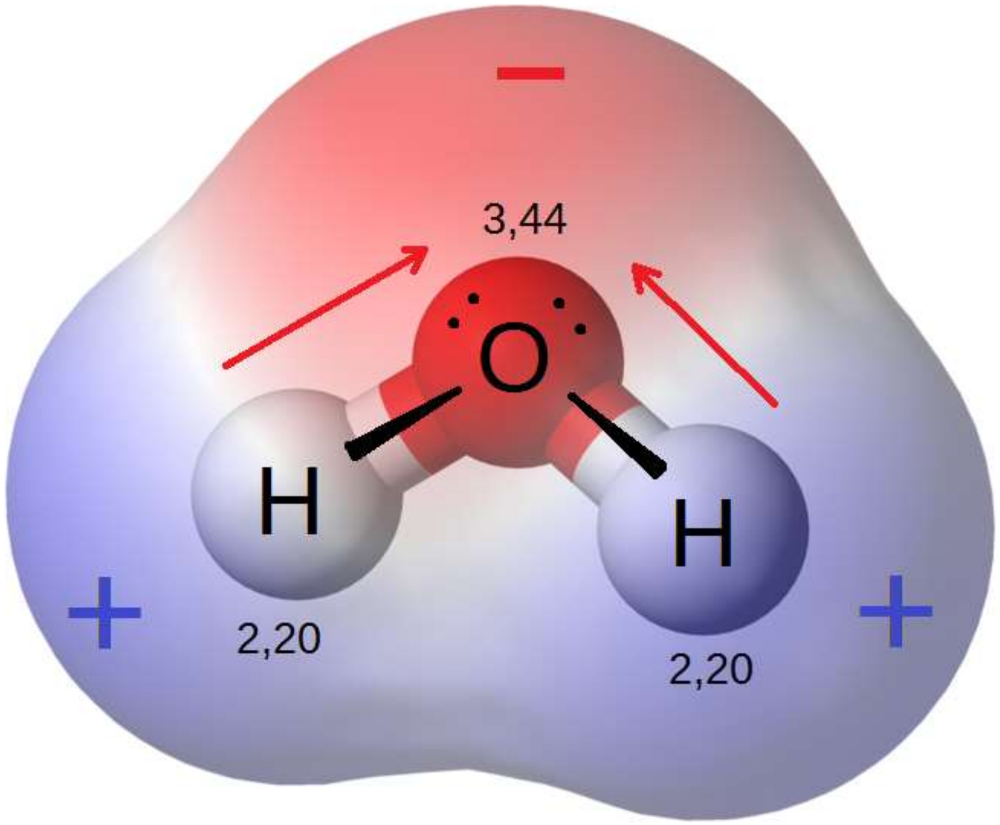
(a) Positive rod polarizes molecules in an insulator. Electrons within each molecule shift.

(b) Negative rod polarizes insulator in the opposite way.

(c) Negative rod near conductor polarizes the entire conductor.



Polar molecules



Video

- Video about static electricity and water:

  https://youtu.be/VhWQ-r1LYXY

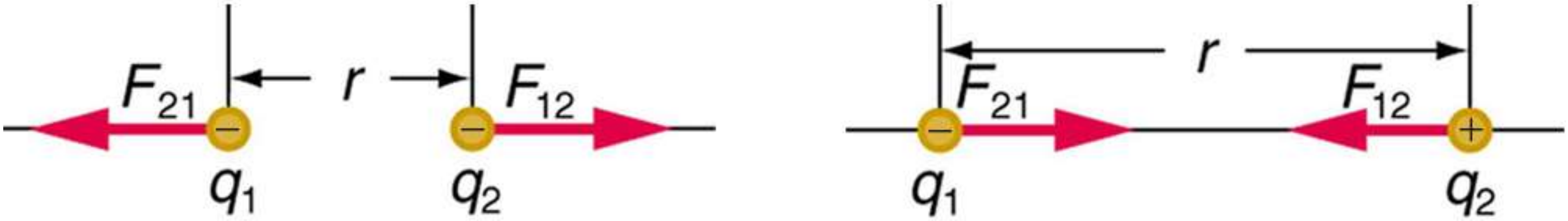
18.3 Coulomb's Law

- Coulomb's law:

$F \equiv |\mathbf{F}| = k\frac{|q_1 q_2|}{r^2}$ (force along the line between the two charges q_1, q_2)

- Opposite charges attract, like charges repel.
- Coulomb's constant:

$$k \approx 8.987\,551\,792\,3(14) \times 10^9\ \mathrm{N \cdot m^2/C^2}$$



- Where does Coulomb's constant come from?
- Fine-structure constant (experimentally determined):

$$\alpha \approx 7.297\,352\,5643(11) \times 10^{-3}$$

  - Note: Dimensionless! Does not depend on any system of units. So cannot be defined.
  - Quantifies the strength of the electromagnetic interaction.
- Vacuum permeability (defined in terms of α):

$$\mu_0 \equiv \frac{2\alpha h}{e^2 c} \approx 1.256\,637\,061\,27(20) \times 10^{-6}\ \mathrm{N \cdot A^{-2}}$$

  - h is Planck's constant (ch. 29), c is the speed of light, both are exactly defined.
  - Quantifies the strength of the magnetic field induced by an electric current (see ch. 22).
- Vacuum permittivity (defined in terms of μ_0):

$$\varepsilon_0 \equiv \frac{1}{\mu_0 c^2} = \frac{e^2}{2\alpha h c} \approx 8.854\,187\,8188(14) \times 10^{-12}\ \mathrm{F \cdot m^{-1}}$$

  - $\mathrm{F} \equiv \mathrm{kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2}$ is farad (see ch. 19).
  - Quantifies the electric field density "permitted" to form in response to electric charges.
- Coulomb's constant (defined in terms of ε_0):

$$k \equiv \frac{1}{4\pi\varepsilon_0} \approx 8.987\,551\,792\,3(14) \times 10^{9}\ \mathrm{N \cdot m^2 \cdot C^{-2}}$$

- **Problem:** Consider an electron and proton separated by 10^{-10} m. Compare the electrostatic force with the gravitational force.
- **Solution:**
  - Electrostatic force:

$$|\boldsymbol{F}| = k\frac{|q_e q_p|}{r^2} \approx 10^{-8}\ \mathrm{N}$$

  - Gravitational force (always attractive), $m_e \approx 10^{-30}$ kg, $m_p \approx 10^{-27}$ kg:

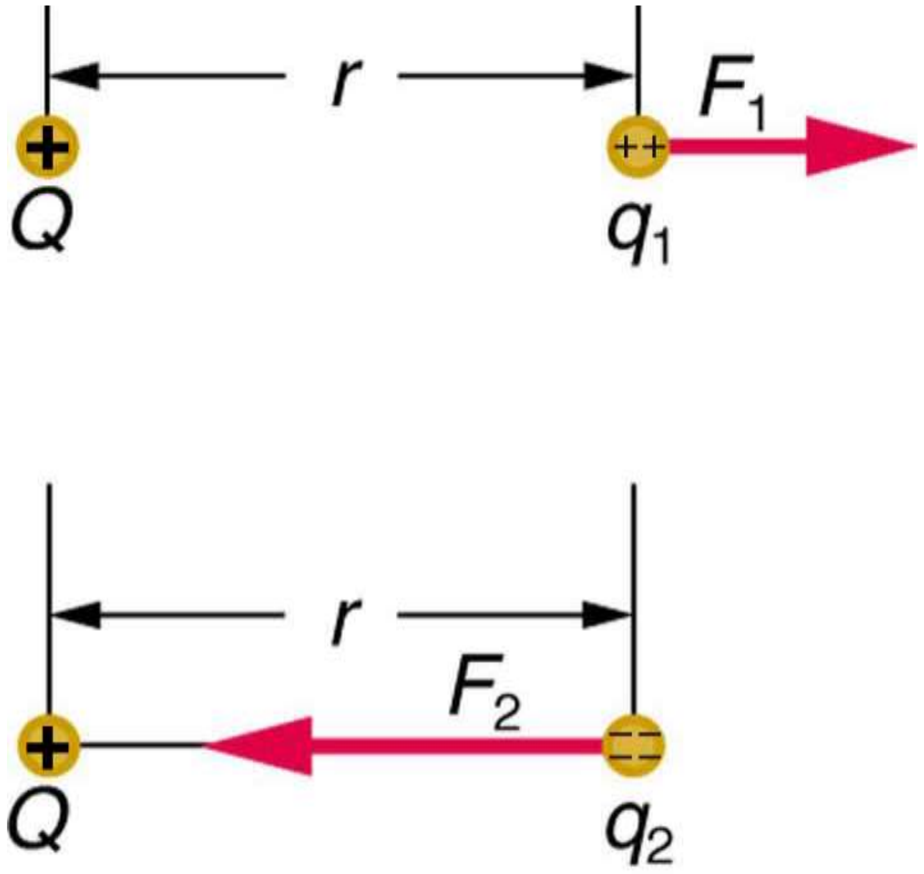
$$|\boldsymbol{F}| = G\frac{|m_e m_p|}{r^2} \approx 10^{-47}\ \mathrm{N}$$

- Difference: 39 orders of magnitude!

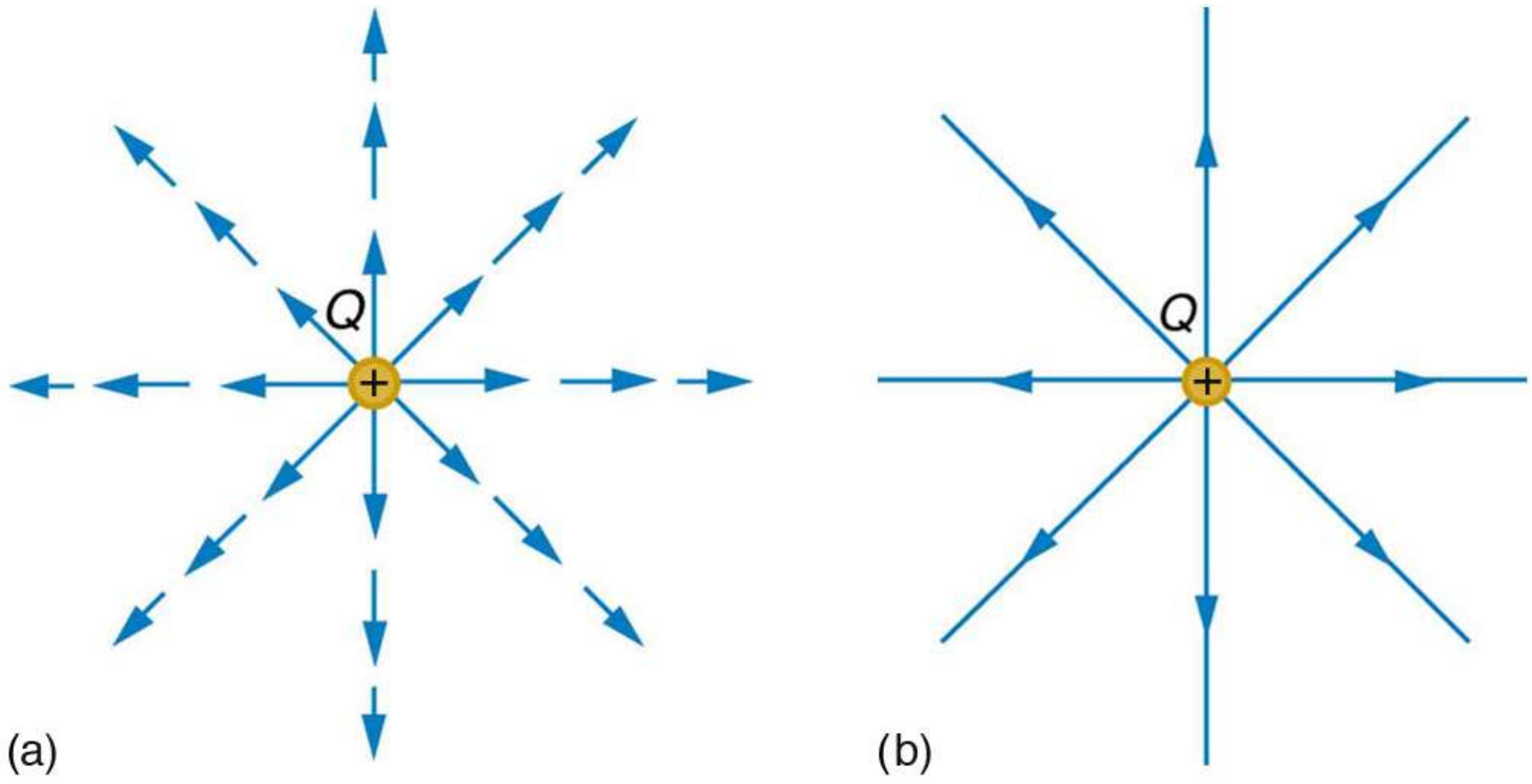
18.4 Electric Field: Concept of a Field Revisited

Definition of electric field

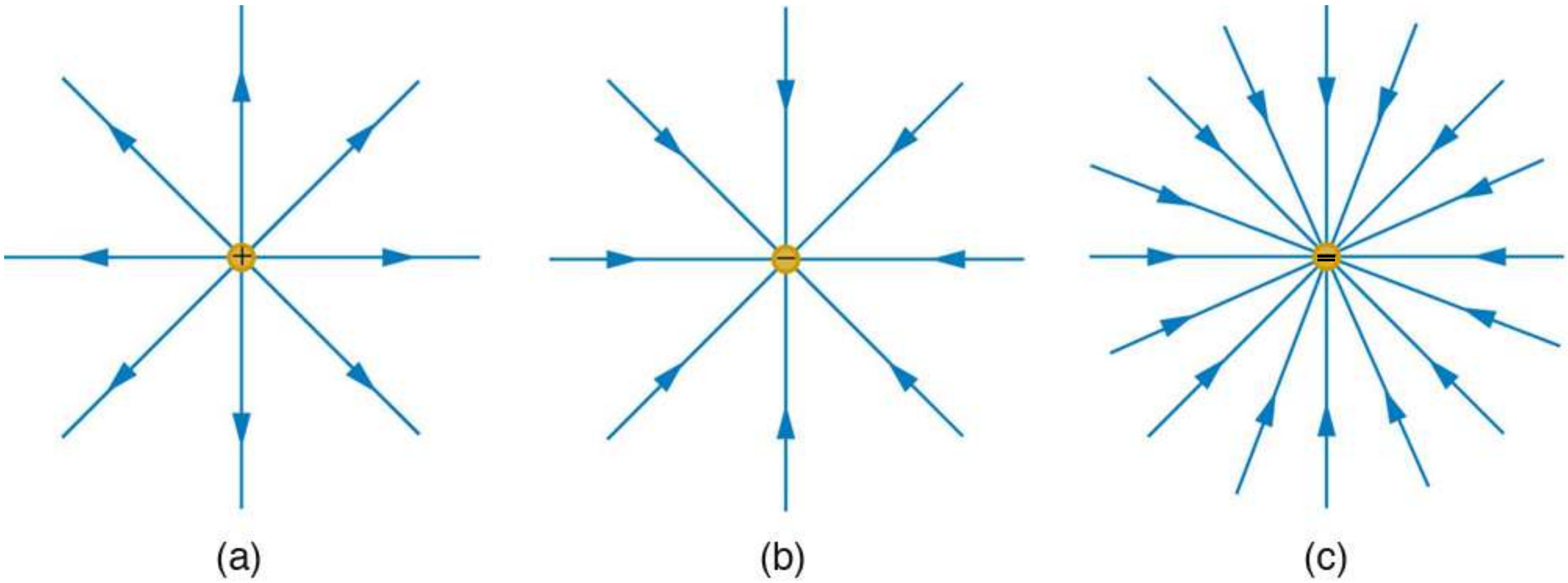
- Electric field is force per unit charge:
$$\boldsymbol{E} \equiv \frac{\boldsymbol{F}}{q}$$
- q is a "test charge". Force is acting on this charge. Size or sign of q don't matter.
- For example, point charge Q, test charge q:
$$|\boldsymbol{F}| = k\frac{|qQ|}{r^2}$$
$$|\boldsymbol{E}| = \frac{|\boldsymbol{F}|}{q} = k\frac{|Q|}{r^2}$$
- Force on charge in an electric field:
$$\boldsymbol{F} = q\boldsymbol{E}$$



18.5 Electric Field Lines: Multiple Charges

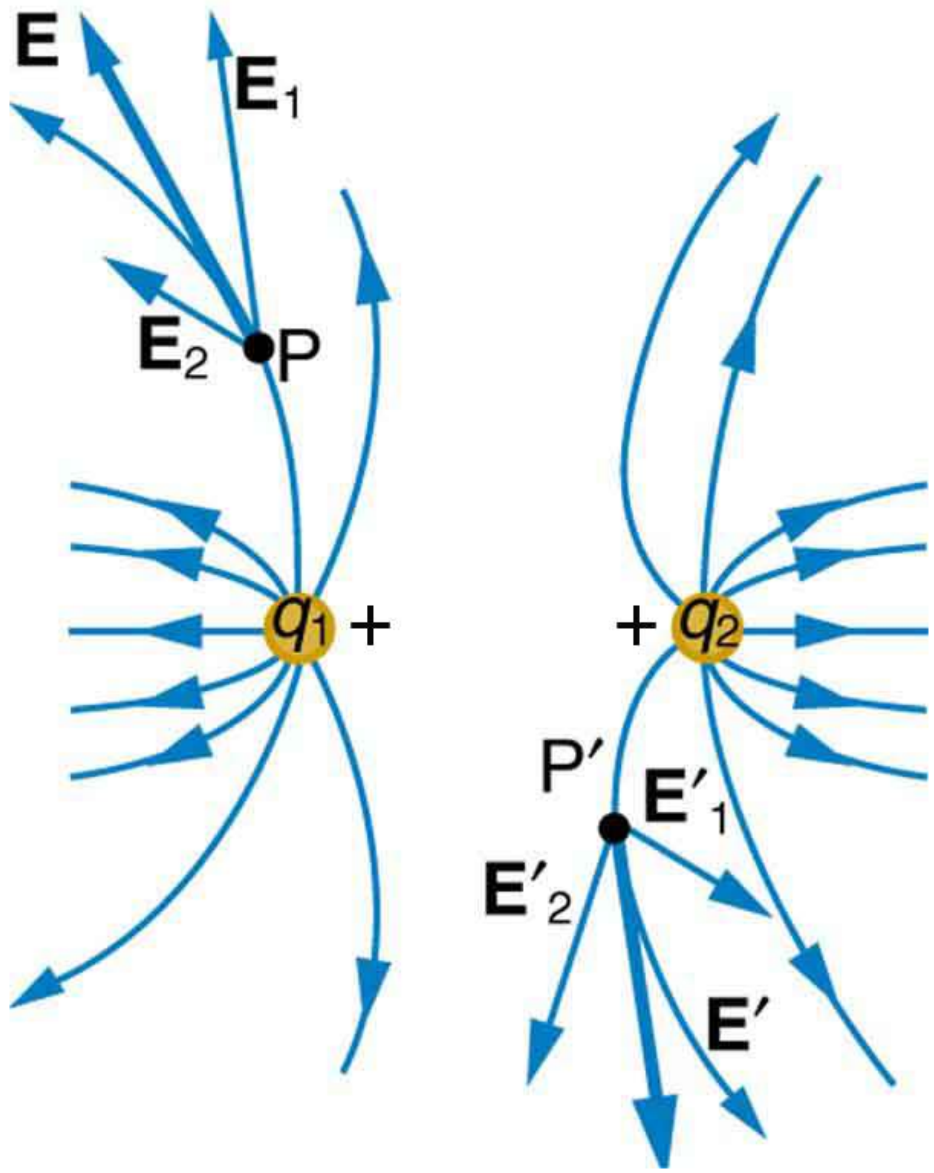


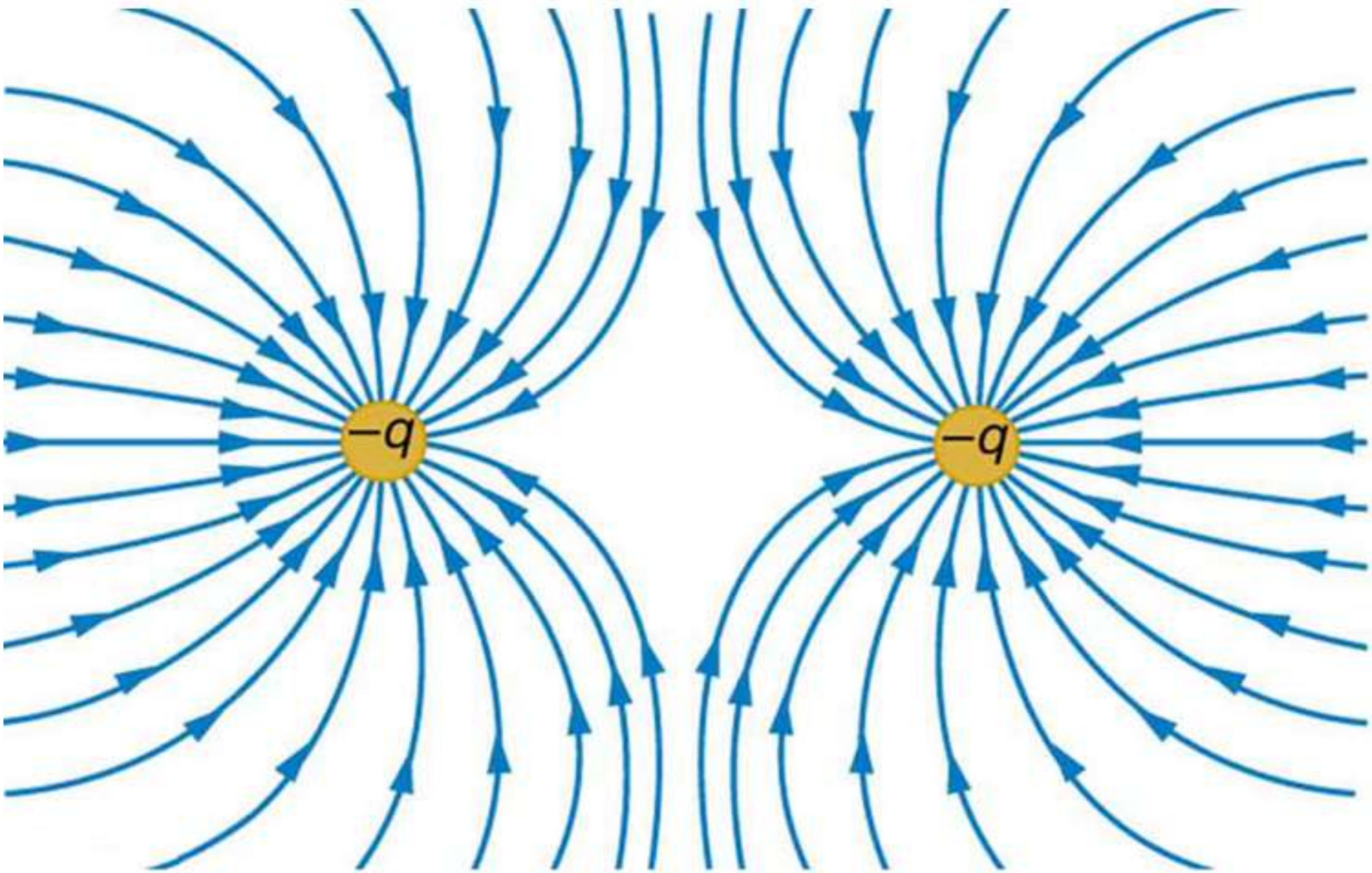
Replace arrows representing the electric field vector (a) with continuous field lines (b).



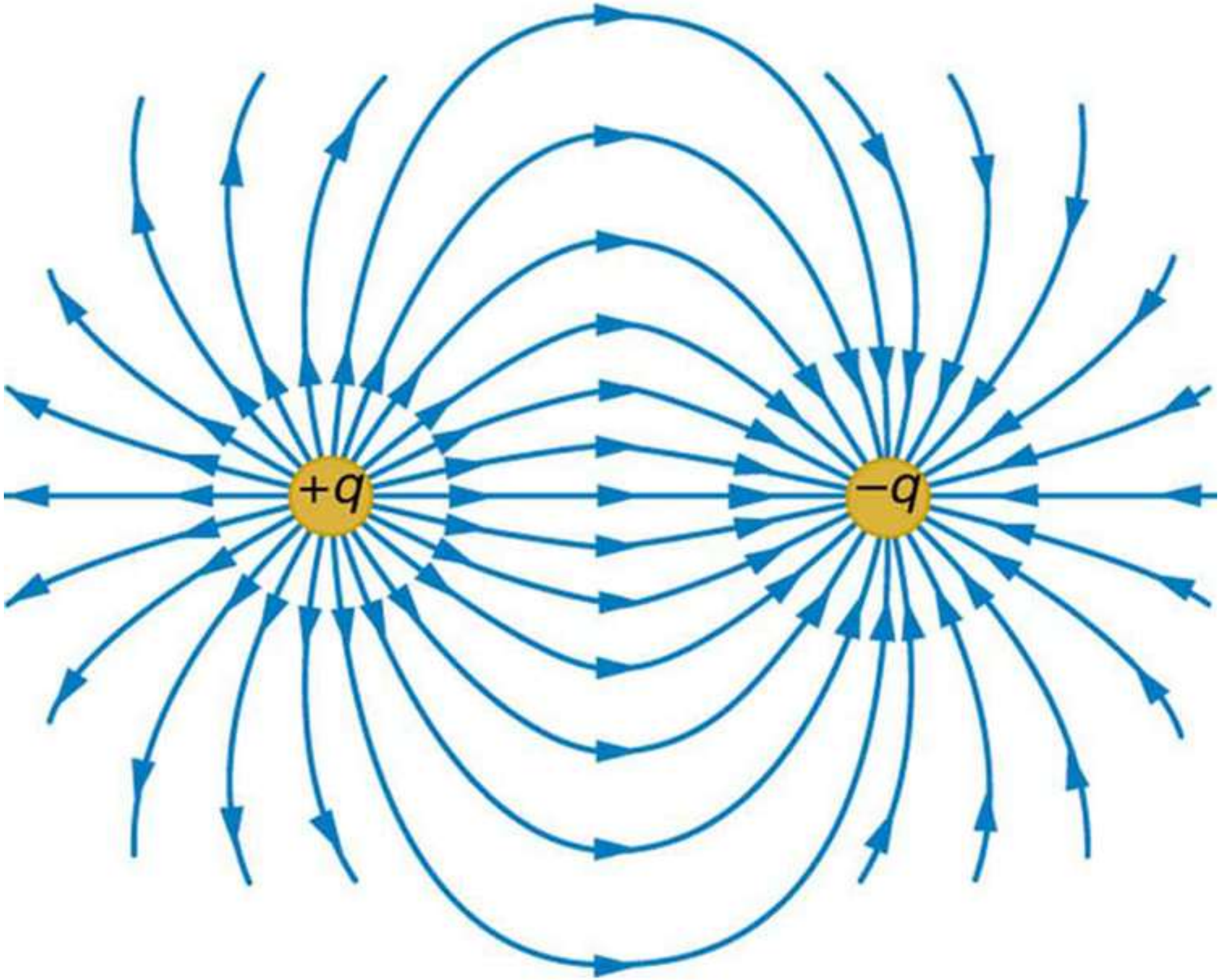
(a) Positive charge.
(b) Negative charge of same magnitude.
(c) Negative charge of larger magnitude.
Magnitude is proportional to density of field lines.

More than one charge: need to add the electric field vectors.





Two negative charges.



Two opposite charges.

Rules for electric field lines

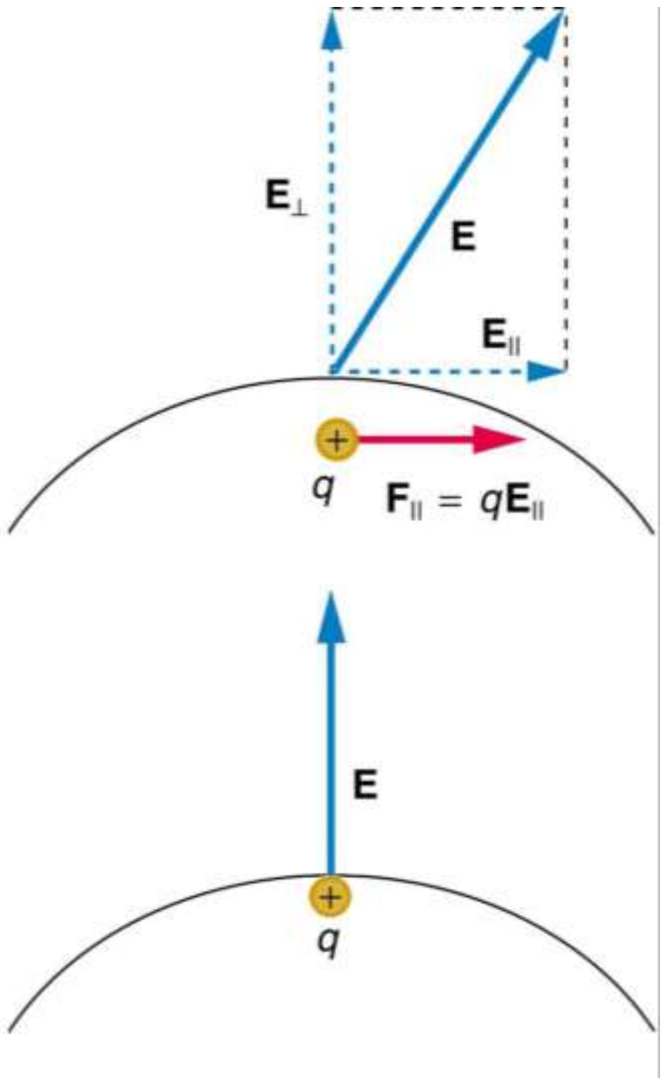
1. Field lines must begin on positive charges or infinity and terminate on negative charges or infinity.
2. The number of field lines connected to a charge is proportional to the magnitude of the charge.
3. The field strength is proportional to the density of the lines.
4. The field direction is tangent to the field line at any point.
5. Field lines can never cross.
   - Pop quiz: why?
   - Answer: The vector field **E** is a **vector** at each point. A vector can only point in one direction. The field lines must point in that direction.

(Skipping section 18.6)

18.7 Conductors and Electric Fields in Static Equilibrium

Electrostatic equilibrium

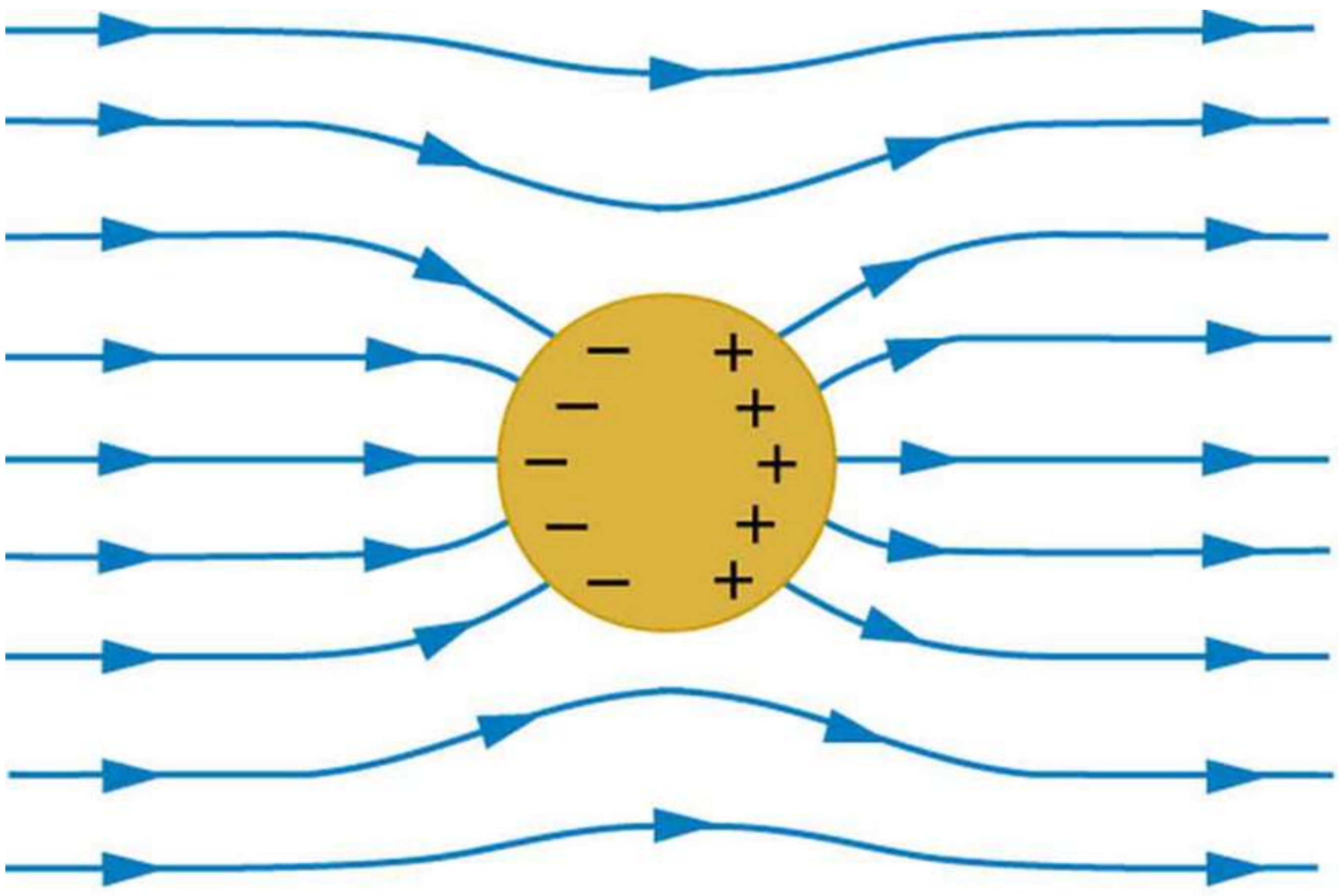
- Conductor in electric field: parallel field components $\mathbf{E}_{\parallel}$ will move the free charges.
- When the field is perpendicular to the surface, there are no parallel components, so it will stop moving.
- The charges will reach electrostatic equilibrium.



Conductor polarized by electric field

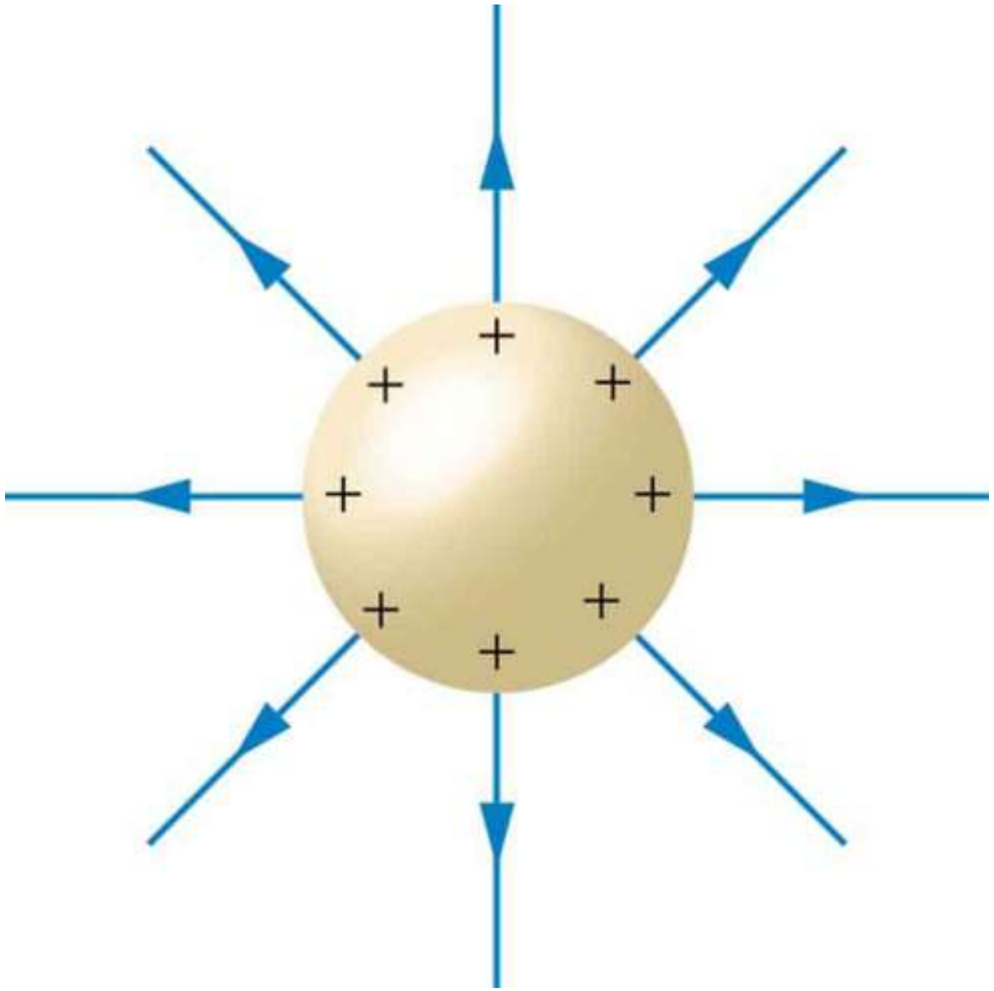
The field **E** was originally uniform (straight lines).

All field lines are now perpendicular to the surface.



Electric field vanishes inside a conductor

- The charges will distribute uniformly on the surface of the conductor until the electric field inside vanishes.
- There are no excess charges inside, only on the surface.
- The electric field outside is identical to that of a point charge at the center of the conductor (for a spherical conductor).
- The field lines are perpendicular to the surface.

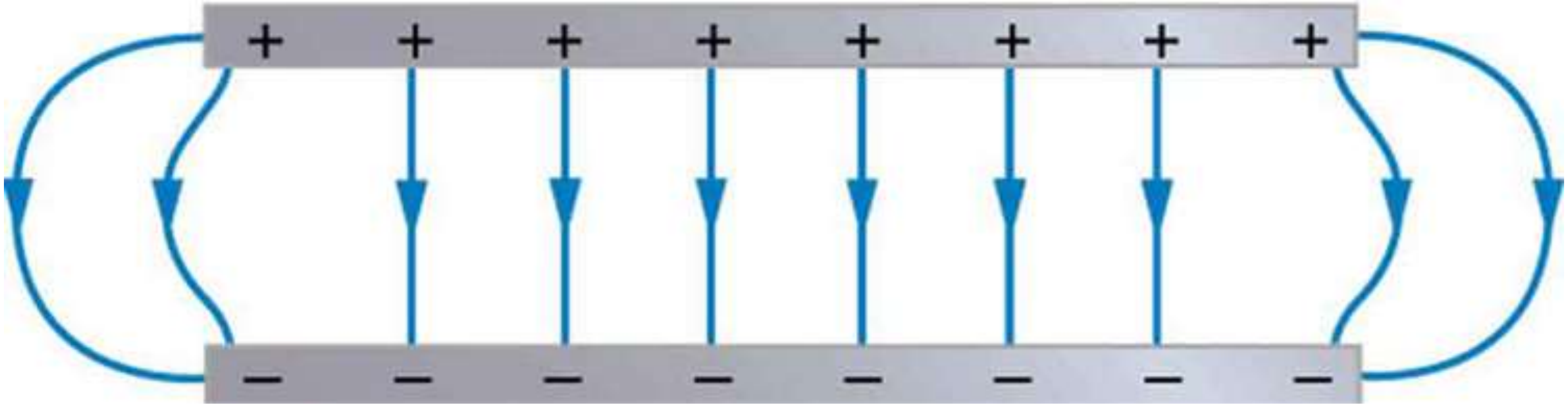


Properties of conductor in equilibrium

- Summary:
  - The electric field is zero inside a conductor.
  - Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
  - Any excess charge resides entirely on the surface or surfaces of a conductor.

Creating a uniform electric field

- Two metal plates with equal but opposite excess charges.
- The field is uniform, but not near the edges.



Electric field on uneven surface

(a) Forces **F** are identical, but parallel components $\mathbf{F}_{||}$ are not.

(b) Excess charges will concentrate on the sharpest points.

(c) Uncharged conductor becomes polarized in electric field.