PHYS 1P22/92

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20. Electric Current & Resistance

20.1 Current

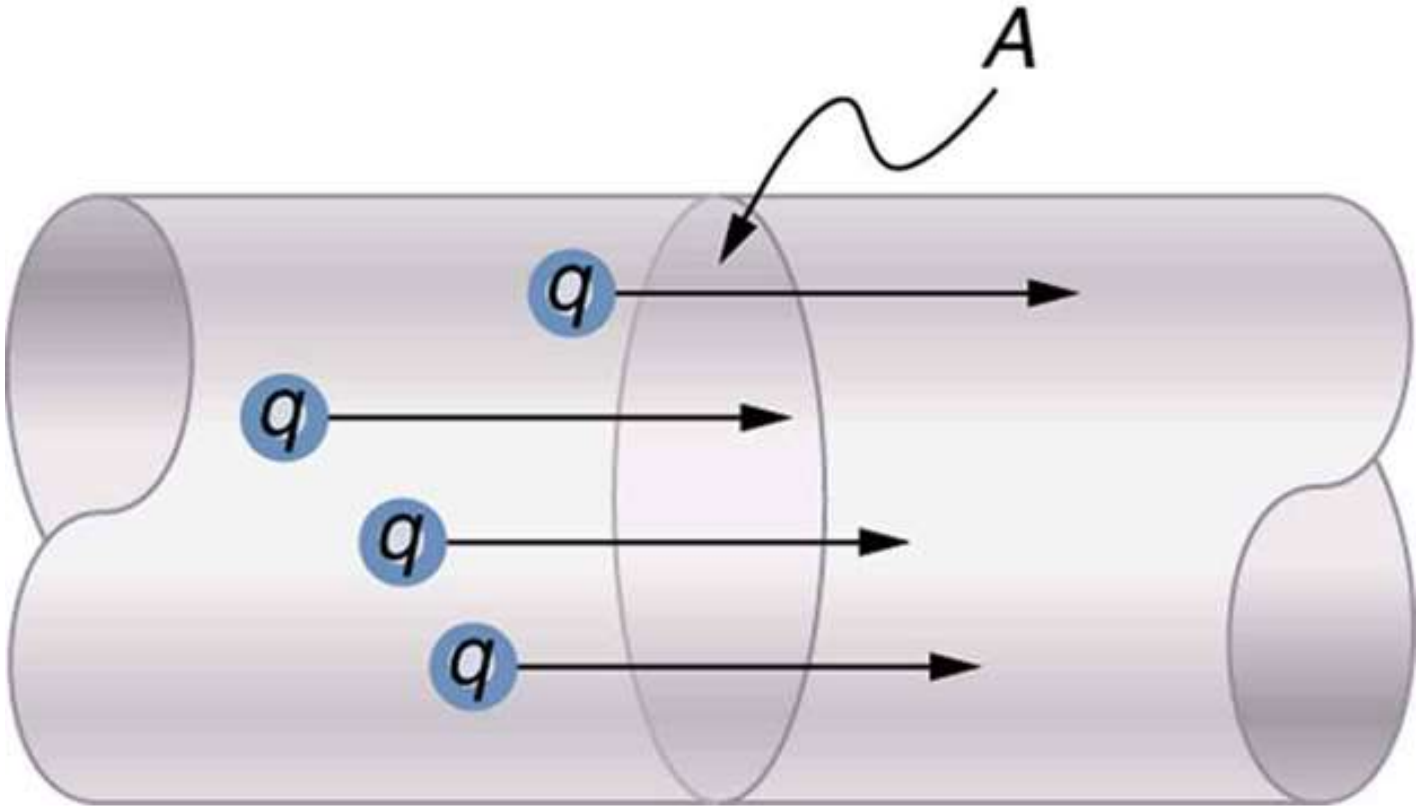
Definition

- Current (I) = rate of flow of charge through a given area:

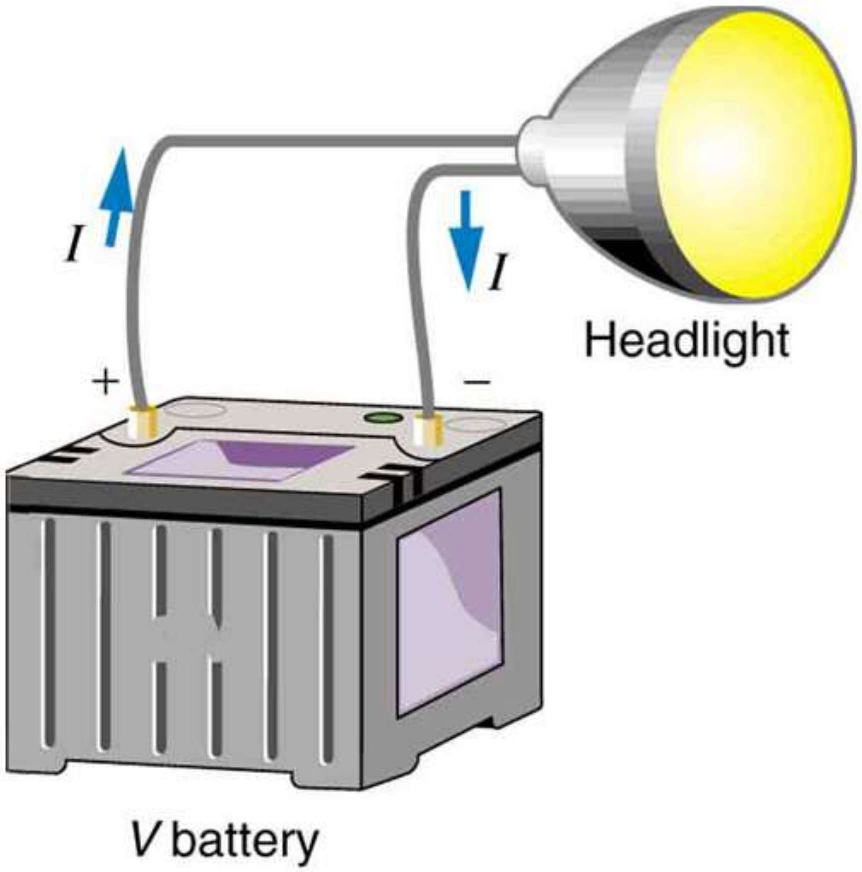
$$I \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \approx \frac{\Delta Q}{\Delta t}$$

- Unit: ampere (A) = coulomb per second:

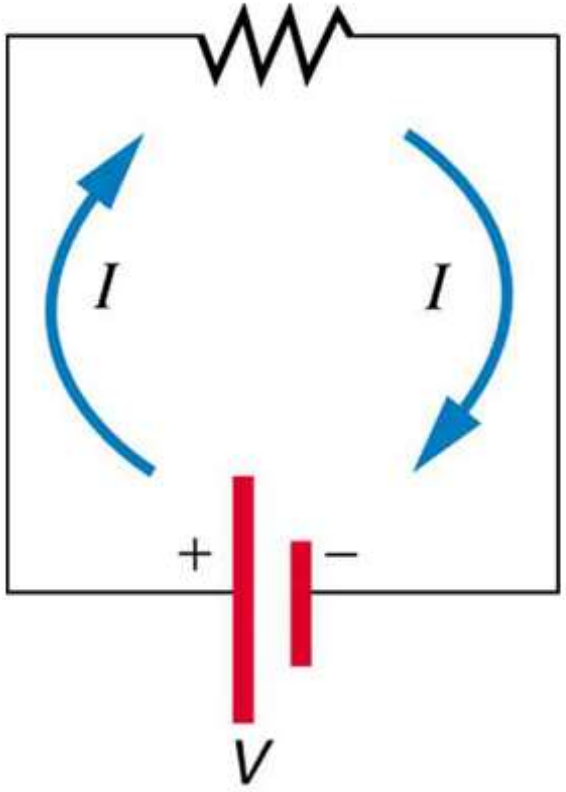
$$\mathrm{A} \equiv \frac{\mathrm{C}}{\mathrm{s}}$$



Circuits



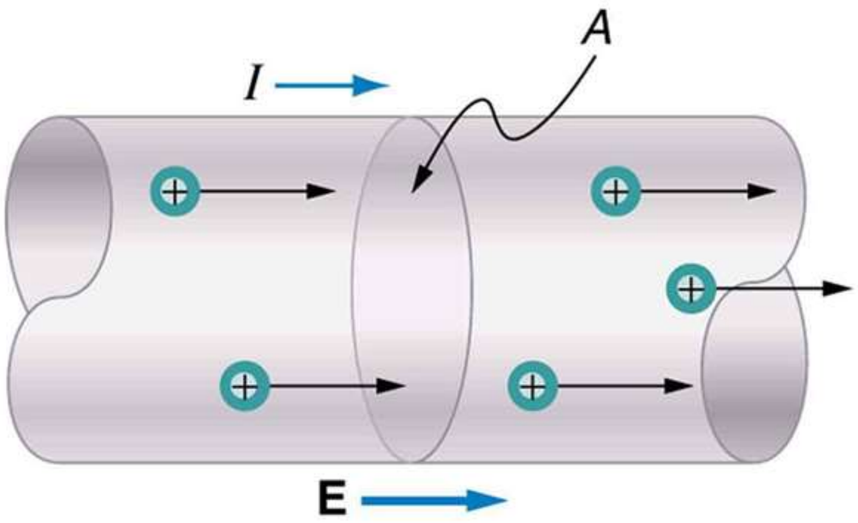
Load (resistor)

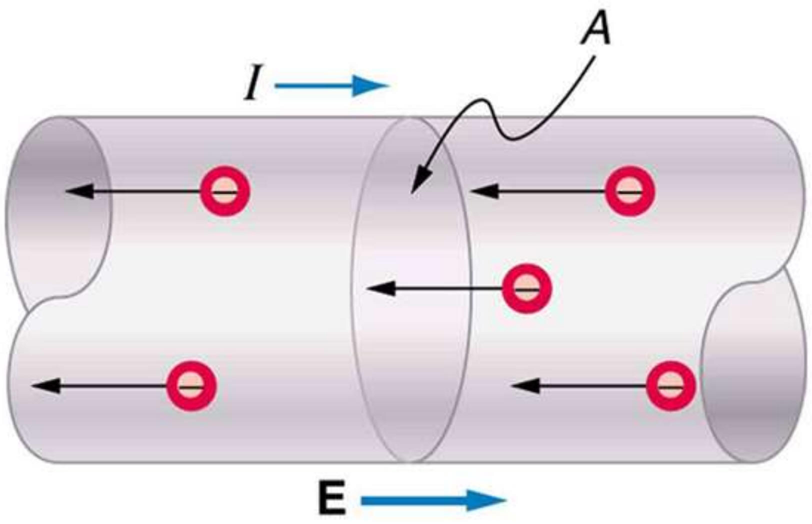


Battery

Flow of charges

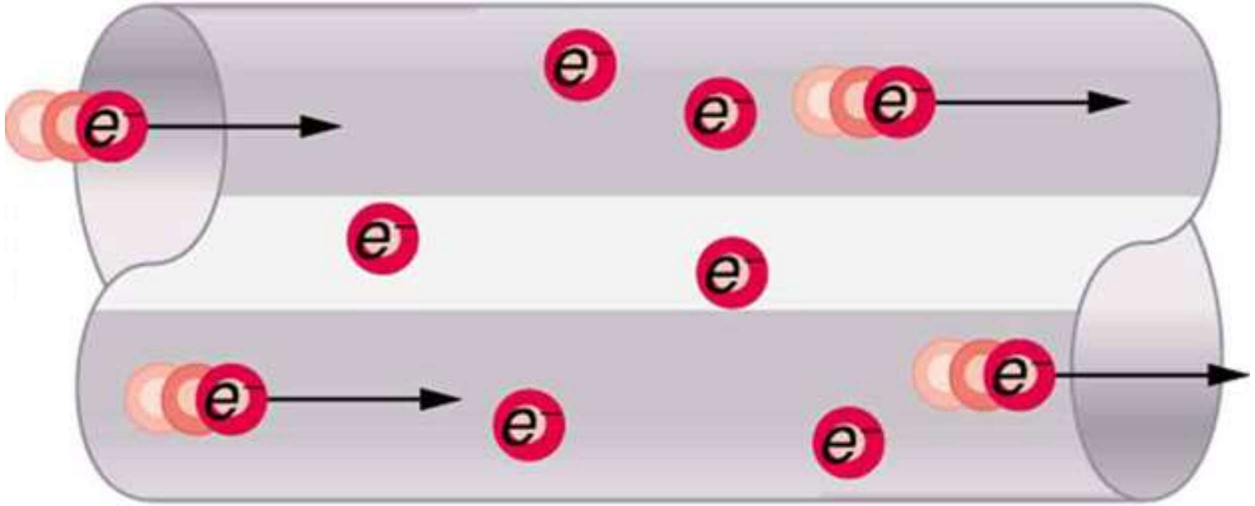
- *I* is always in the direction where positive charges would flow.
- This happens in some cases, e.g. ionic solutions such as salt water.
- However, in most cases, e.g. metal wires, negative charges (electrons) flow. So *I* will be opposite to the real flow.





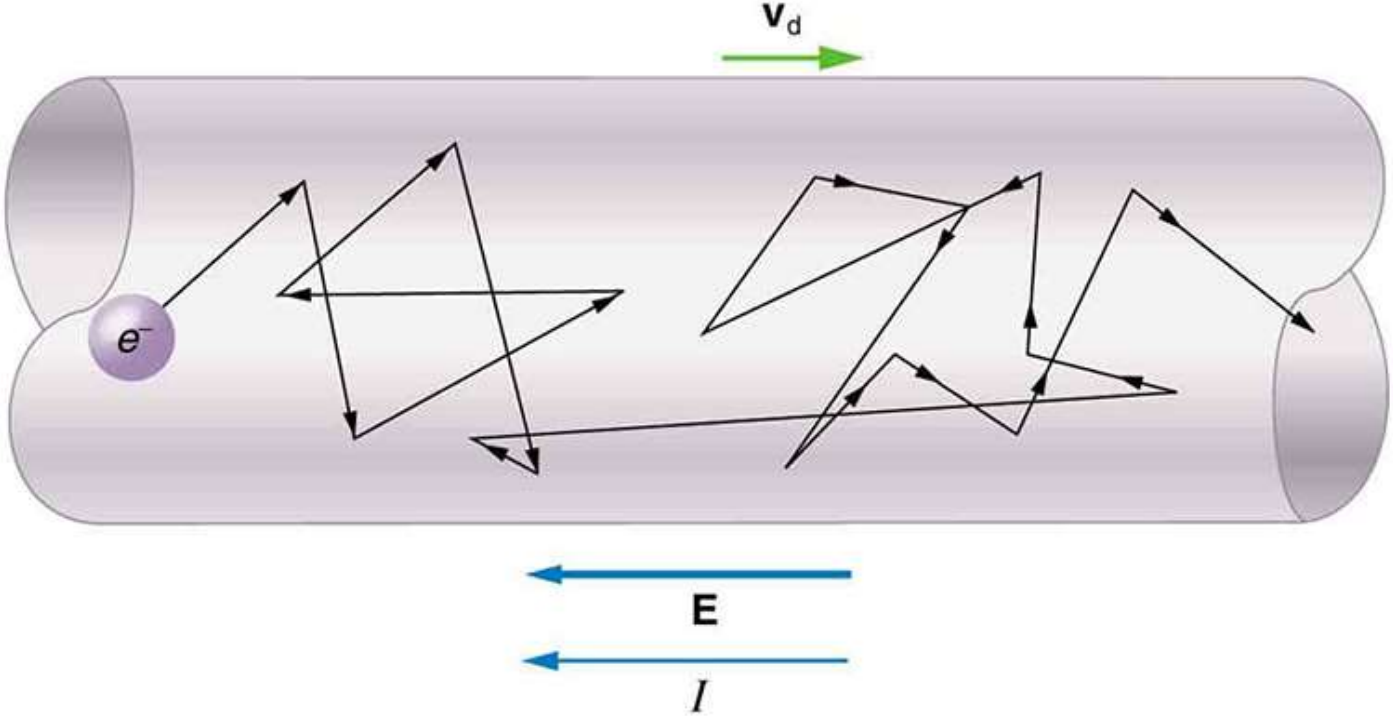
Drift velocity

- Electric signals travel very fast ($\approx 10^8$ m/s), e.g. Internet.
- Electric charges move at a slow drift velocity ($\approx 10^{-4}$ m/s).
- The signal is a wave in the electric field, **not** the individual charges.
- Analogy: charges = gas particles, signals = sound waves.



Current in a conductor

- Electrons in a conductor (e.g. metal) move freely.
- They move randomly, like atoms in a gas. But **on average** they drift at the drift velocity if there is current.



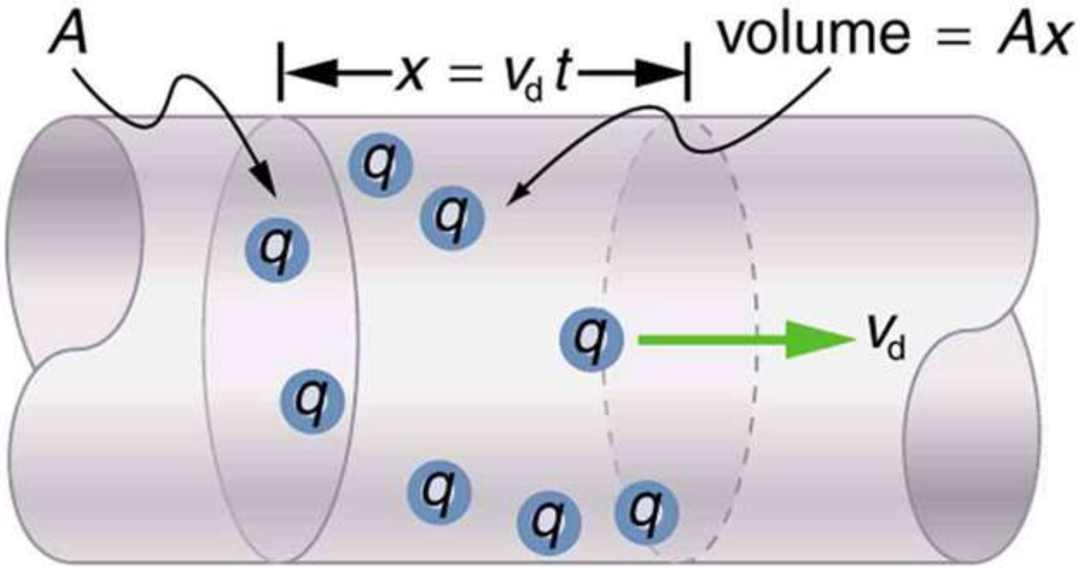
Work inside a conductor

- The electric field does work to “push” the charges, but they don’t keep moving on their own.
- The work is transferred to the atoms in the conductor, producing heat.
- To keep the current going, we need to continuously supply power.
- In a superconductor, resistance (see next section) drops to zero, so current can continue flowing indefinitely on its own.

Calculating the drift velocity

- v_d = drift velocity. Charges flow distance $x = v_d \Delta t$ in time Δt.
- n = charges q per unit volume. In time Δt the volume is Ax.

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqAx}{\Delta t} = nqAv_d.$$



20.2 Ohm’s Law: Resistance and Simple Circuits

Ohm’s law

$$I = \frac{V}{R}$$

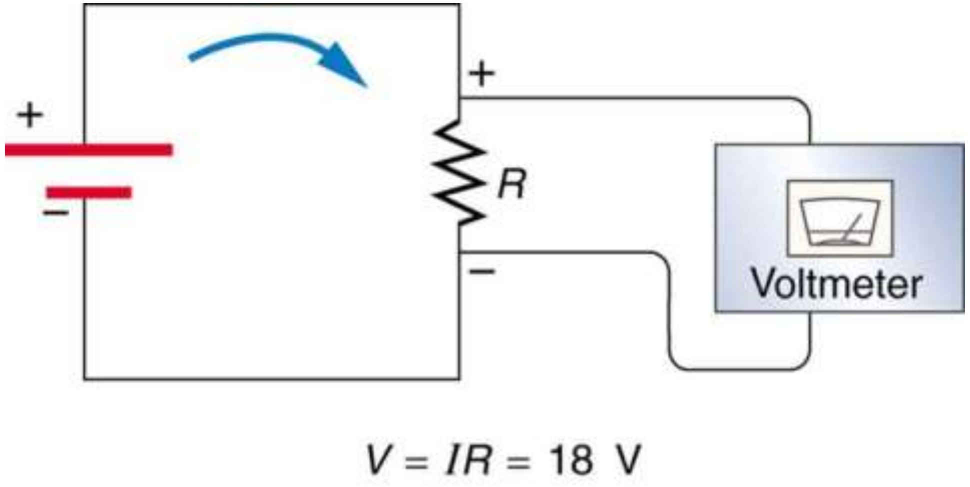
- I = current.
- V = voltage.
- R = resistance.
- Holds only in “ohmic” substances (mostly conductors).
- Units for resistance: ohm (Ω)

$$R = \frac{V}{I}, \qquad \Omega \equiv \frac{\mathrm{V}}{\mathrm{A}}$$

Typical resistances

- Ceramic insulators: $10^{12}\ \Omega$ or more.
- Dry person: $10^{5}\ \Omega$.
- Human heart: $10^{3}\ \Omega$.
- 1 meter of copper wire: $10^{-5}\ \Omega$.
- Superconductor: $0\ \Omega$.

Voltage drop



Voltage supplied by source = voltage drop across resistor.

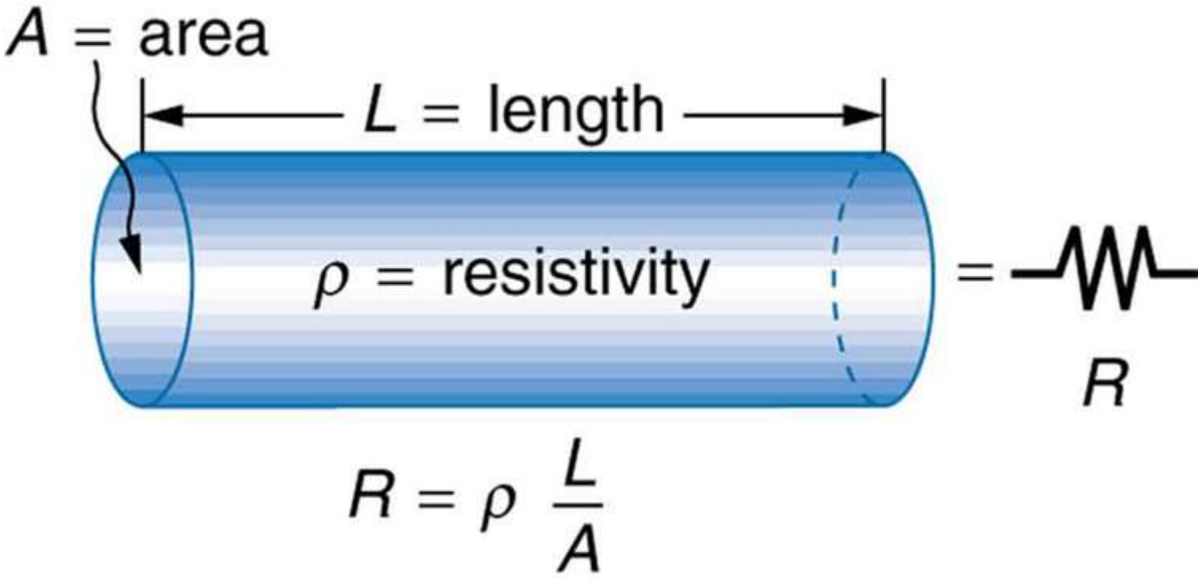
20.3 Resistance and Resistivity

Material and shape dependence

$$R = \frac{\rho L}{A}$$

- $R =$ resistance.
- $\rho =$ **resistivity** of material.
- $L =$ length of cylinder.
- $A =$ cross-sectional area.
- Pop quiz: Units of ρ?
- Answer:

$$\rho = \frac{RA}{L} \implies [\rho] = \frac{\Omega \cdot \mathrm{m}^2}{\mathrm{m}} = \Omega \cdot \mathrm{m}$$



Conductors

Material	Resistivity ρ $(10^{-8}\ \Omega\cdot\mathrm{m})$
Silver	1.59
Copper	1.72
Gold	2.44
Aluminum	2.65
Tungsten	5.6
Iron	9.71
Platinum	10.6
Steel	20
Lead	22
Manganin (Cu, Mn, Ni alloy)	44
Constantan (Cu, Ni alloy)	49
Mercury	96
Nichrome (Ni, Fe, Cr alloy)	100

Semiconductors

(Between conductors and insulators)

Material	Resistivity ρ ($\Omega\cdot$m)
Carbon (pure)	3.5×10^{-5}
Carbon	$(3.5 - 60) \times 10^{-5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1 - 600) \times 10^{-3}$
Silicon (pure)	2300
Silicon	$0.1 – 2300$

Insulators

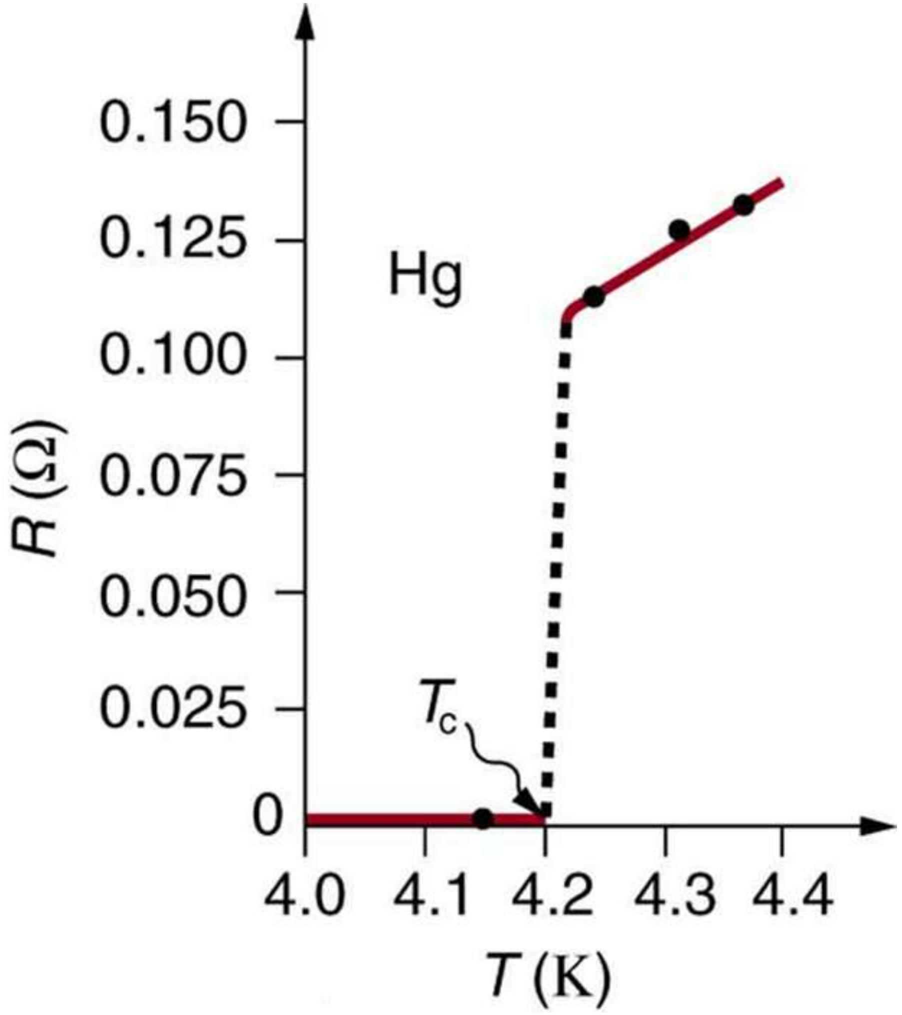
Material	Resistivity ρ (Ω·m)
Amber	5×10^{14}
Glass	$10^{9} - 10^{14}$
Lucite	$> 10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	7.5×10^{17}
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}
Teflon	$> 10^{13}$
Wood	$10^{8} - 10^{11}$

Temperature dependence

$$\rho = \rho_0(1 + \alpha\Delta T)$$

- $\rho =$ new resistivity.
- $\rho_0 =$ original resistivity.
- $\alpha =$ temperature coefficient of resistivity.
    - Units: $1/\mathrm{K}$ or K^{-1}.
    - Typical value: around 10^{-3} for conductors.
- $\Delta T =$ change in temperature.

Example: mercury (Hg) is a superconductor below 4.2 K. Above that, there is a linear dependence on temperature with $\alpha \approx 0.89 \times 10^{-3}\ \mathrm{K}^{-1}$.



Temperature dependence

Since $R \propto \rho$:

$$R = R_0(1 + \alpha\Delta T)$$

- R = new resistance.
- R_0 = original resistance.

Application: thermometers

- **Thermistor:** material with resistance that strongly depends on temperature.
- Temperature is obtained by measuring the resistance.

20.4 Electric Power and Energy

Power in a circuit

- Potential energy:

$$E_p = qV$$

  - $q =$ charge
  - $V =$ voltage (potential difference)
- Power:

$$P \equiv \frac{\mathrm{d}E}{\mathrm{d}t} \approx \frac{E}{t} = \frac{qV}{t}$$

- Current:

$$I \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \approx \frac{q}{t}$$

- Therefore:

$$P = IV$$

Equivalent expressions

$$P = IV$$

- Using Ohm's law:
  - $I = V/R$:

$$P = \frac{V^2}{R}$$

  - $V = IR$:

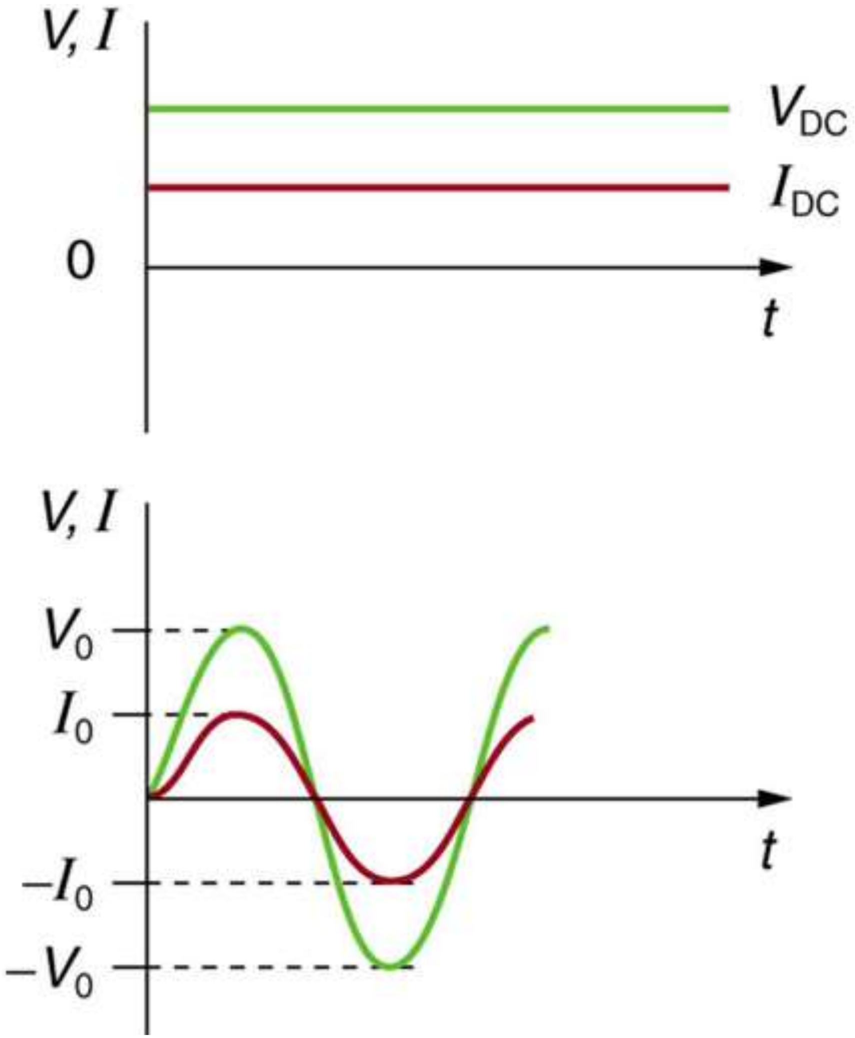
$$P = I^2 R$$

(Valid only when Ohm's law is valid.)

20.5 Alternating Current versus Direct Current

DC vs. AC

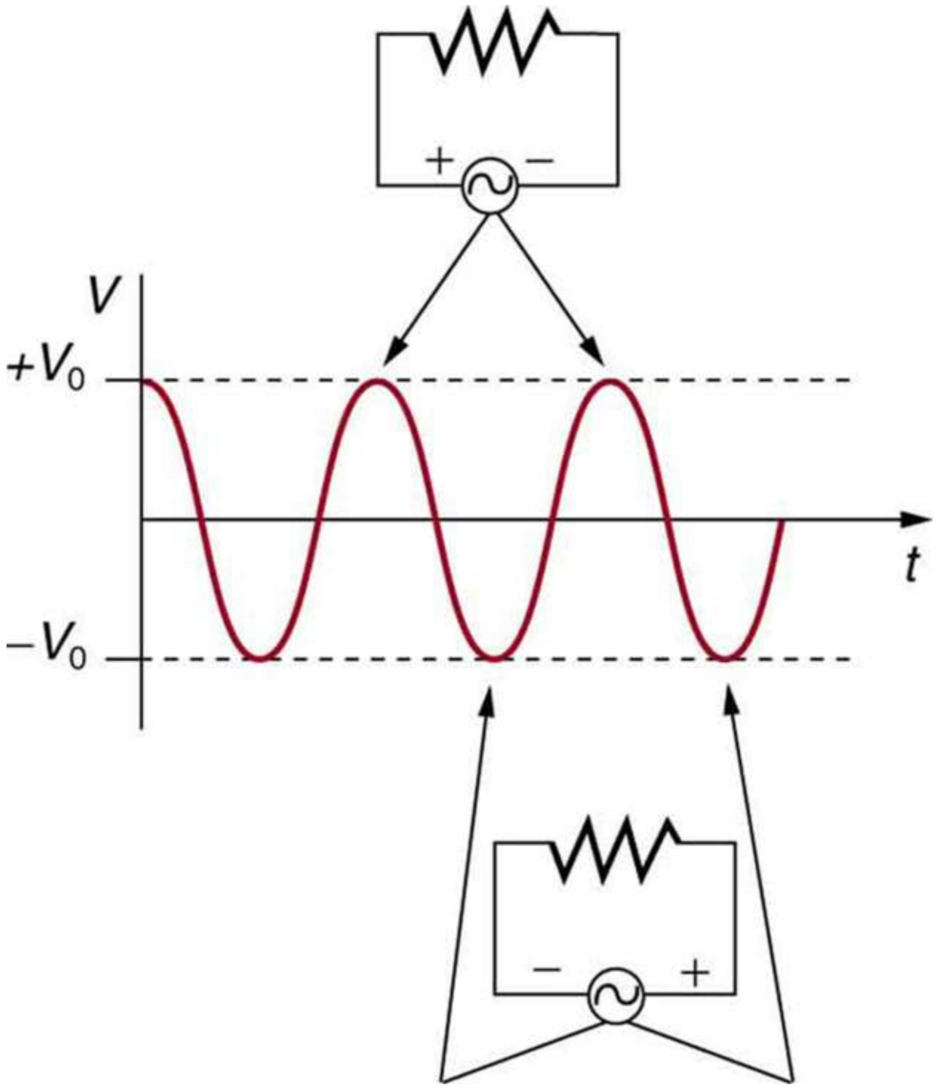
- **Direct current (DC):** charge flowing in one direction.
  - Example: battery.
- **Alternating current (AC):** direction of flow is oscillating.
  - Example: wall outlet.



Oscillating voltage

$$V(t) = V_0 \sin(2\pi f t)$$

- $V(t)$ = voltage at time t.
- V_0 = **peak** voltage (in Canada: 120 V).
- f = frequency (in Canada: 60 Hz).



Oscillating current

- Using Ohm's law:
  - $I = V/R$
  - $I_0 = V_0/R$

So

$$I(t) = I_0 \sin(2\pi f t)$$

- Example: fluorescent light bulb brightens and dims 120 times every second.

Oscillating power

- Using $P = IV$:

$$P(t) = I_0 V_0 \sin^2(2\pi f t)$$

- Average power:

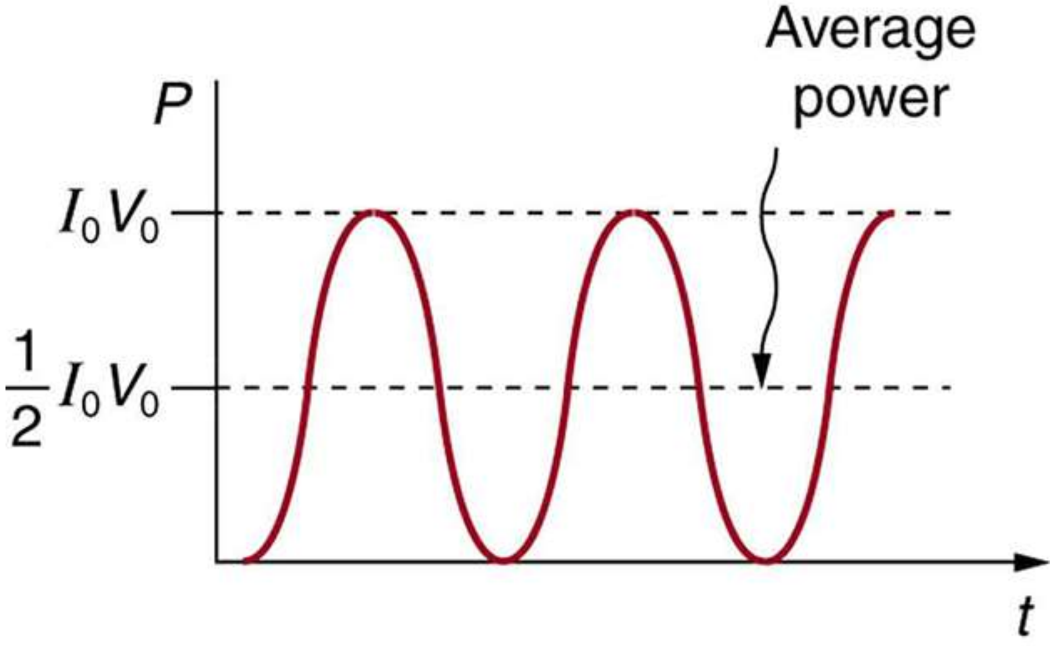
$$P_{\text{ave}} = \frac{1}{2} I_0 V_0$$

- rms (root mean square):

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

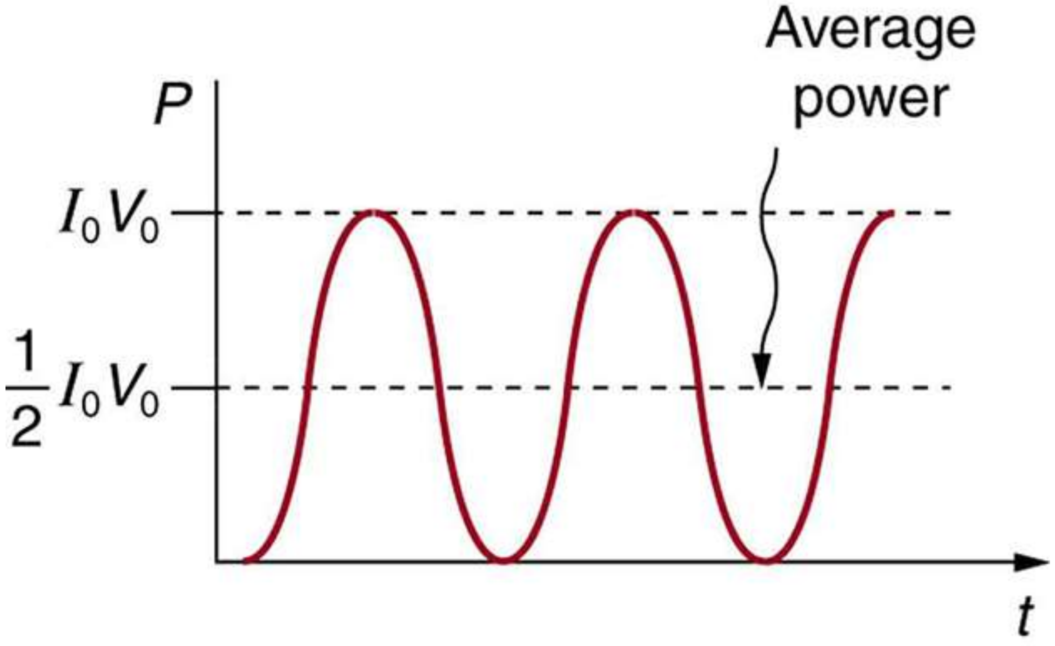
$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$$



rms identities

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R$$



Power loss in power lines

- $I = P_{\text{transmit}}/V$, so if V is larger, I is smaller.
- $P_{\text{lost}} = I^2 R$ so power lost is quadratic in I.
- Therefore, larger V means less power lost during transmission.

AC for power distribution

- When sending power from power plant, energy loss must be minimized.
- So power lines use high voltage (e.g. 330,000 V) = less power lost.
- Needs to be decreased to 120 V for safety reasons.
- Easier to increase/decrease AC than DC.

Transformer