PHYS 1P22/92

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24. Electromagnetic Waves

24.1 Maxwell’s Equations: Electromagnetic Waves Predicted and Observed

Maxwell’s equations

- A collection of 4 equations describing **all** of electromagnetism.
- Each equation has a **differential** form and an **integral** form.
  - We will only talk about the differential form, since it’s simpler.
- You are **not** expected to understand the math for this course, but you **are** expected to understand the basic concepts.
  - Actually, the math isn’t as complicated as it looks! You will understand it if you study vector calculus and differential equations (in physics, math, engineering, and other programs).

1st equation: Gauss's law

$$\nabla \cdot \mathbf{E} = \rho$$

- $\mathbf{E} =$ electric field
- $\nabla \cdot =$ divergence (a type of derivative), measures density and direction of field lines
- $\rho =$ electric charge (per unit volume)
- Describes the relationship between the electric field and electric charge:
  - The force between two charges (Coulomb's law)
  - Fields due to different charges add up (interfere destructively or constructively)
  - Field lines point from positive to negative charges
  - Number of field lines is proportional to the charge
  - Density of field lines is proportional to the magnitude of the field
- All this information comes from just one simple equation!

2nd equation: Gauss's law for magnetism

$$\mathbf{\nabla} \cdot \mathbf{B} = 0$$

- $\mathbf{B} =$ magnetic field
- Describes the magnetic field:
  - There are no magnetic monopoles (no "magnetic charge" on the RHS)
  - Field lines never begin or end (they form loops or extend to infinity)
  - Density of field lines is proportional to the magnitude of the field
- If magnetic monopoles exist, the equation will change to $\mathbf{\nabla} \cdot \mathbf{B} = \rho_m$ where ρ_m is the magnetic "charge" per unit volume.

3rd equation: Faraday's law of induction

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- $\nabla \times$ = curl (a type of derivative), measures how much a field rotates and in which direction
- On the right side: derivative = **rate of change** of magnetic field over time.
- A changing magnetic field induces an electric field around it.
- Faster change = stronger field.
- Not part of the course material, see chapter 23 for more information.
- Demonstration: https://phet.colorado.edu/sims/html/faradays-law/latest/faradays-law_en.html
- Live demonstration: eddy currents (or video: https://youtu.be/YELQNyair7Q)

4th equation: Ampere-Maxwell equation

$$\mathbf{\nabla} \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}$$

- $\mathbf{J} =$ electric current (per unit volume)
- A magnetic field can be generated in two ways:
  - Via an electric current **J** (this is Ampere's Law, section 22.9)
  - Via a changing electric field (this is new)
- Symmetric with Faraday's law:
  - Faraday's law: Changing **magnetic** field creates **electric** field
  - Ampere-Maxwell equation: Changing **electric** field creates **magnetic** field
- This allows electromagnetic waves, as we will see.

Summary of Maxwell’s equations

Gauss’s law

$$\mathbf{\nabla} \cdot \mathbf{E} = \rho$$

Gauss’s law for magnetism

$$\mathbf{\nabla} \cdot \mathbf{B} = 0$$

Faraday's law of induction

$$\mathbf{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere-Maxwell equation

$$\mathbf{\nabla} \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}$$

With magnetic monopoles – more symmetric, but unrealistic!

Gauss's law

$$\nabla \cdot \mathbf{E} = \rho_e$$

Gauss's law for magnetism

$$\nabla \cdot \mathbf{B} = \rho_m$$

Faraday's law of induction

$$\nabla \times \mathbf{E} = -\mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}$$

Ampere-Maxwell equation

$$\nabla \times \mathbf{B} = +\mathbf{J}_e + \frac{\partial \mathbf{E}}{\partial t}$$

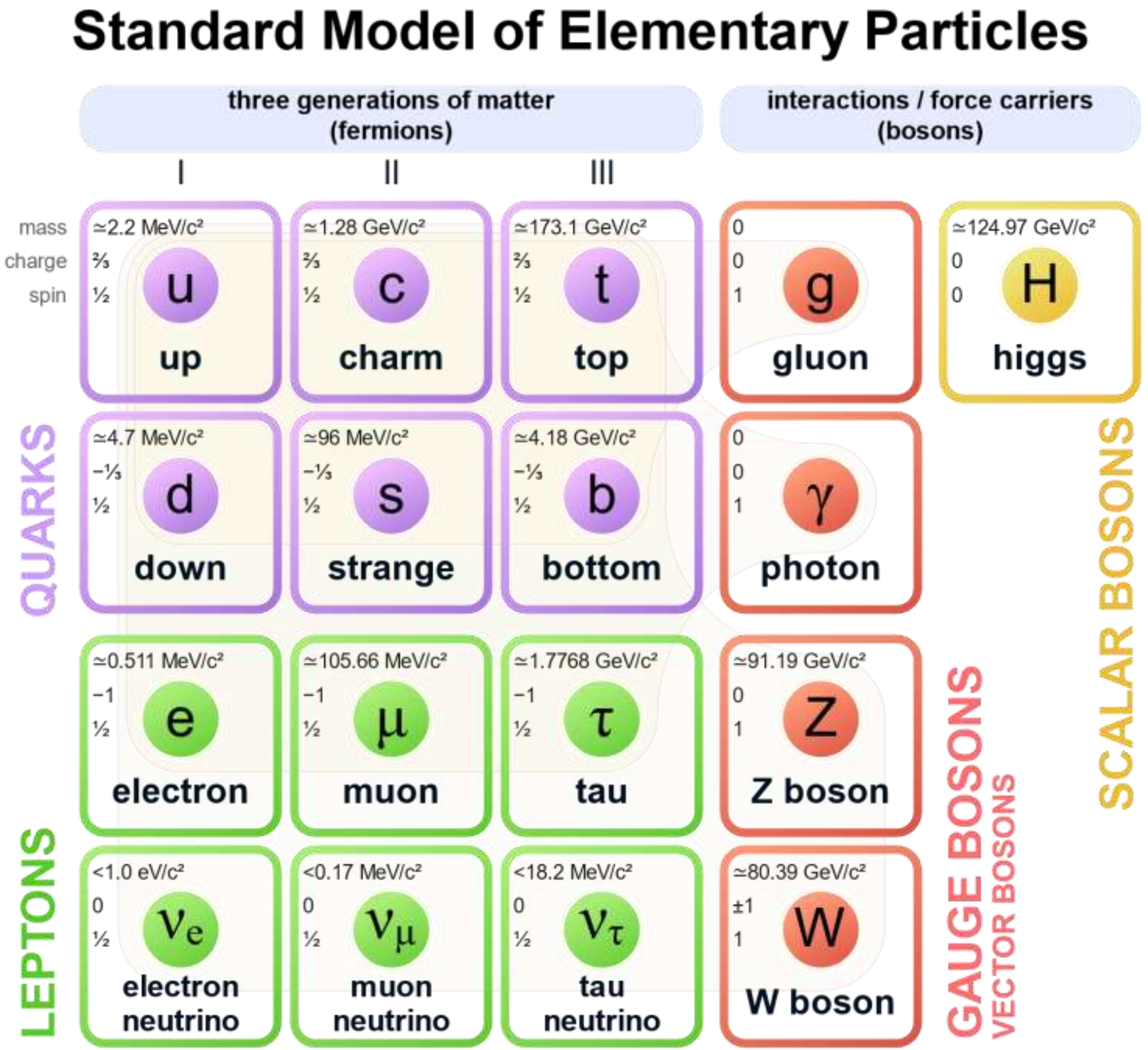
Unification of electromagnetic force

- We see from Maxwell’s equations that the electric and magnetic forces are not separate, they work in similar ways and influence each other.
- The two forces can be **unified** into a single **electromagnetic force**.
- Then Maxwell’s equations can be written in terms of one field, the **electromagnetic field**:

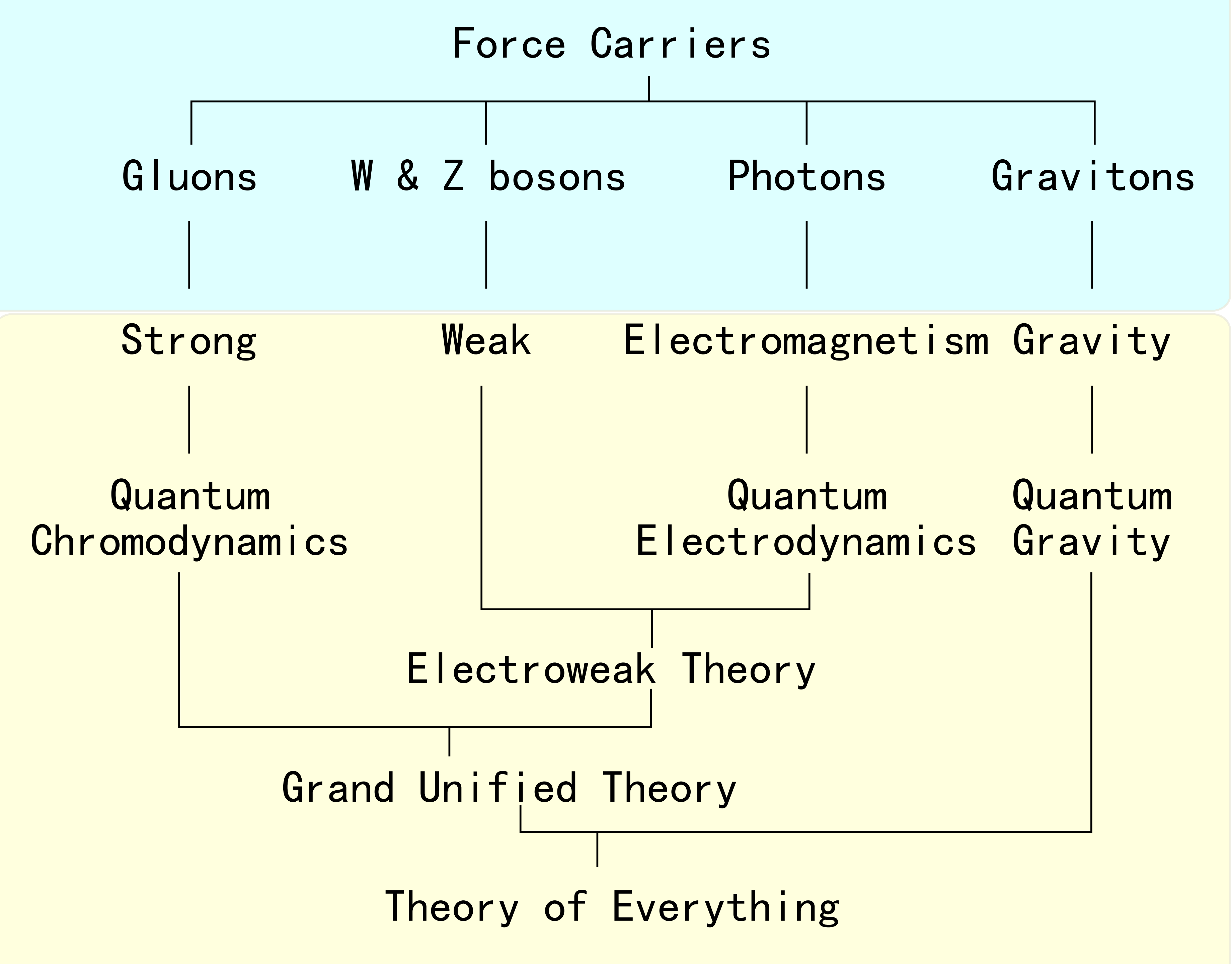
$$\mathrm{d}F = 0, \qquad \mathrm{d} \star F = \star J$$

  - $F =$ electromagnetic field (represented by a matrix)
  - $J =$ electric current
  - $\mathrm{d} =$ divergence (corresponds to first two Maxwell’s equations)
  - $\mathrm{d}\star =$ curl (corresponds to other two Maxwell’s equations)

Unification of the four fundamental forces



Unification of the four fundamental forces



Electromagnetic waves

- An oscillating electric field produces an oscillating magnetic field and vice versa.
- This allows a disturbance in the electromagnetic field to travel in space – an electromagnetic wave.
- The speed of propagation is the speed of light:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \equiv 299{,}792{,}458 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$$

- We've seen ε_0 and μ_0 before:
  - $\varepsilon_0 = \text{vacuum permittivity} \approx 8.854\,187\,8188(14) \times 10^{-12} \text{ F/m}$
  - $\mu_0 = \text{vacuum permeability} \approx 1.256\,637\,061\,27(20) \times 10^{-6} \text{ N/A}^2$
- The numerical values don't matter because they have no physical meaning.

Dimensionful constants

- Any constant in physics that has units (is not a pure number) is called a dimensionful constant.
- Any such constant is **meaningless**; just a conversion factor from one unit to another. So it depends on the choice of units.
- For example, the speed of light:
  - $c \approx 3.0 \times 10^8$ in meters per second
  - $c \approx 1.1 \times 10^7$ in miles per minute
  - $c \approx 170$ astronomical units (distance from Earth to the Sun) per day
  - $c = 1$ light-year per year
- Each value depends on the choice of units. So the number itself is meaningless. The universe doesn't care what units humans choose to measure things with!

The speed of light

- In SI units, c is **defined** to be exactly 299,792,458 m/s.
- This **defines** what a meter is:

  "A meter is the distance traveled by light in 1/299,792,458 seconds."

- So the number itself has no meaning, it's just a choice made by humans.
- Theoretical physicists use units of light-second to measure distance, so c is exactly equal to

$$c = 1 \text{ light—second/second}$$

- This simplifies a lot of equations and calculations since c disappears from the equation, e.g.:

$$E = mc^2 \quad \text{becomes} \quad E = m$$

Dimensionless constants

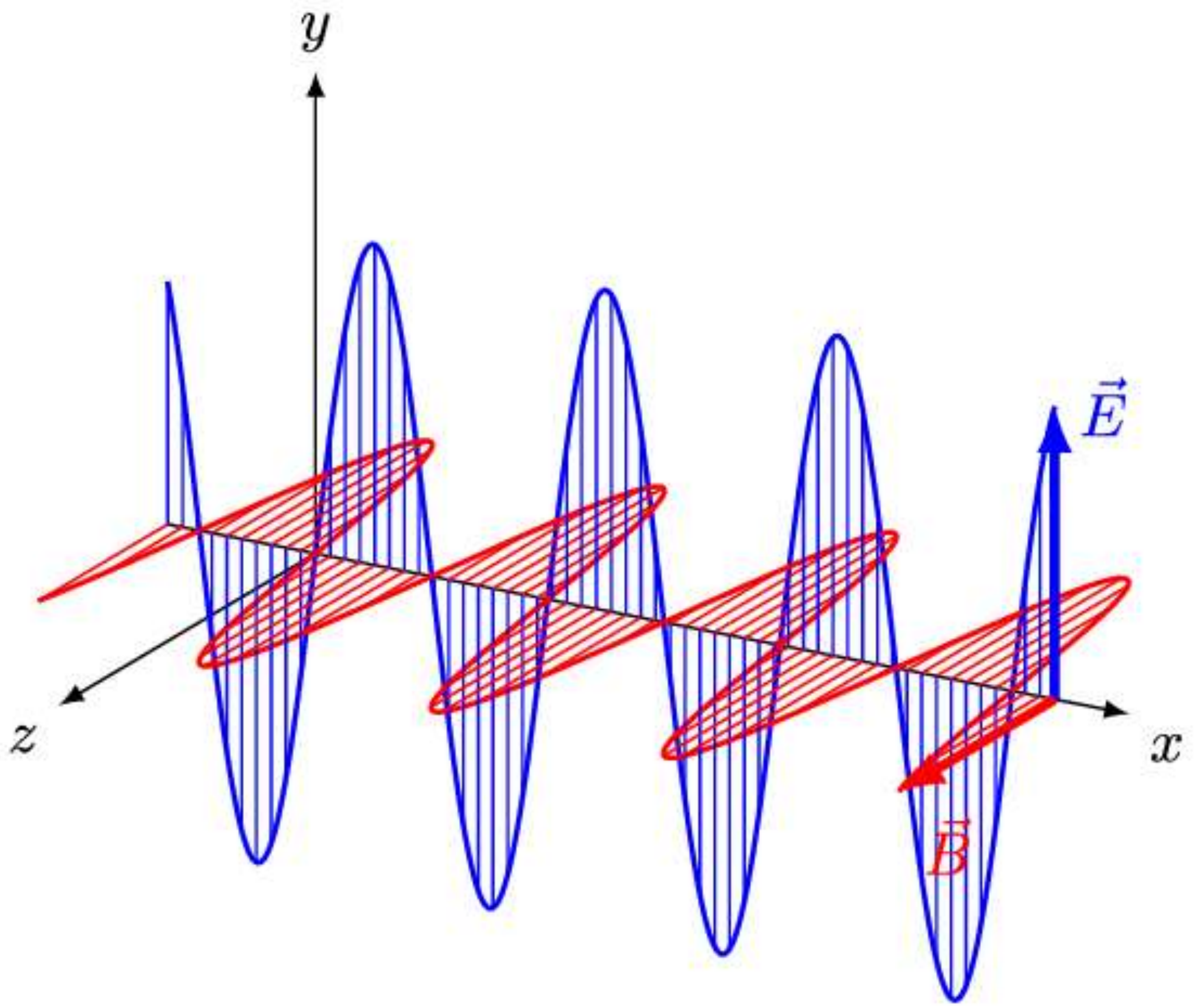
- The only constants that have meaning are dimensionless constants: **pure numbers with no units.**
- Example: the fine-structure constant

$$\alpha \approx 7.297\,352\,5643(11) \times 10^{-3}$$

- This constant quantifies the strength of the electromagnetic interaction. It has the same value in any system of units.
- Must be determined experimentally, not defined.

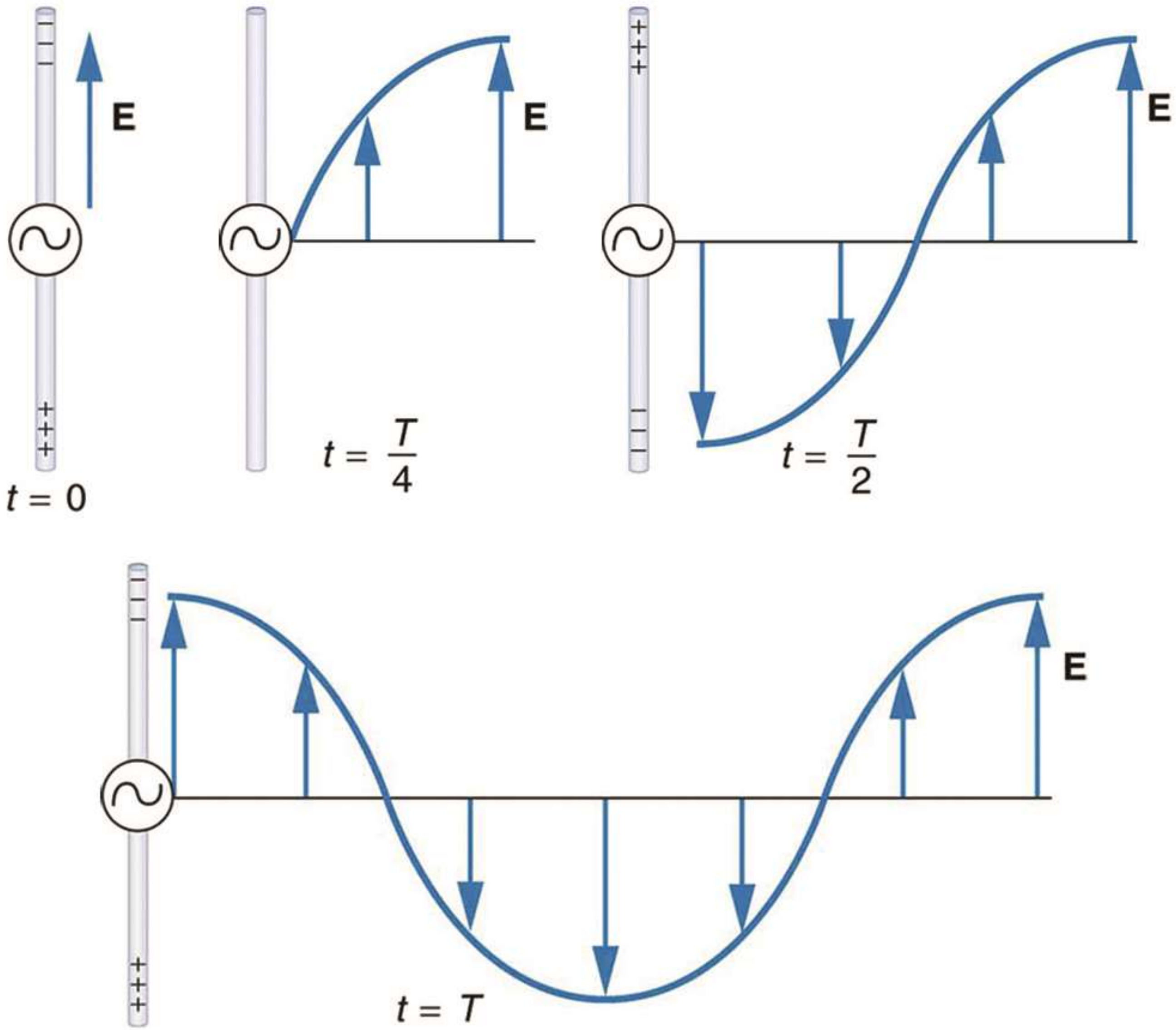
24.2 Production of Electromagnetic Waves

Oscillating electric and magnetic fields

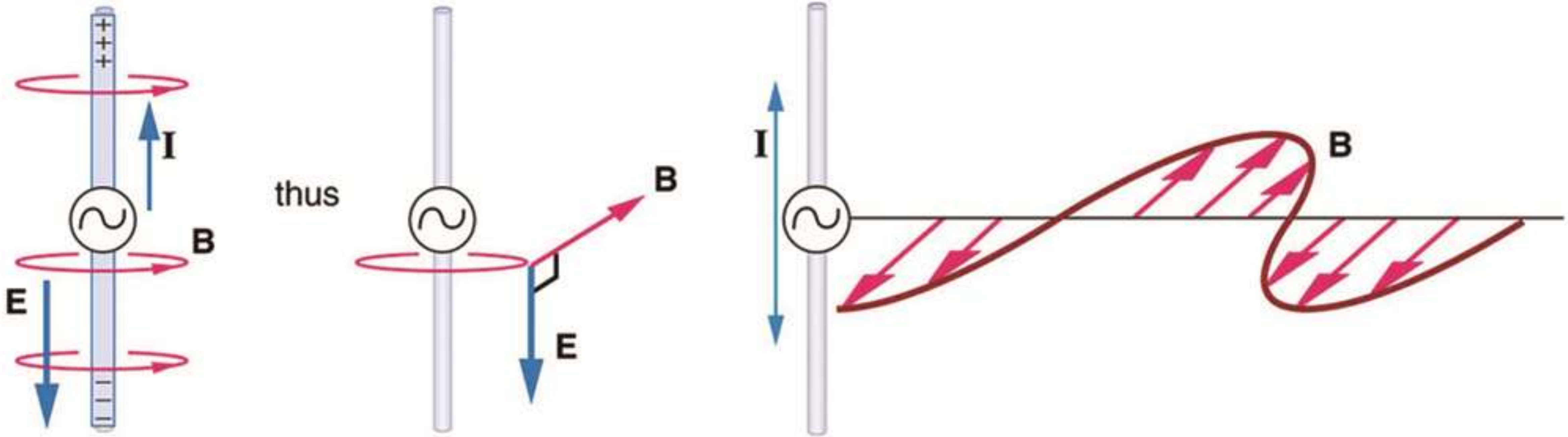


AC current in long wire (antenna)

Electric field propagates away at speed c



The magnetic field

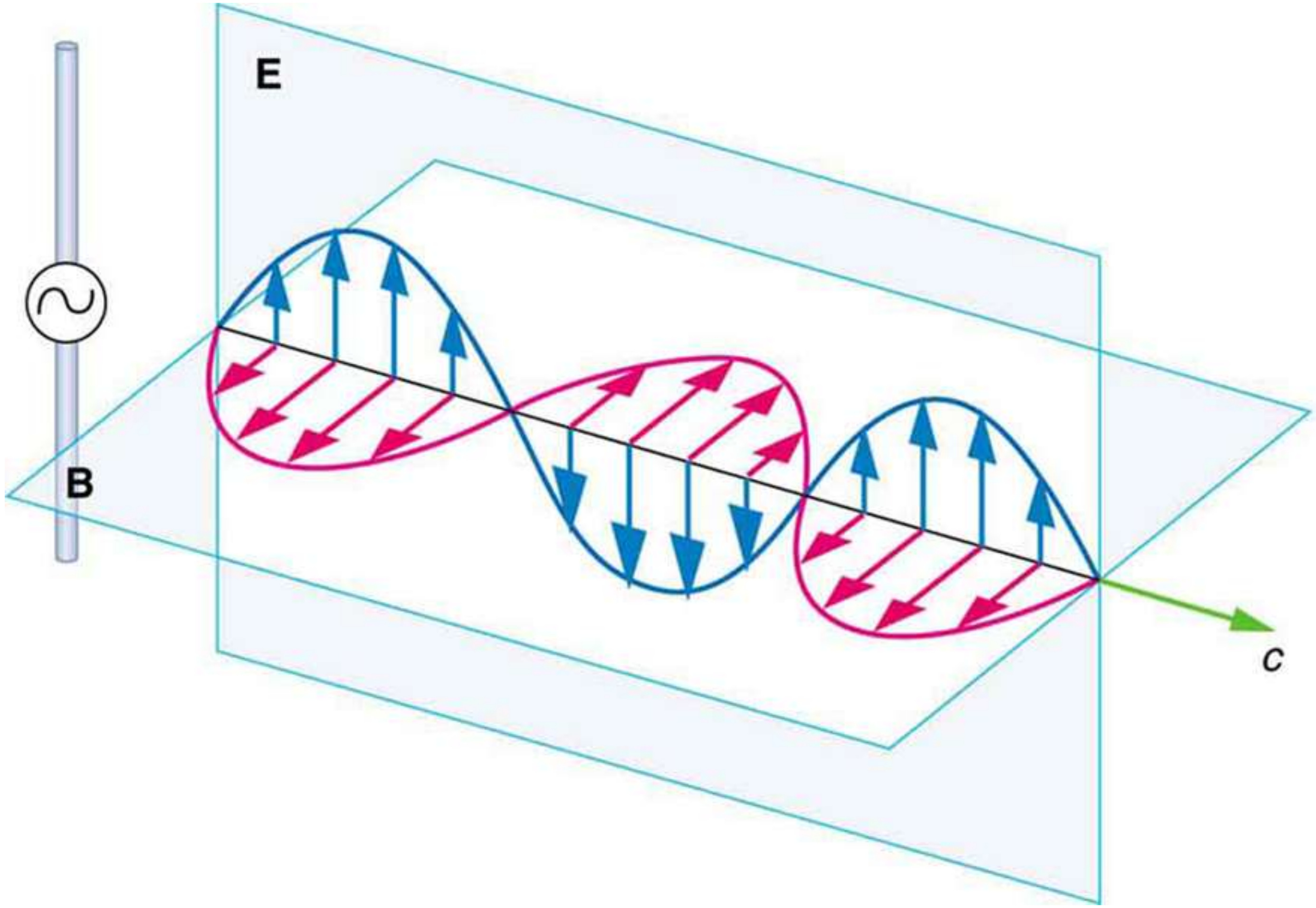


Current produces a circular magnetic field (Ampere's law)

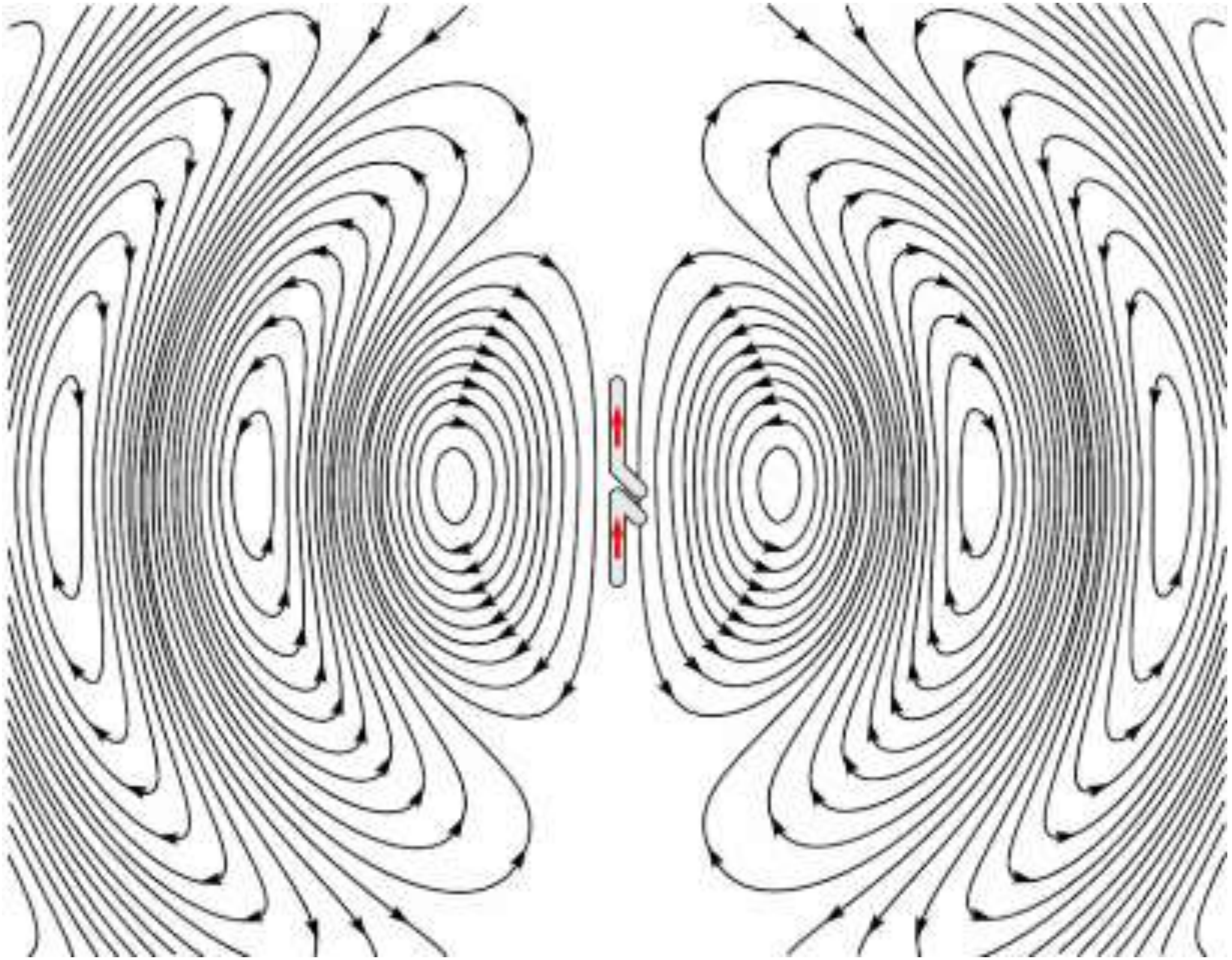
Electric and magnetic fields are perpendicular

Magnetic field propagates away at speed *c*

Both fields



Electric field lines



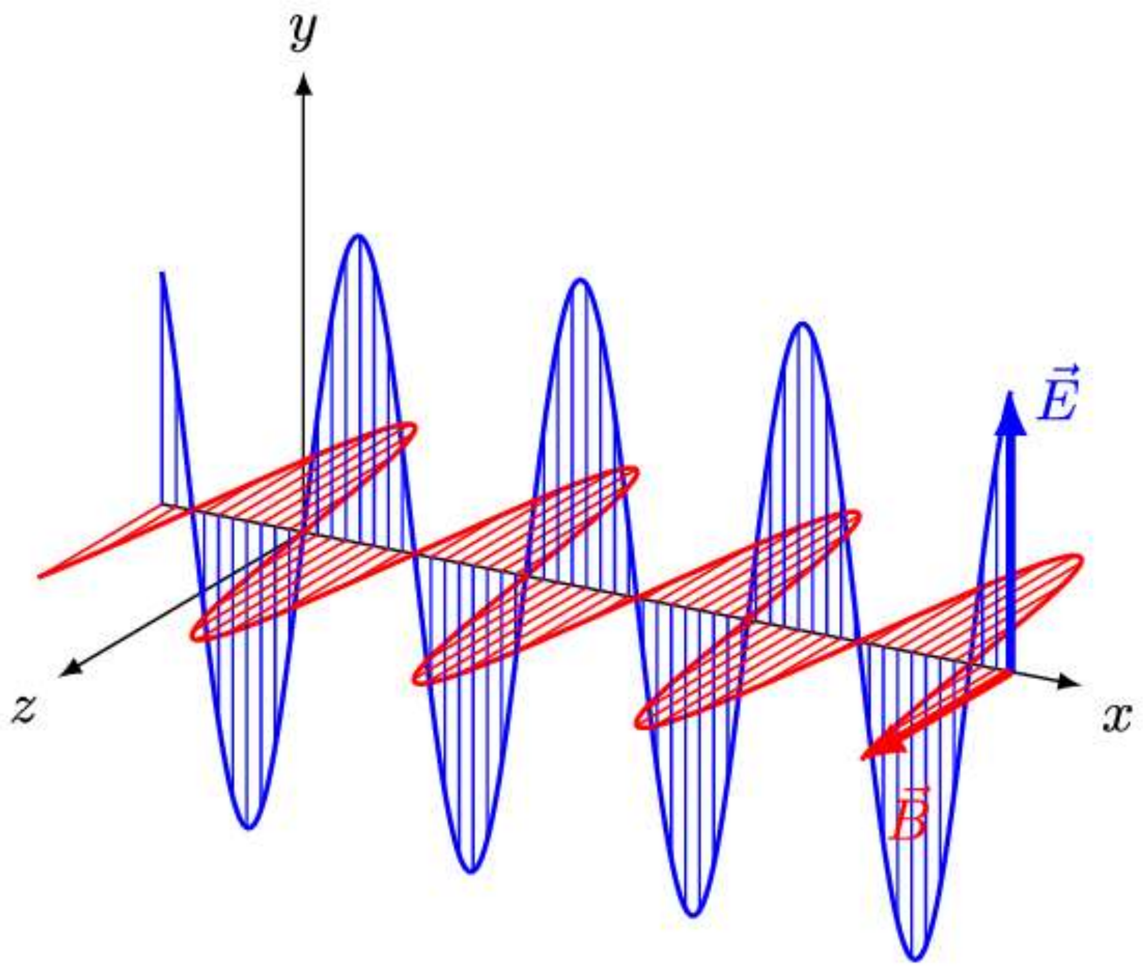
Receiving electromagnetic waves

- The electromagnetic wave produced by the antenna using AC power propagates in space.
- Another antenna receives the wave. The electric and magnetic fields accelerate electrons in the antenna.
- This generates an electric signal that can be converted to audio, video, or digital data within the device (radio, phone, etc.).
- A dish can be used to focus the signal. Especially useful for astronomy!

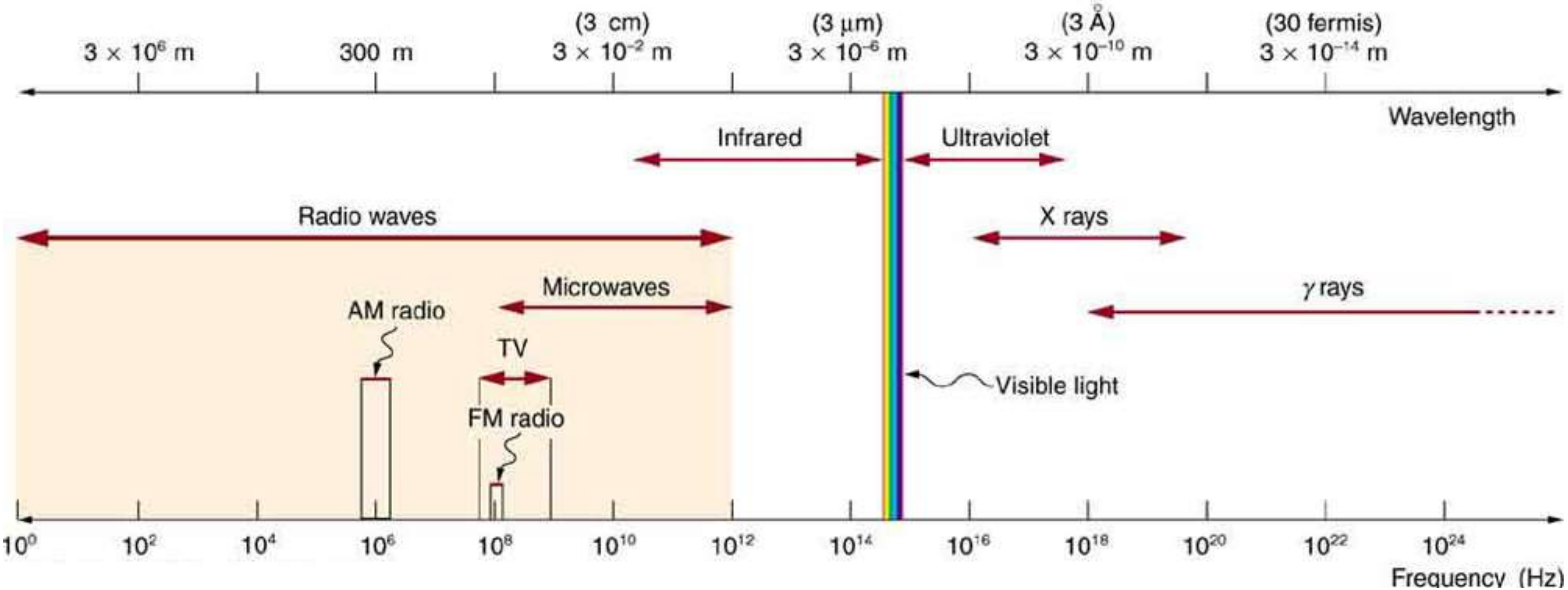
Electric vs. magnetic field strength

$$\frac{E}{B} = c \qquad \Longrightarrow \qquad E = cB$$

The fields are proportional in magnitude.



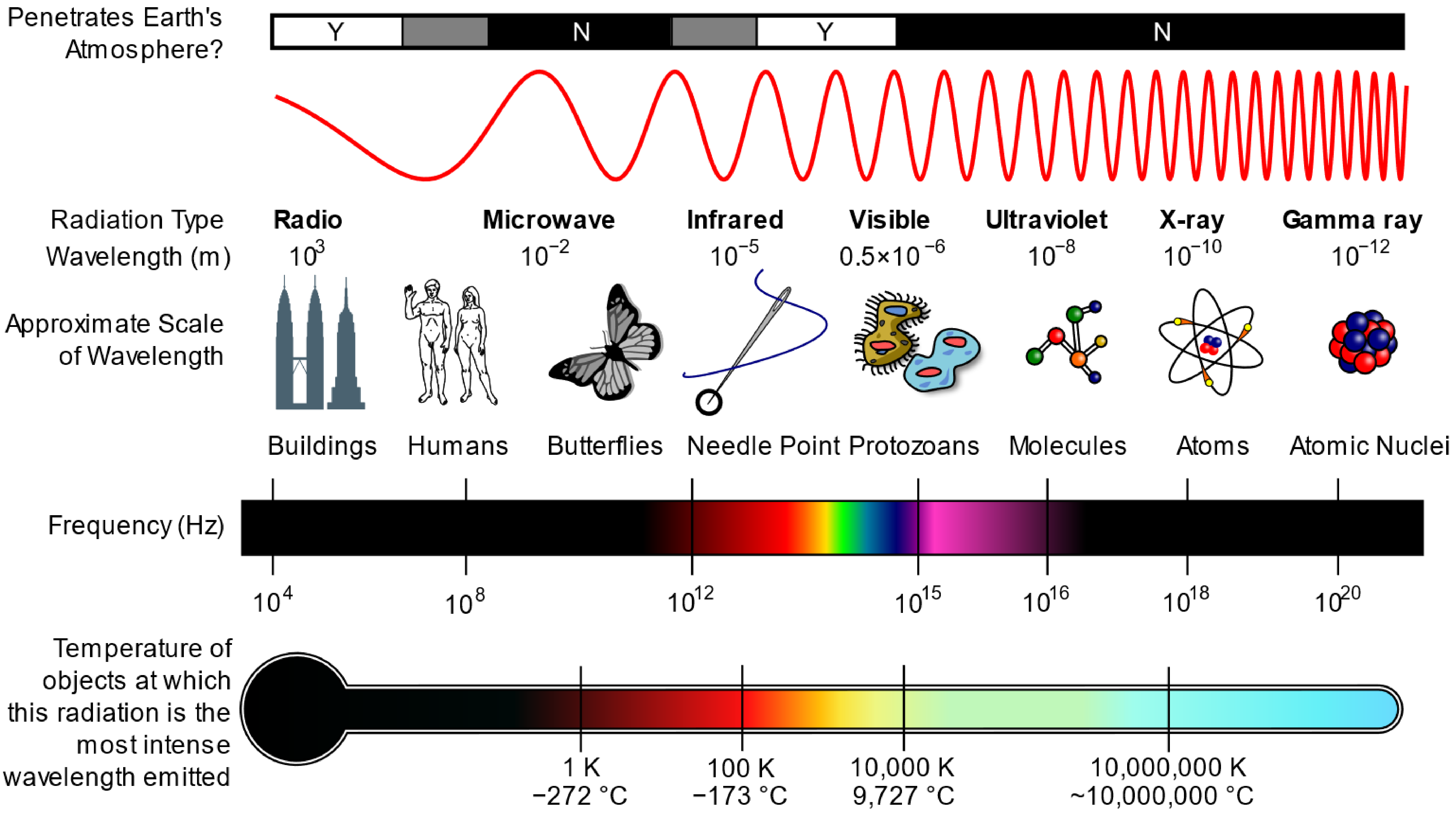
24.3 The Electromagnetic Spectrum



There are many "types" of EM waves, but they're actually all the same thing, just with different frequencies!

$$\lambda = c/f$$

(recall section 16.9)



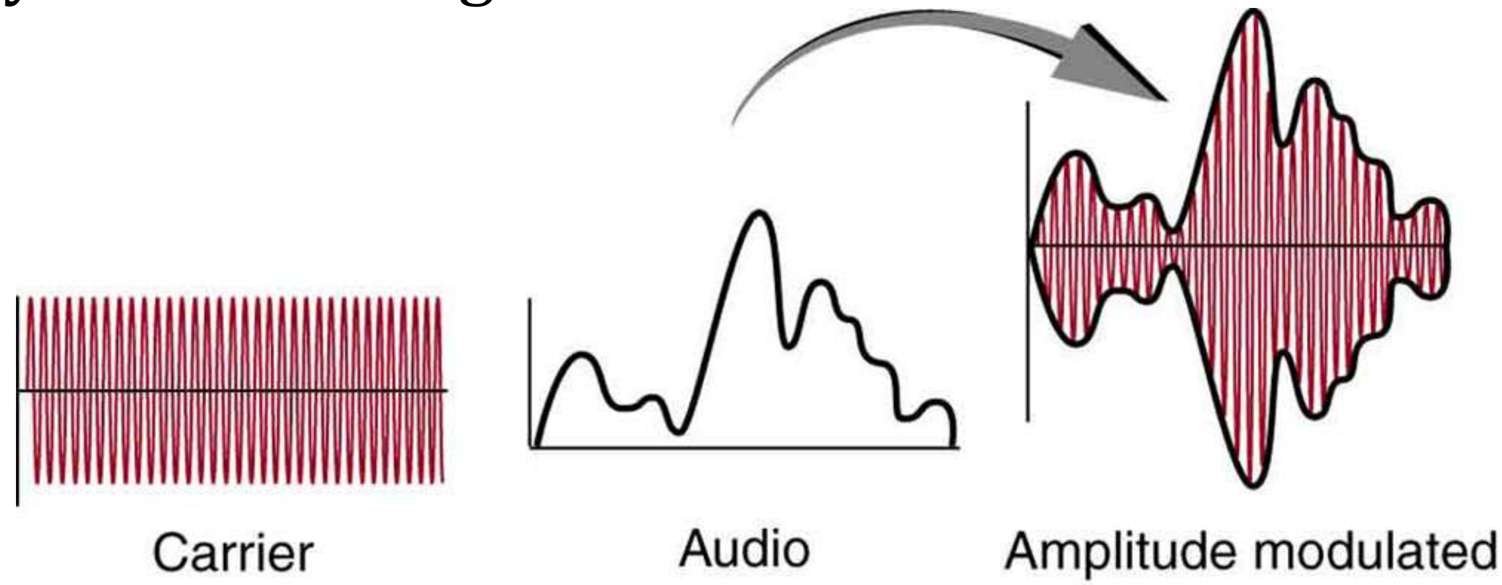
Penetrates Earth's Atmosphere?
Y
N
Y
N
Radiation Type
Wavelength (m)
Radio
10^{3}
Microwave
10^{-2}
Infrared
10^{-5}
Visible
0.5×10^{-6}
Ultraviolet
10^{-8}
X-ray
10^{-10}
Gamma ray
10^{-12}
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^{4}
10^{8}
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of objects at which this radiation is the most intense wavelength emitted
1 K
−272 °C
100 K
−173 °C
10,000 K
9,727 °C
10,000,000 K
~10,000,000 °C

Radio waves

- Smallest frequencies, longest wavelengths
- Range:
  - 3 Hz to 300 GHz (and below)
  - 1 mm to 100,000 km (and above)
- Examples/applications:
  - AC power: 50/60 Hz
  - AM radio: 540 Hz to 1600 kHz
  - FM radio: 88 MHz to 108 MHz
  - 5G mobile network: 600-6000 MHz, 26-47 GHz
    - Higher frequency = higher transfer speeds

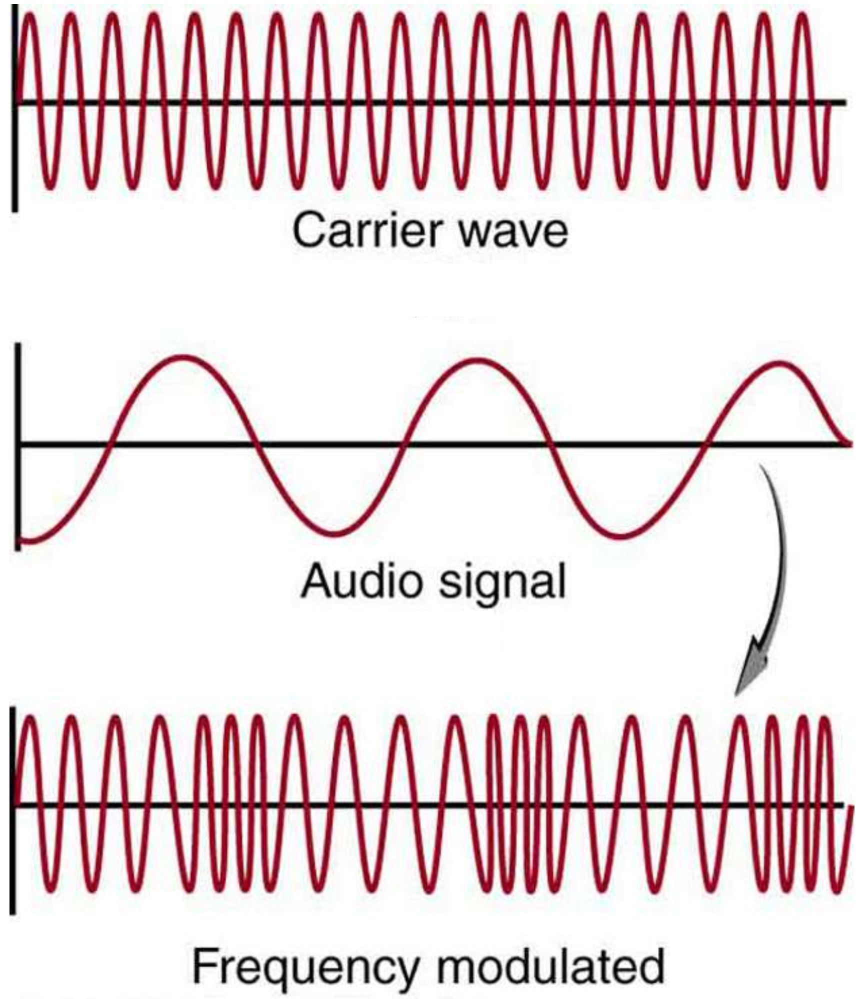
AM radio (amplitude modulation)

- Carrier wave: base frequency of the radio station (e.g. 1500 kHz)
- The audio signal modulates the **amplitude** of the carrier wave.
- Frequency doesn’t change.



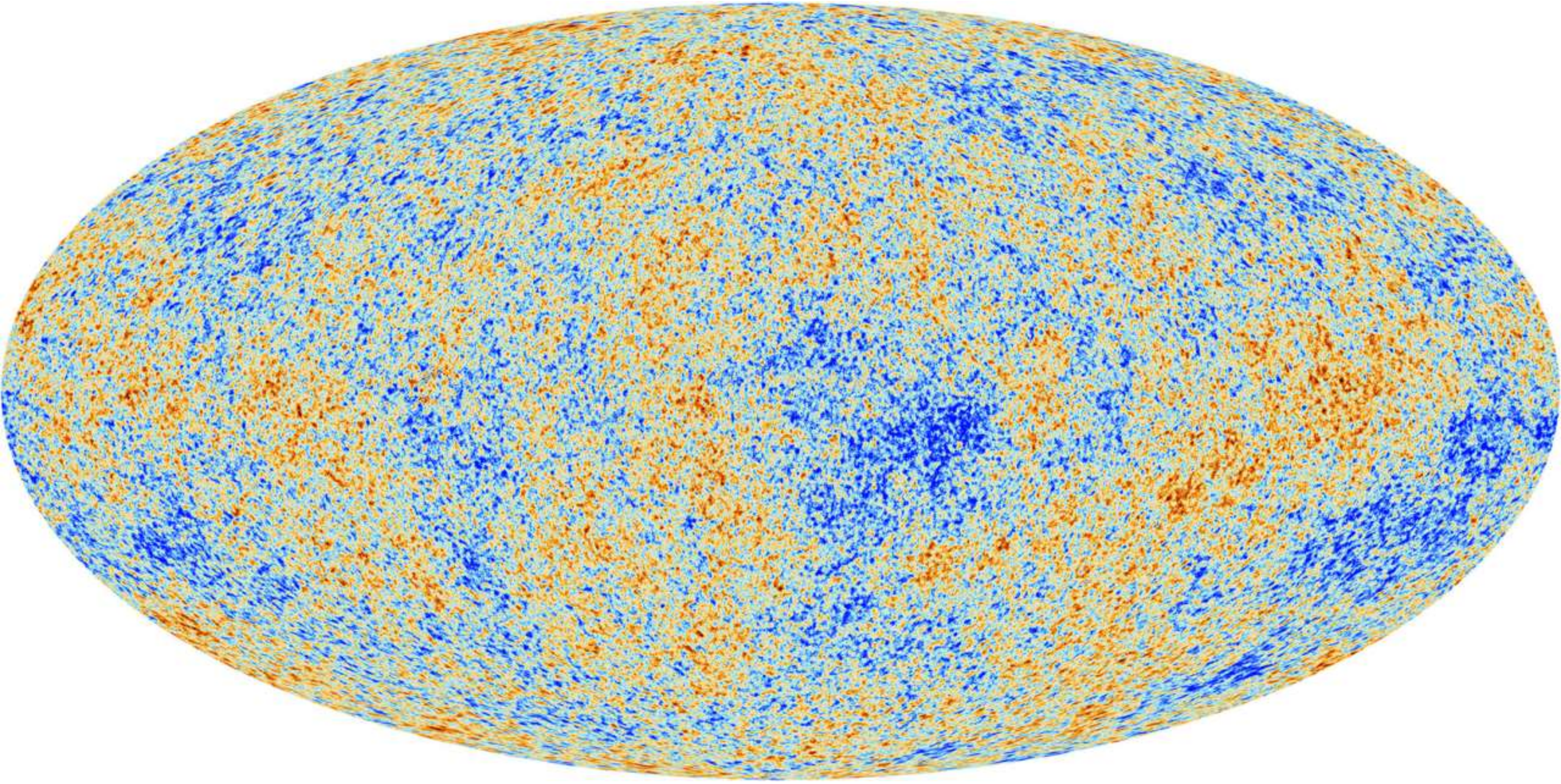
FM radio (frequency modulation)

- Carrier wave: base frequency of the radio station (e.g. 100 MHz)
- The audio signal modulates the **frequency** of the carrier wave.
- Amplitude doesn’t change.



Microwaves

- Range (in the upper range of radio waves):
  - 300 MHz to 300 GHz
  - 1 mm to 1 m
- Examples/applications:
  - Microwave ovens: 2.45 GHz, causes water molecules (which are polar) to jiggle and increases the food temperature.
    - **Common misconception:** 2.45 GHz is **not** a resonant frequency of water. Other frequencies can be used. 2.45 GHz is just not reserved for communication.
  - Cosmic microwave background: 160 GHz, radiation that exists everywhere. Generated 380,000 years after the Big Bang, when hydrogen atoms first formed and emitted photons. Provides evidence for the Big Bang theory.
    - If you want to know more, take **ASTR 1P01/02**!

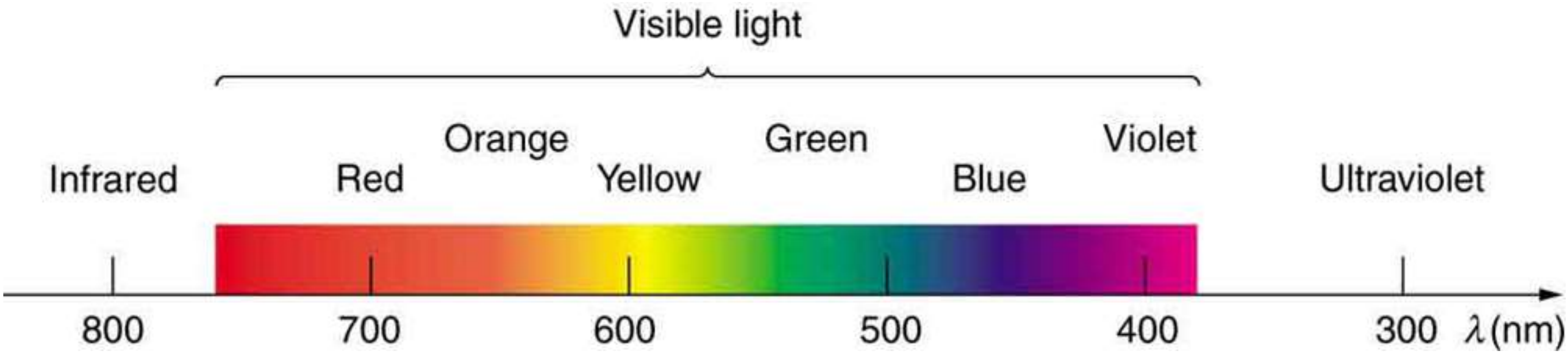


Infrared (IR) radiation

- Range:
  - 300 GHz to 430 THz
  - 700 nm to 1 mm
- Infrared = “below red” in the visible spectrum (i.e. lower frequency than red).
- Examples/applications:
  - Black-body radiation from objects near room temperature, including humans.
    - We’ll learn about black-body radiation when we learn about quantum mechanics.
    - Can be used e.g. for night-vision devices.

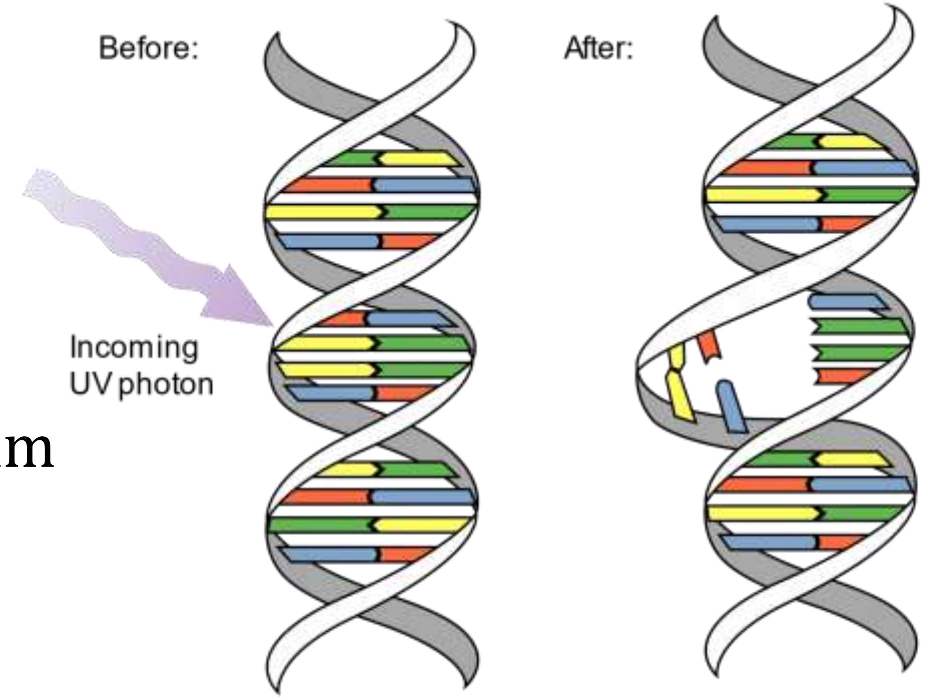
Visible light

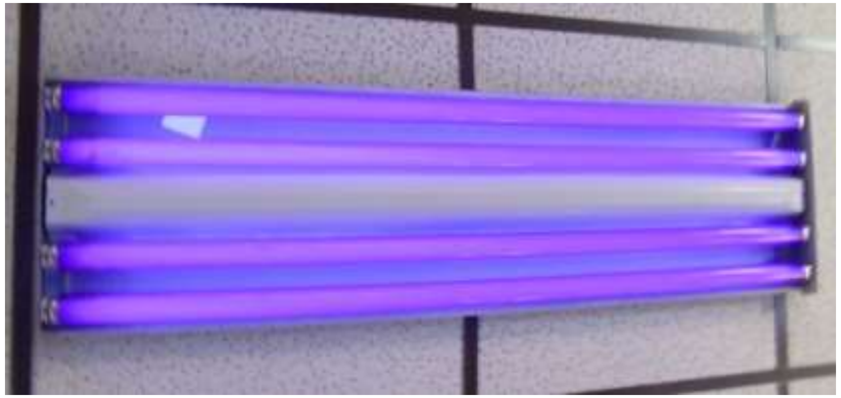
- Range:
  - 430 THz (red) to 750 THz (violet)
  - 400 nm (violet) to 700 nm (red)
- Live demonstration: lasers



Ultraviolet (UV) radiation

- Range:
  - 750 THz to 30 PHz (petahertz = 10^{15} Hz)
  - 10 nm to 400 nm
    - UV-A: 315-400 nm
    - UV-B: 280-315 nm
    - UV-C: 100-280 nm
- Ultraviolet = "above violet" in the visible spectrum (i.e. higher frequency than violet).
- Examples/applications:
  - Suntan and sunburn.
  - UV-A: Blacklight, can be useful for medical diagnosis.
    - Live demonstration
  - UV-B: Can cause skin cancer, but also important for creating vitamin D.
  - UV-C: Germicide.

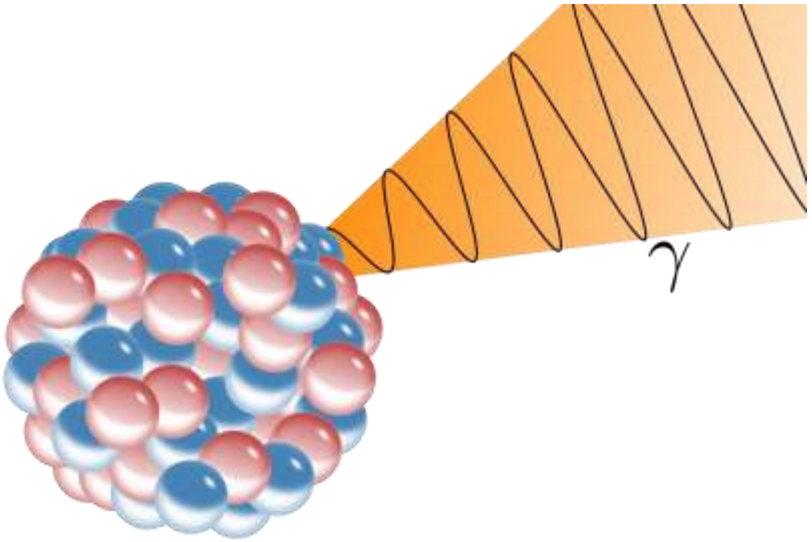




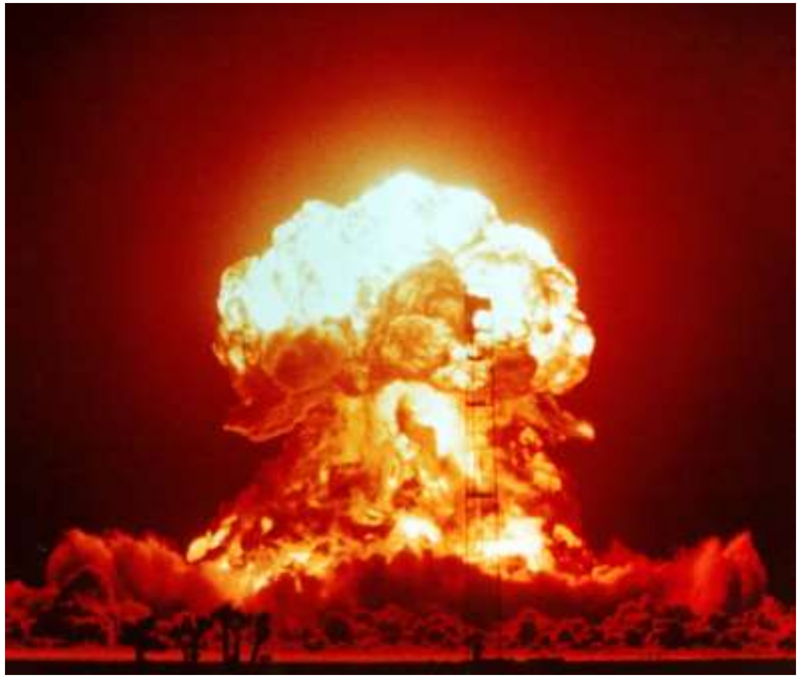
X-rays

- Range:
  - 30 PHz to 30 EHz (exahertz = 10^{18} Hz)
  - 0.01 nm to 10 nm
- Examples/applications:
  - Medical imaging (X-ray radiography, CT). X-rays pass through most objects, but are absorbed e.g. by calcium in the bones.
  - Higher frequency than UV means it's even more dangerous to humans. Smaller wavelength means it can penetrate cells.

Gamma rays



- Range:
  - 30 EHz and above
  - 0.01 nm and below
- Examples/applications:
  - Radioactive decay of atomic nuclei
  - Highest frequency and lowest wavelength, thus the most dangerous.



Ionizing radiation

- Ionizing radiation has sufficient energy to ionize atoms or molecules.
- Frequency is proportional to energy (as we'll see in ch. 29).
  - Anything from high ultraviolet and above is ionizing radiation, including X-rays and gamma rays.
  - Anything below is non-ionizing radiation.
- Electromagnetic waves (photons) are not the only source of ionizing radiation.
  - Other example include neutrons, alpha particles (helium-4 nucleus with 2 protons and 2 neutrons), and "beta particles" (electrons or positrons).

24.4 Energy in Electromagnetic Waves

Energy of electromagnetic waves

- Recall that wave energy is proportional to amplitude squared.
- In electromagnetic waves, the amplitude is the magnitude of the electric and magnetic fields.
- Average intensity (power per unit area, recall section 16.11):

$$I_{\text{ave}} = \frac{1}{2} c\varepsilon_0 E_0^2 = \frac{cB_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0}$$

- E_0 = maximum electric field magnitude.
- B_0 = maximum magnetic field magnitude.
- The three expressions are related using $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and $E_0 = cB_0$.
- Peak intensity (since wave is sinusoidal):

$$I_{\text{peak}} = 2I_{\text{ave}}$$

Live demonstration

Crookes radiometer