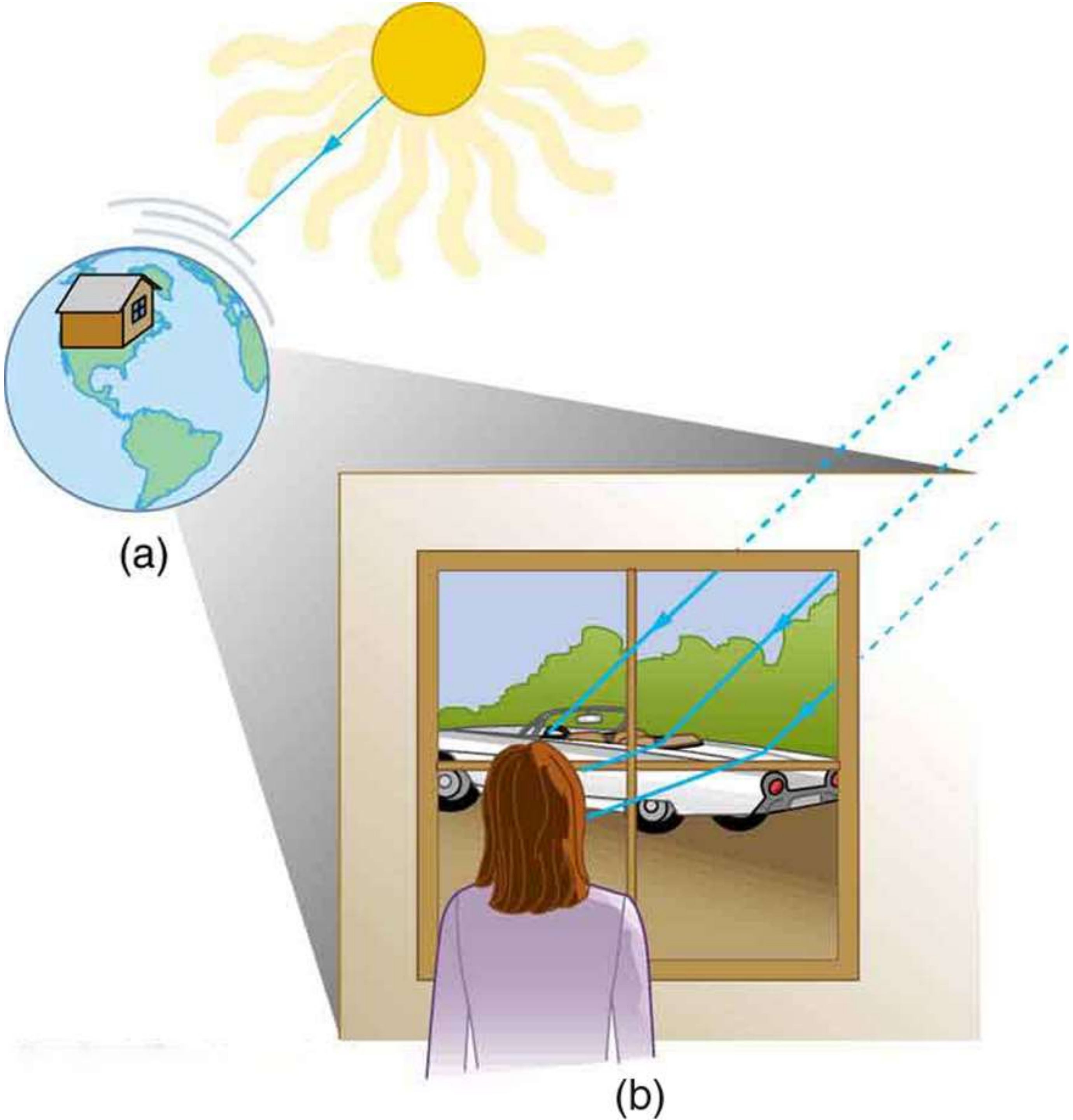
PHYS 1P22/92

Prof. Barak Shoshany

Spring 2024

25. Geometric Optics

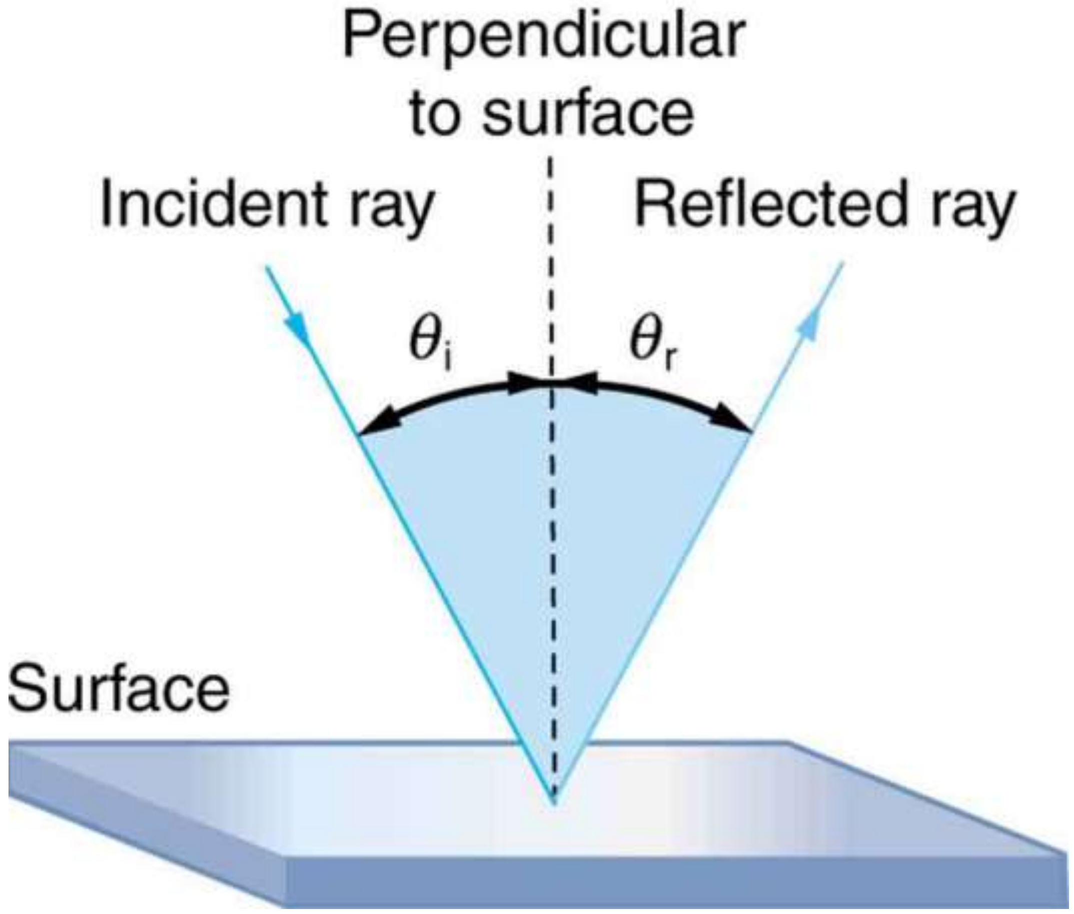
25.1 The Ray Aspect of Light


(a)
(b)

25.2 The Law of Reflection

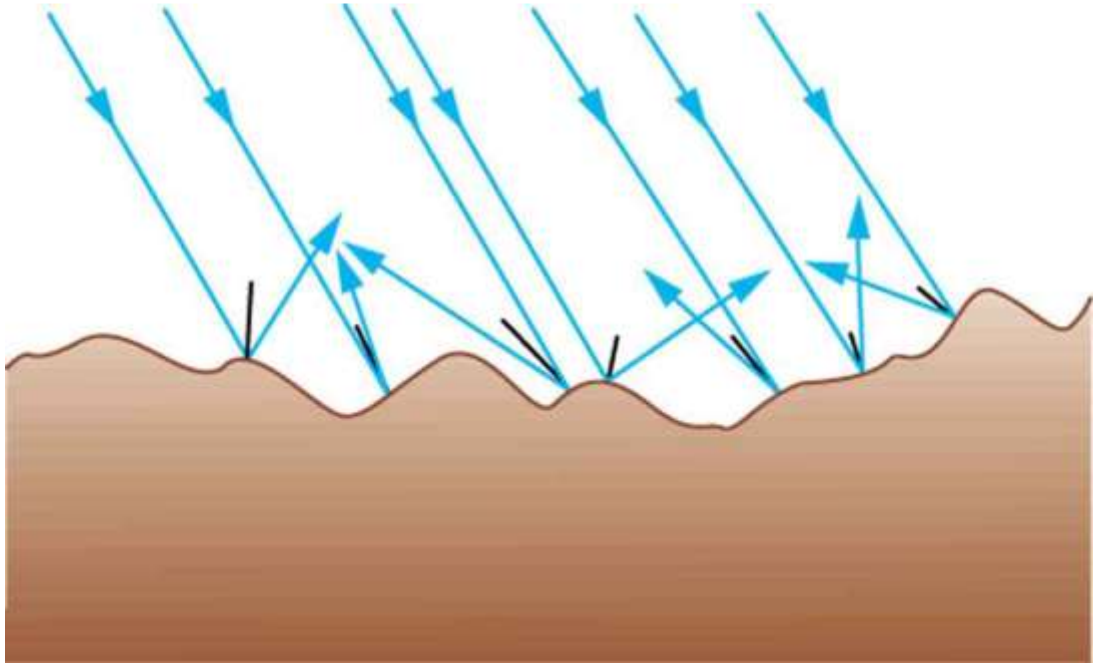
Reflection from a mirror

- Law of reflection: The angle of reflection θ_r equals the angle of incidence θ_i.
- Angles are measured relative to the perpendicular to the surface.



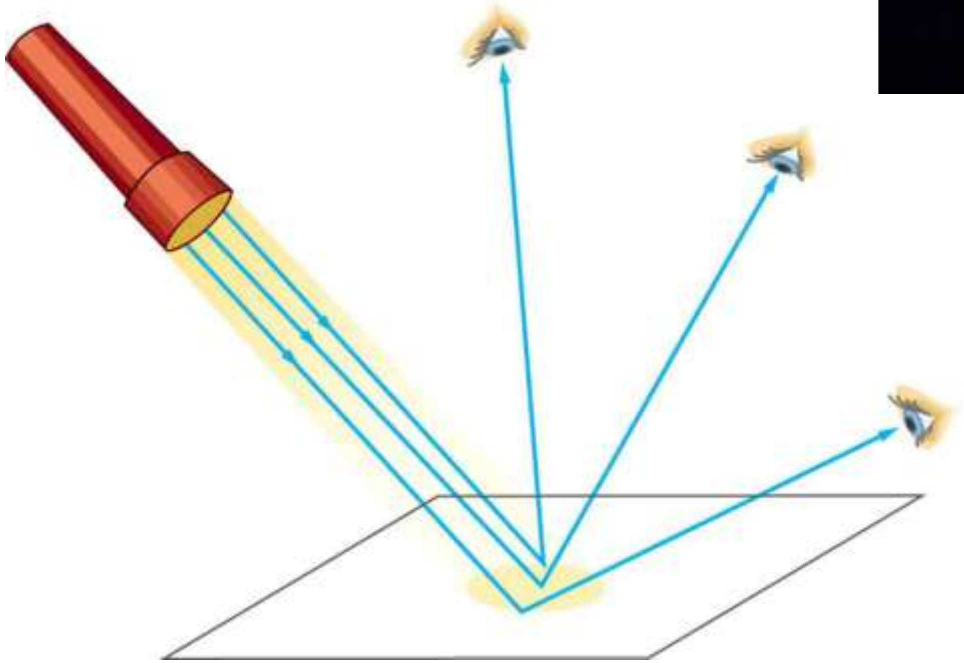
Reflection from a non-even surface

- Rays come in parallel, but reflect in different directions since the perpendiculars are different at each point.



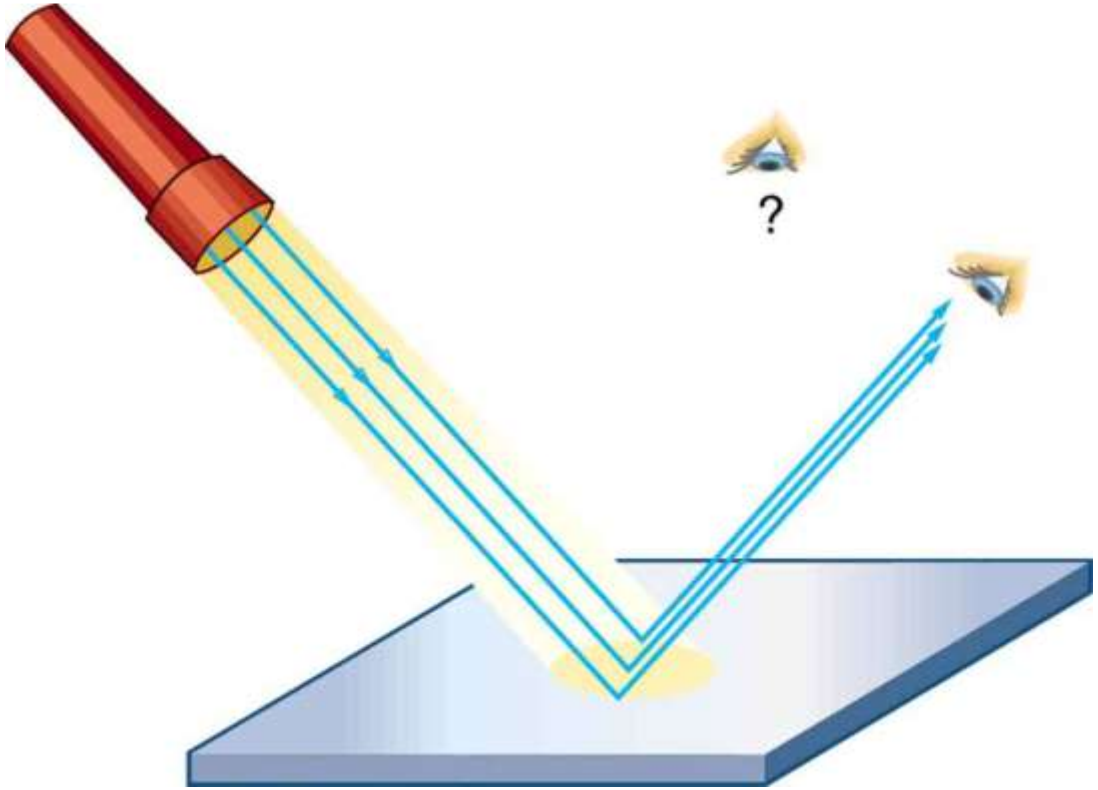
Diffusion

- Rough surface (sheet of paper or surface of a lake) diffuses the parallel rays so they reflect in all directions.



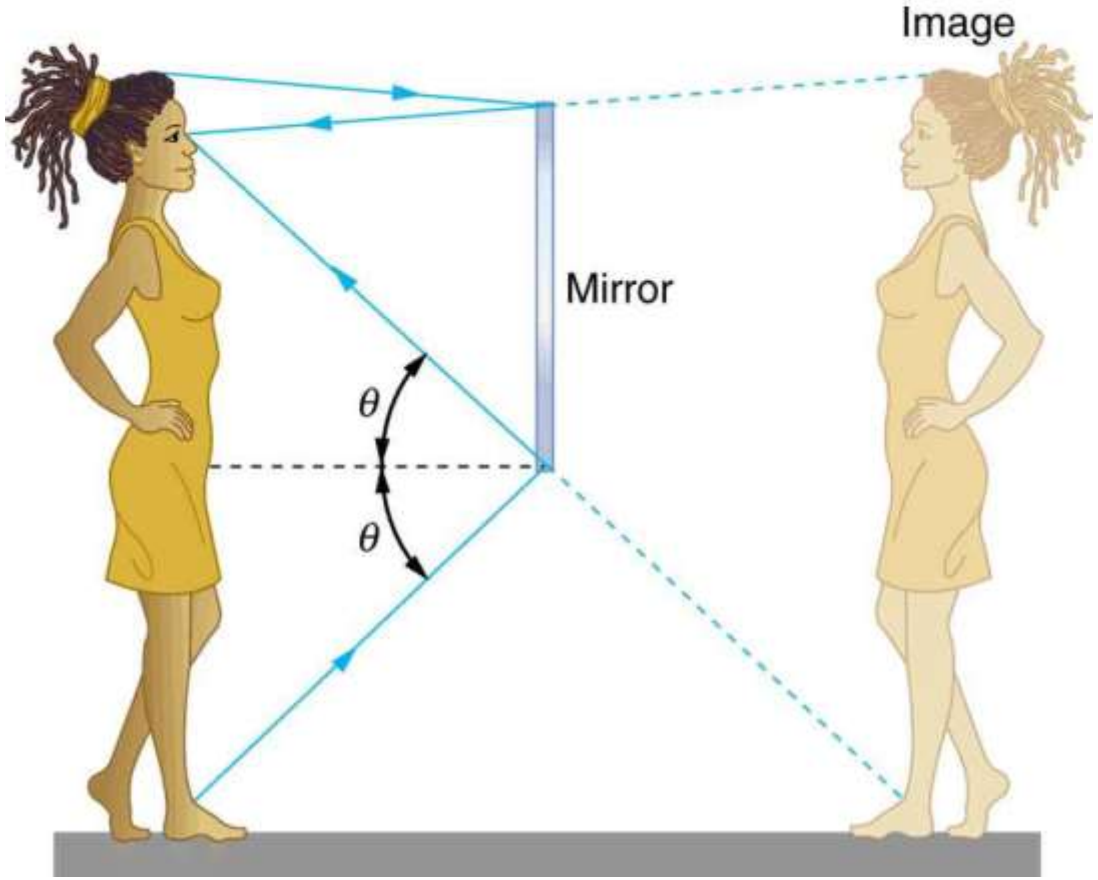
No diffusion

- Mirror reflects the parallel rays in one direction only.



Images in a mirror

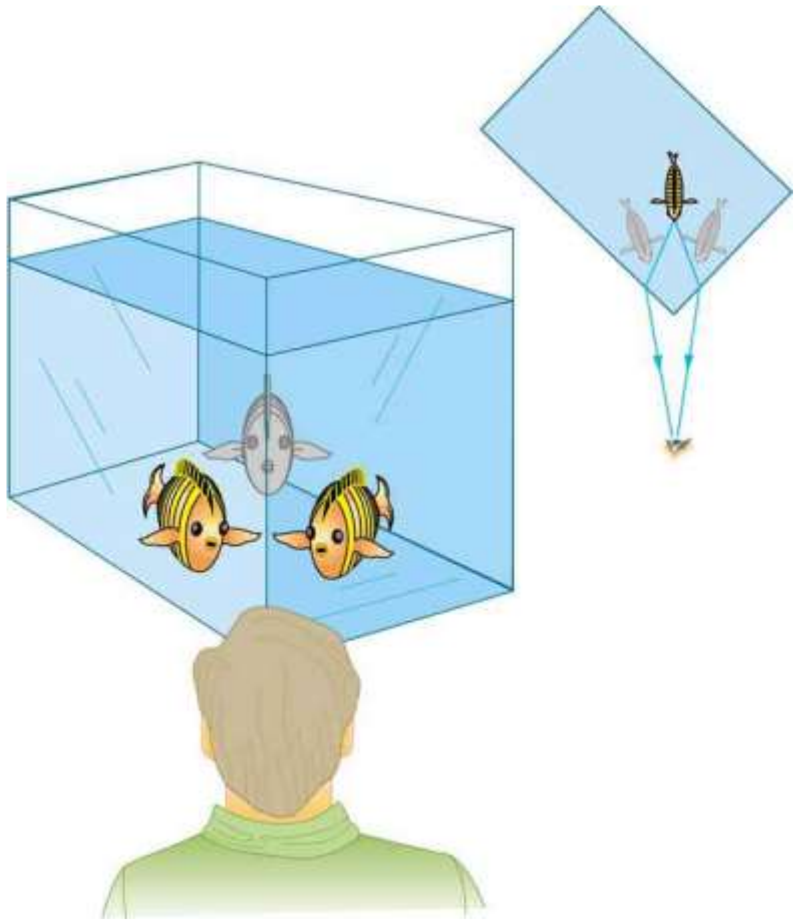
- Image looks as if it's "behind" the mirror.
- Each ray obeys the law of reflection.



25.3 The Law of Refraction

Refraction

- Refraction is the change in direction of a light ray when it passes between two different mediums.
- Example: the fish appears to be in two different locations because the light refracts when it passes from water to air.



Index of refraction

- The index of refraction for a material is:

$$n \equiv \frac{c}{v}$$

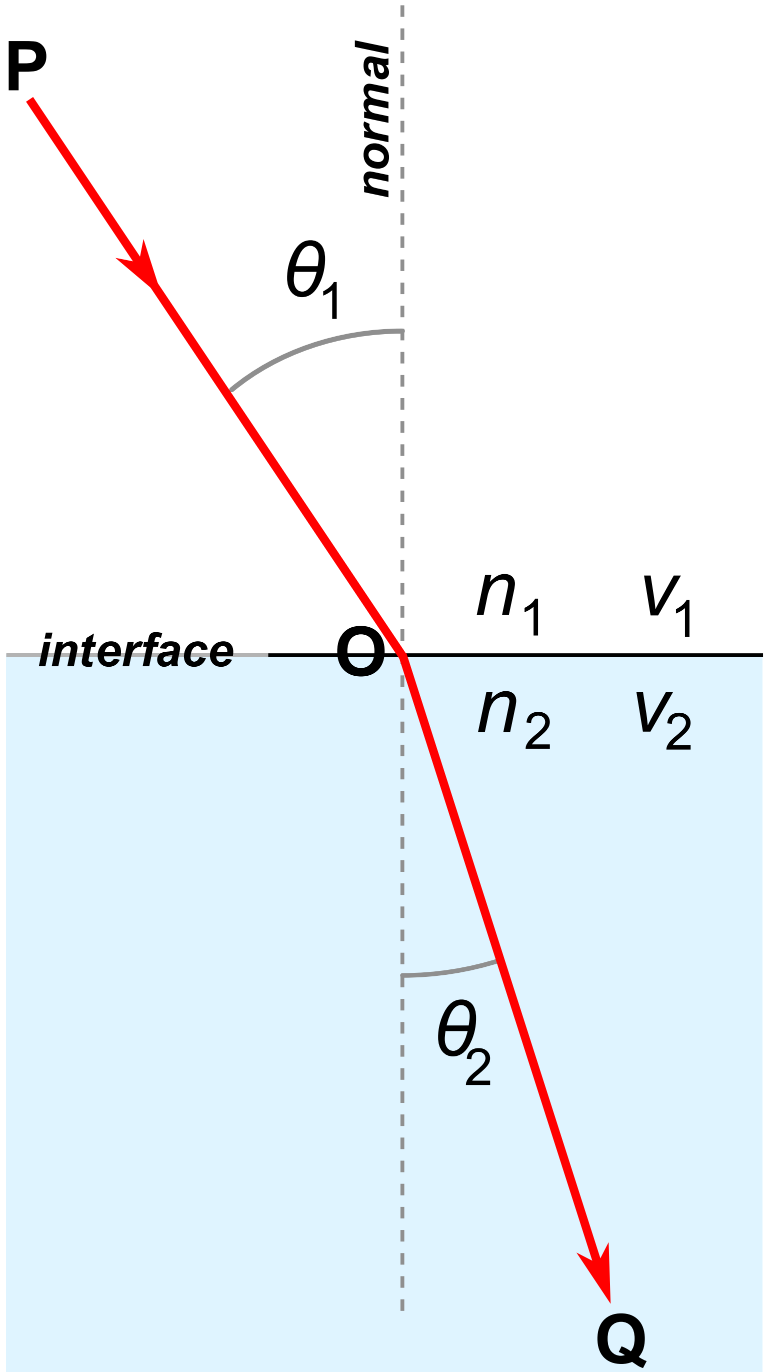
- $c \equiv 299{,}792{,}458 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$ = speed of light **in vacuum.**
- v = speed of light **in the material.**
- Examples:
  - $n = 1$ in vacuum ($v = c$).
  - $n \approx 1.0003$ in air ($v \approx c$).
  - $n \approx 1.333$ in fresh water.
  - $n \approx 2.419$ in diamond.

Law of refraction (Snell's law)

- When light passes from medium 1 to medium 2:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- n_1 = index of refraction for medium 1
- n_2 = index of refraction for medium 2
- θ_1 = angle between ray and perpendicular in medium 1
- θ_2 = angle between ray and perpendicular in medium 2



Class problem:

- Medium 1: air, $n_1 \approx 1.000$
- Medium 2: diamond, $n_2 \approx 2.419$
- Incident angle: $\theta_1 \approx 30.00° = \frac{\pi}{6}\ rad$

What is the angle of refraction θ_2?

Solution: Isolate angle from Snell's law:

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$

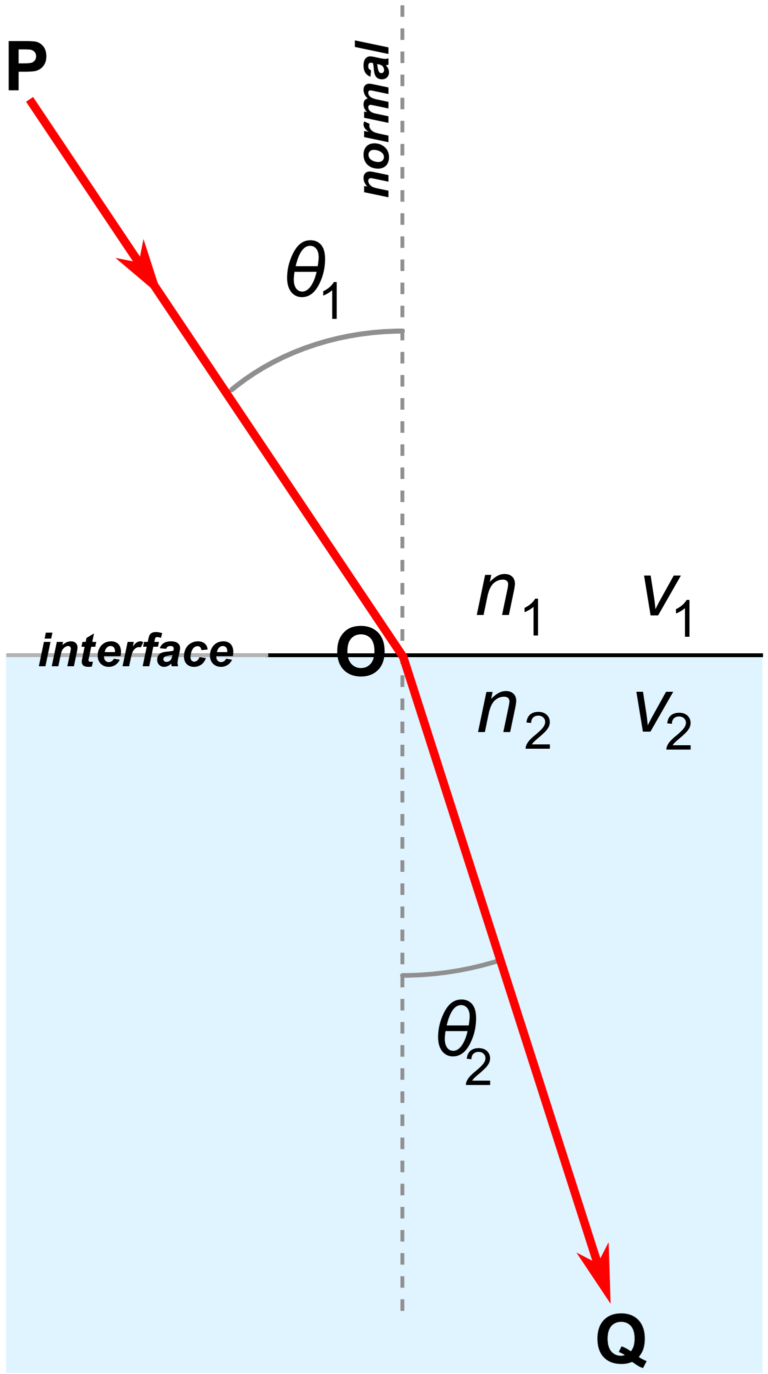
$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\theta_2 = \arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

Plug in the numbers:

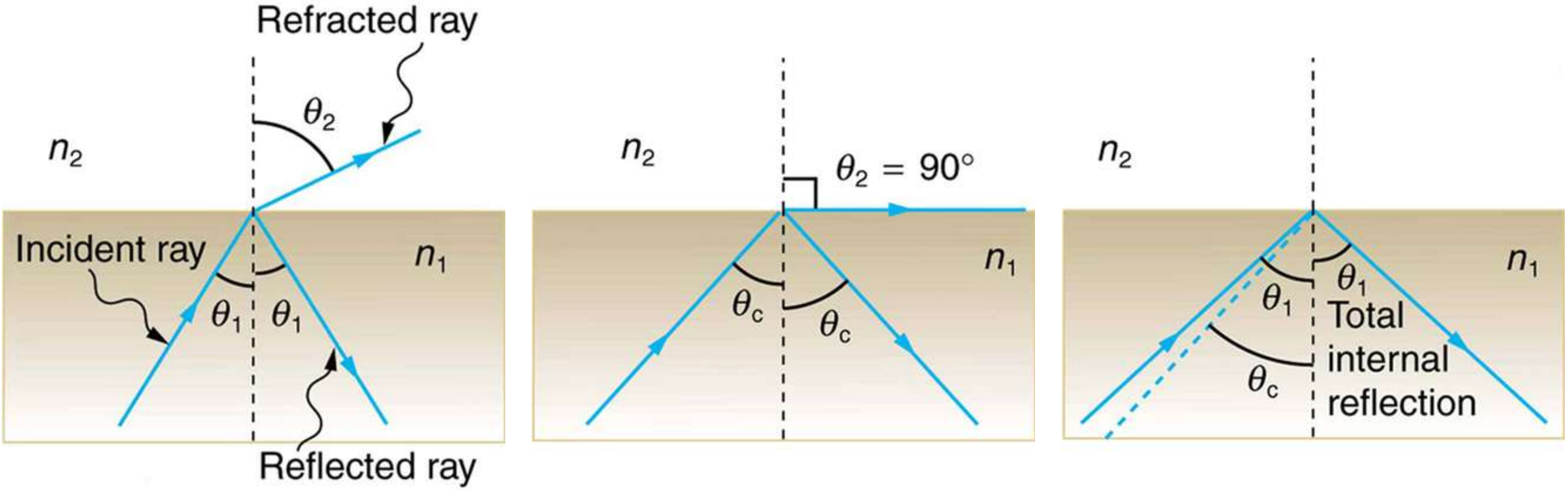
$$\theta_2 = \arcsin\left(\frac{1.000}{2.419}\sin\frac{\pi}{6}\right) \approx \arcsin 0.2067$$

$$\approx 0.2082 \text{ rad} \approx 11.93°$$



25.4 Total Internal Reflection

Critical angle θ_c



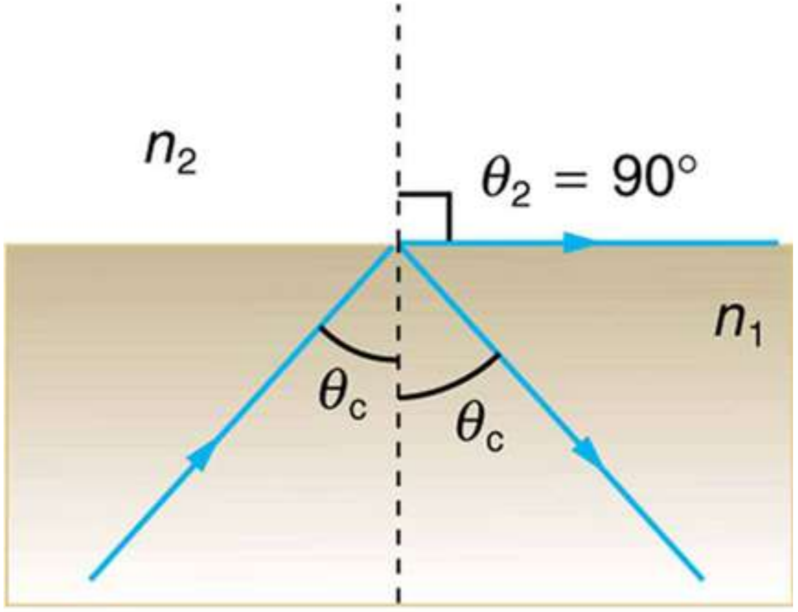
Normal refraction
$\theta_2 < 90°$

Critical refraction
$\theta_2 = 90°$
$\theta_1 = \theta_c$

Total internal reflection
$\theta_1 > \theta_c$

Derivation of the critical angle

- Start from Snell's law:
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
- If $\theta_2 = 90°$ then $\sin\theta_2 = 1$:
$$n_1 \sin\theta_1 = n_2$$
- Isolate $\theta_1 = \theta_c$:
$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\theta_c = \arcsin\frac{n_2}{n_1}$$
- This only works if $n_1 > n_2$ (why?)



Demonstrations

Online demonstration:

https://phet.colorado.edu/sims/html/bending-light/latest/bending-light_en.html

(Go to “More Tools” and enable angles)

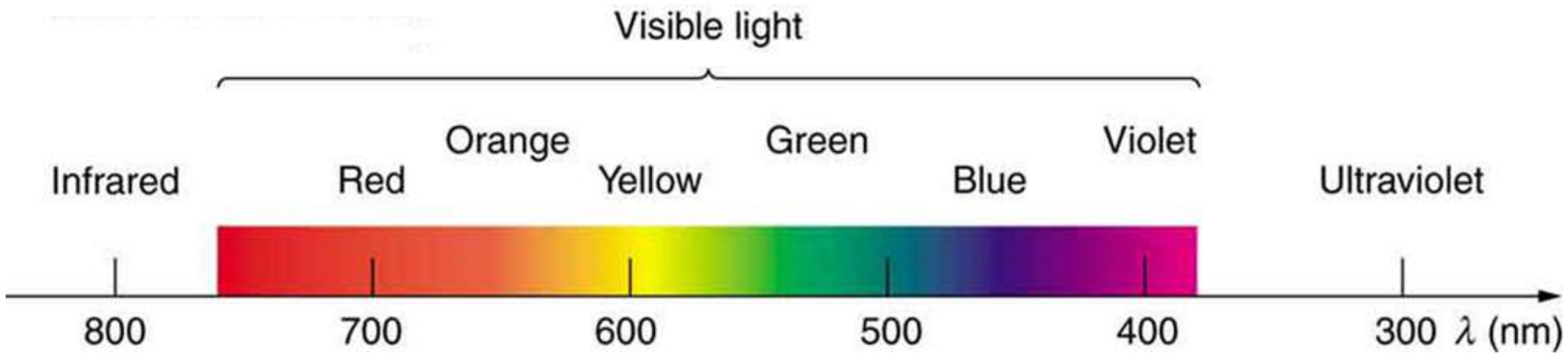
Live demonstration:

Total internal reflection with laser

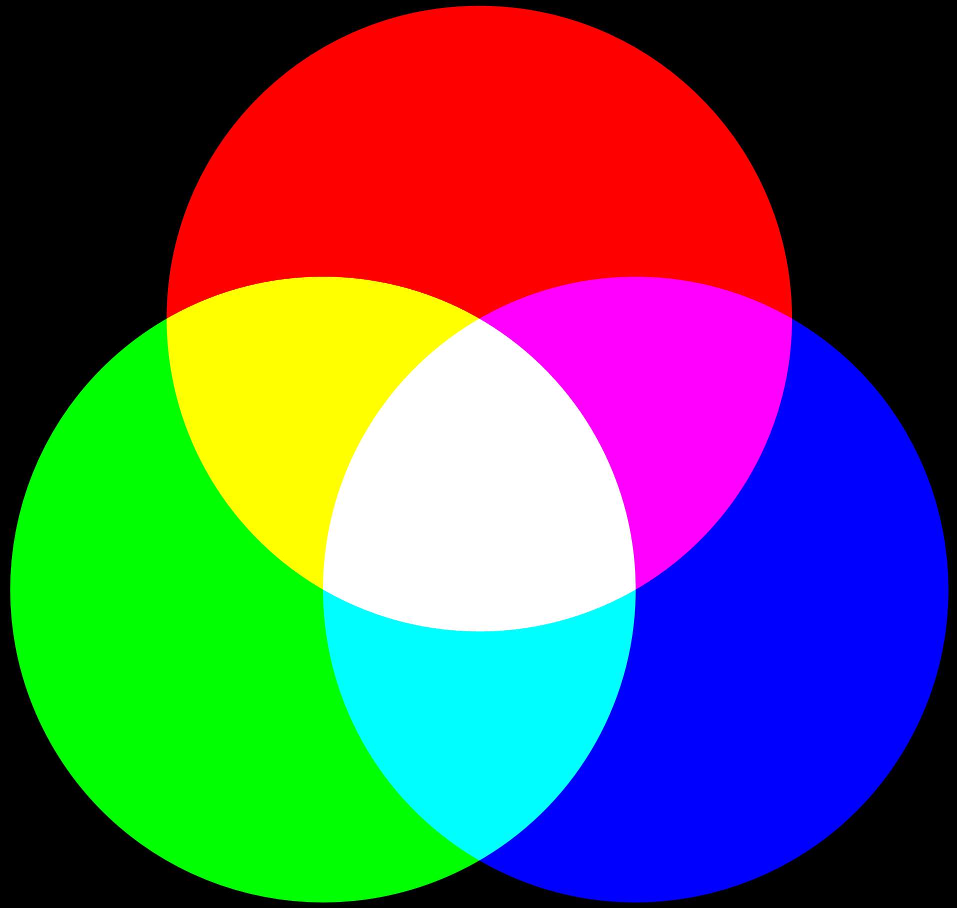
25.5 Dispersion: The Rainbow and Prisms

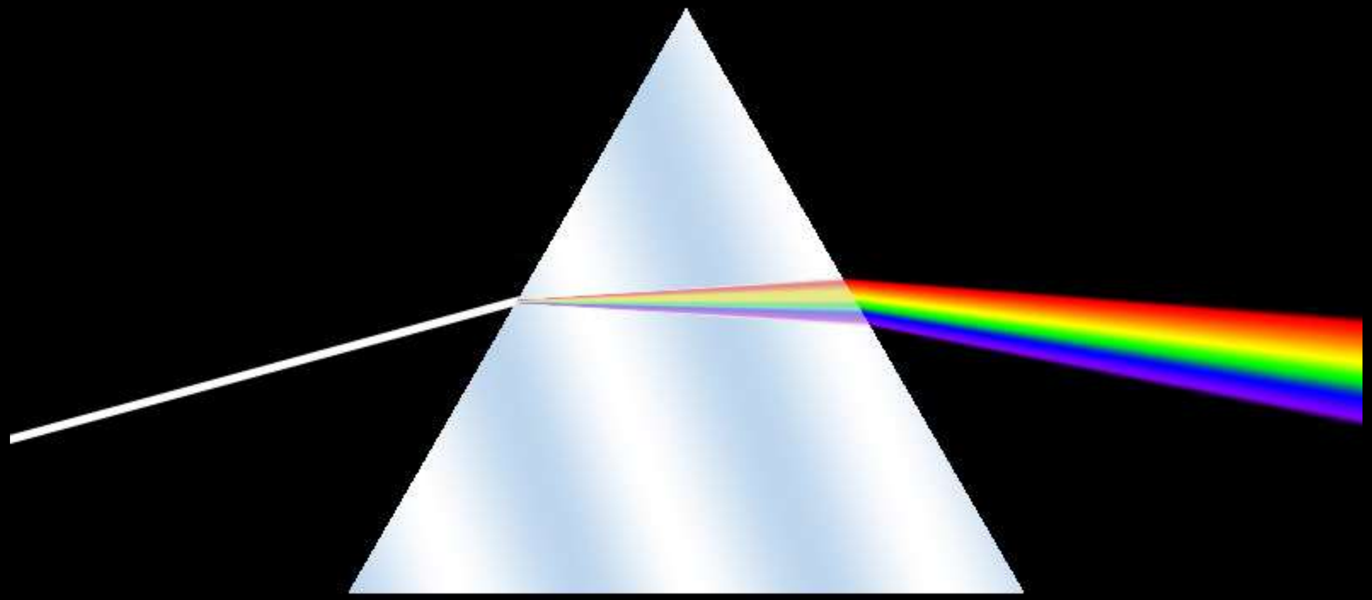
Sunlight

- The Sun emits wavelengths from the entire electromagnetic spectrum, from radio waves to X rays, but mostly infrared, visible, and ultraviolet.
- The most powerful wavelengths, in watt per unit area, are visible light (that's why our eyes evolved to see these wavelengths).



The color “white” isn’t a specific wavelength; it’s a uniform mixture of some or all visible wavelengths.

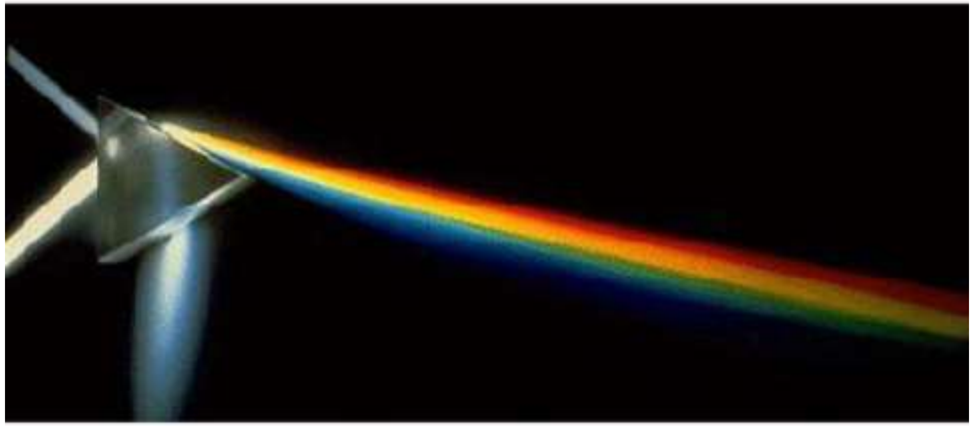




Dispersion

- Dispersion is the spreading of white light into its full spectrum of wavelengths.
- We can see examples of dispersion in both rainbows (a) and prisms (b).

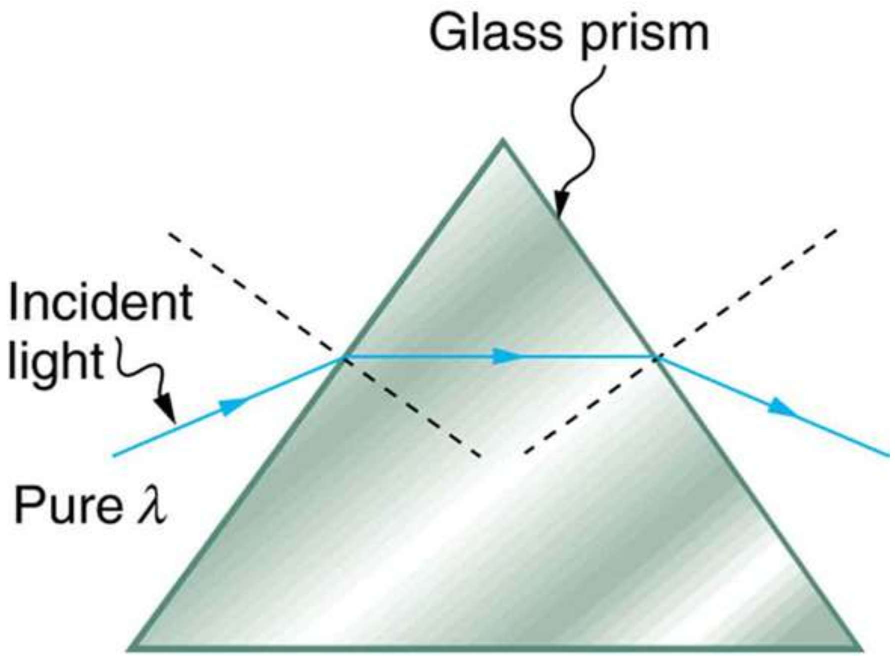
(a)

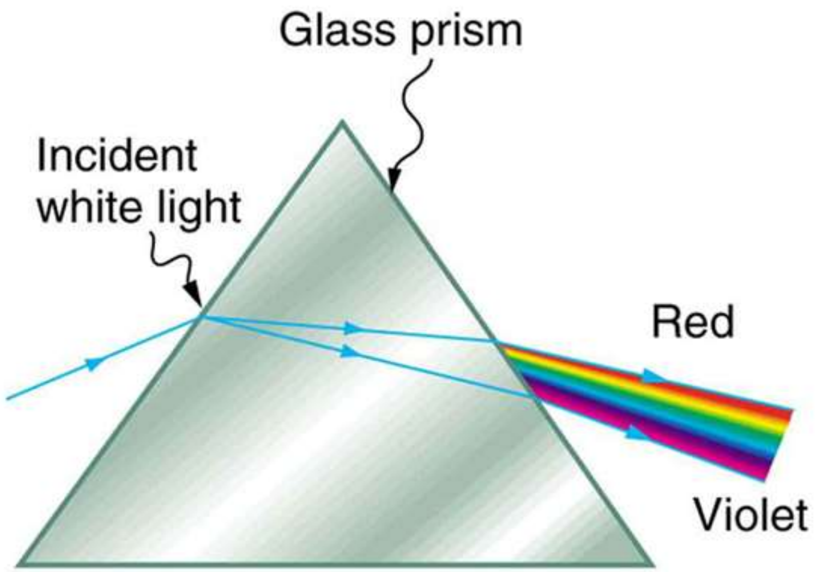


(b)

- Dispersion happens because different wavelengths have slightly different indices of refraction within the same medium.
- For example, in water:

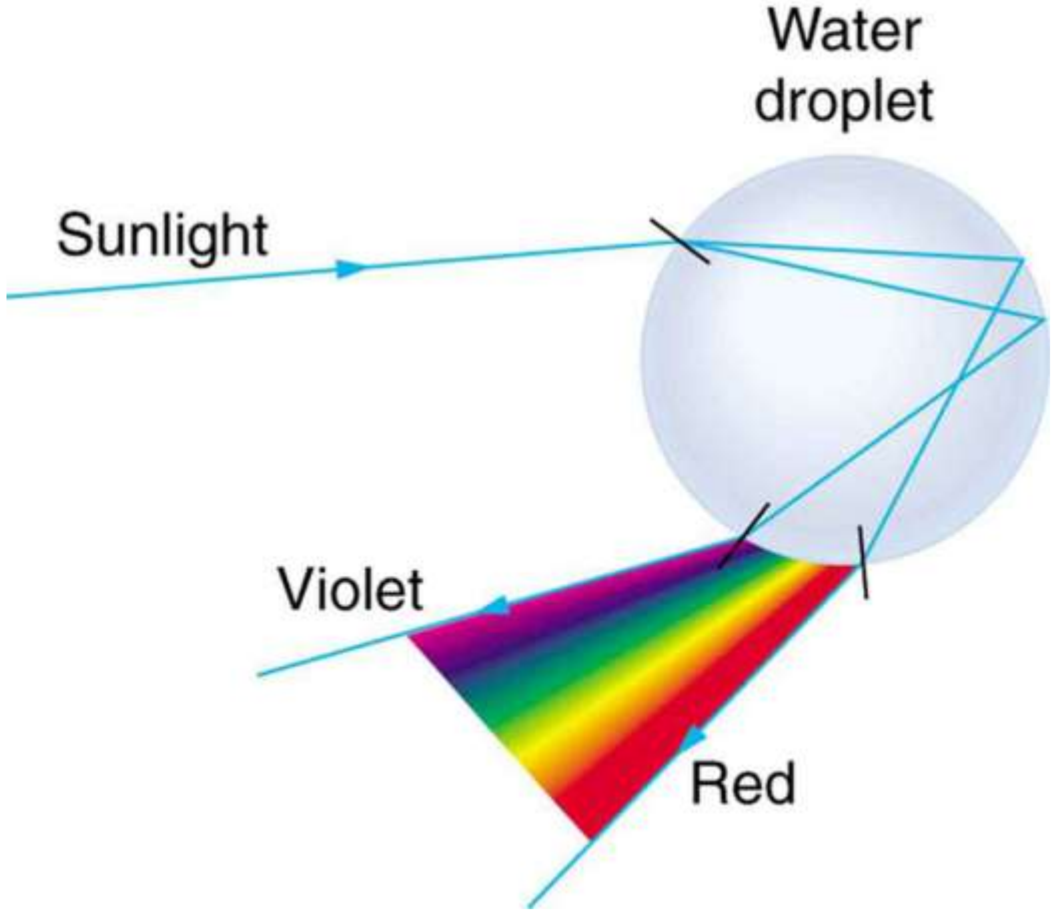
Red (660 nm)	Orange (610 nm)
1.331	1.332
Yellow (580 nm)	Green (550 nm)
1.333	1.335
Blue (470 nm)	Violet (410 nm)
1.338	1.342





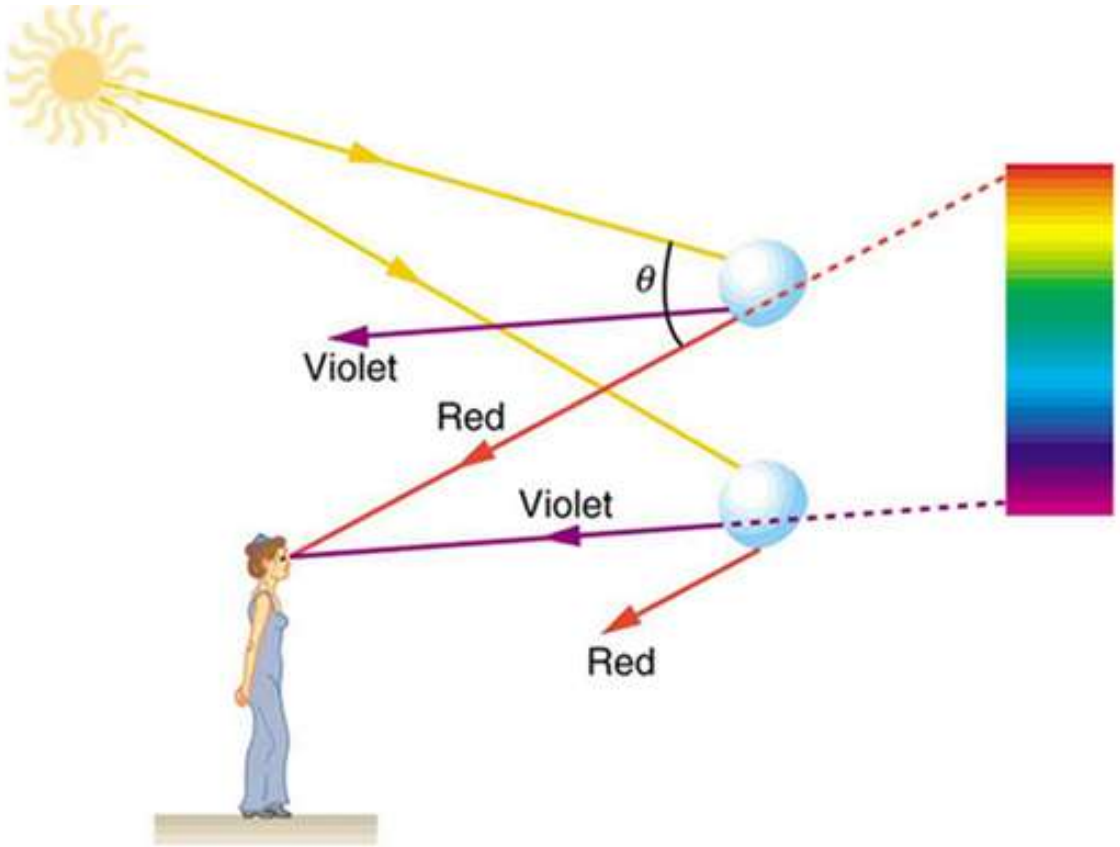
Rainbows

- Light is reflected from the back of the drop.
- It is refracted and dispersed both when it enters and leaves the drop.
- Reflection does not depend on wavelength, so there's no dispersion due to the reflection.



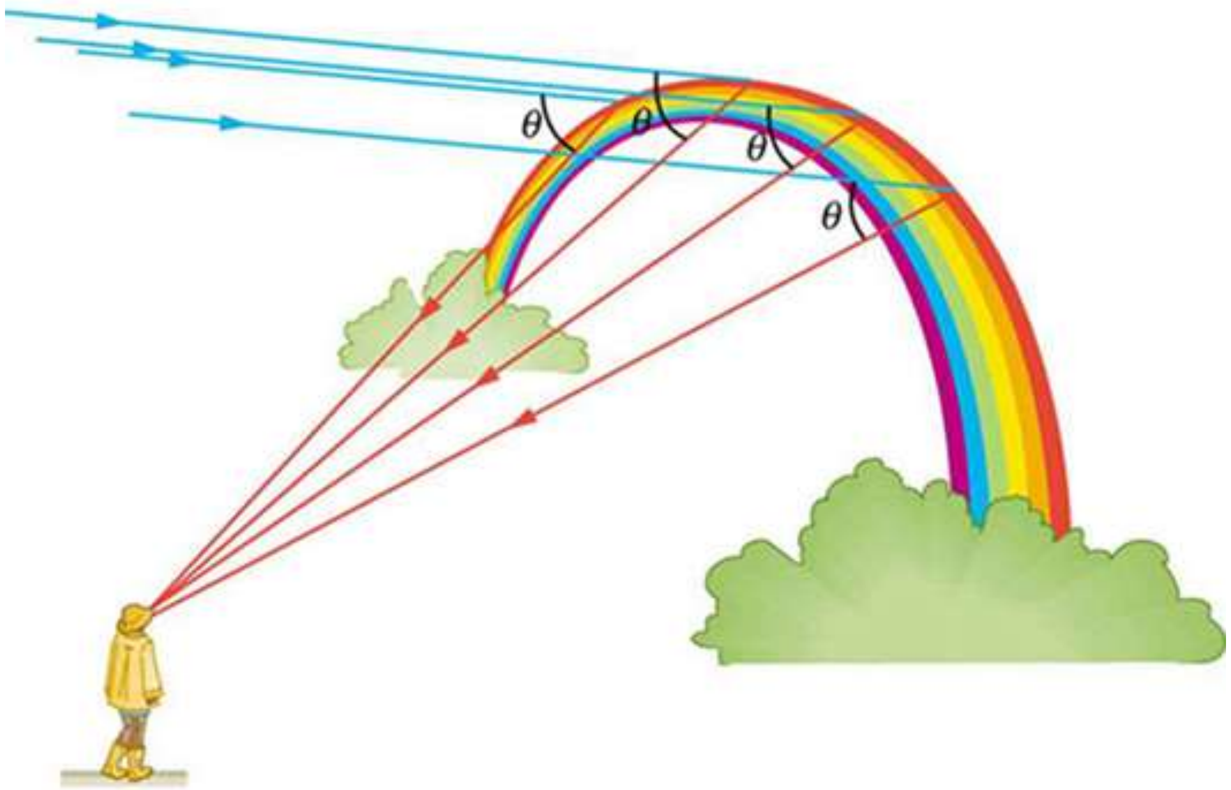
Rainbows

- Rays are refracted and reflected from different drops of water at different places.



Rainbows

- The rainbow looks like an arc because each line between the observer and the rainbow must make the same angle with the parallel rays of sunlight to receive the refracted rays.

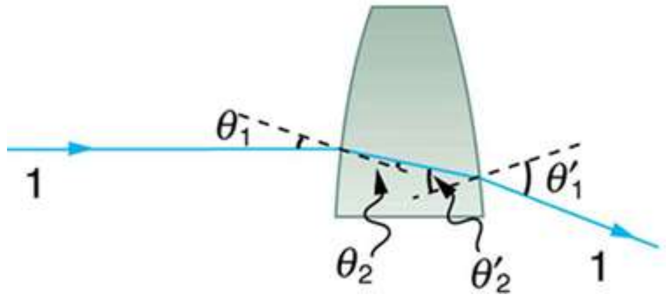


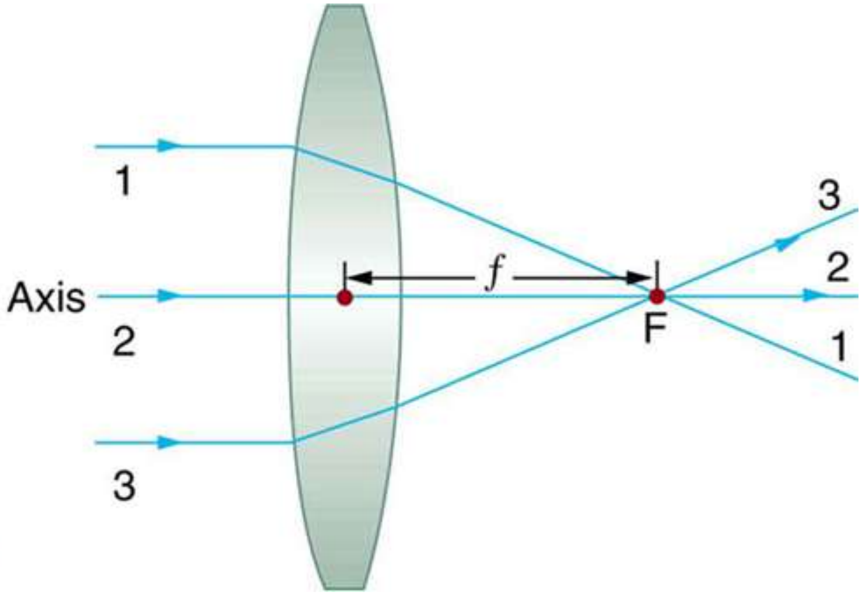
Rainbows are actually full circles! However, usually only the upper arc is above ground.

25.6 Image Formation by Lenses

Converging (convex) lens

- Light rays that enter the lens parallel to its axis cross one another, converging at a single point on the opposite side.
- F = **focal point**, the point where the rays **cross** on the **other** side.
- f = **focal length**, the distance from the center of the lens to F.





Converging (convex) lens

- Example: magnifying glass.
- Focusing sunlight can burn paper at the focal point.

Power of a lens

- Power is the inverse of the focal length:

$$P = \frac{1}{f}$$

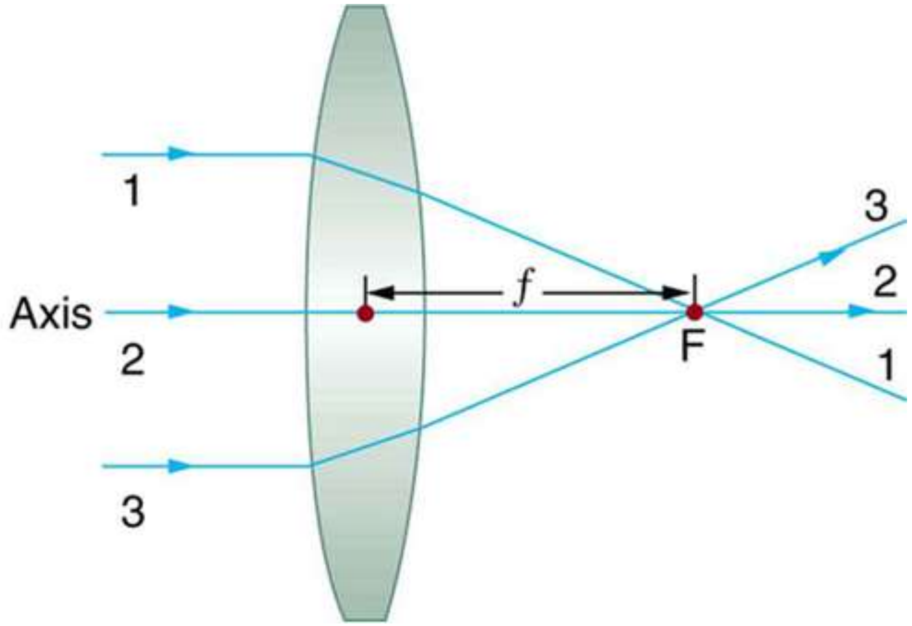
- Unit: inverse meters, also called diopters.

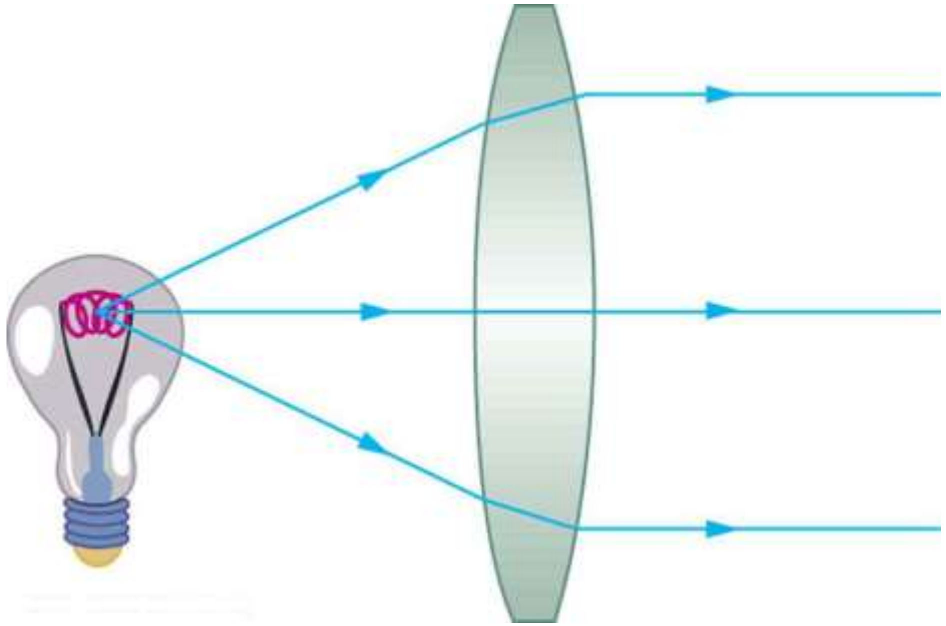
$$\mathrm{D} = \frac{1}{\mathrm{m}}$$

- Note: This is **not** related to the concept of power as energy per unit time, which is measured in watts.

Reversing the paths of light rays

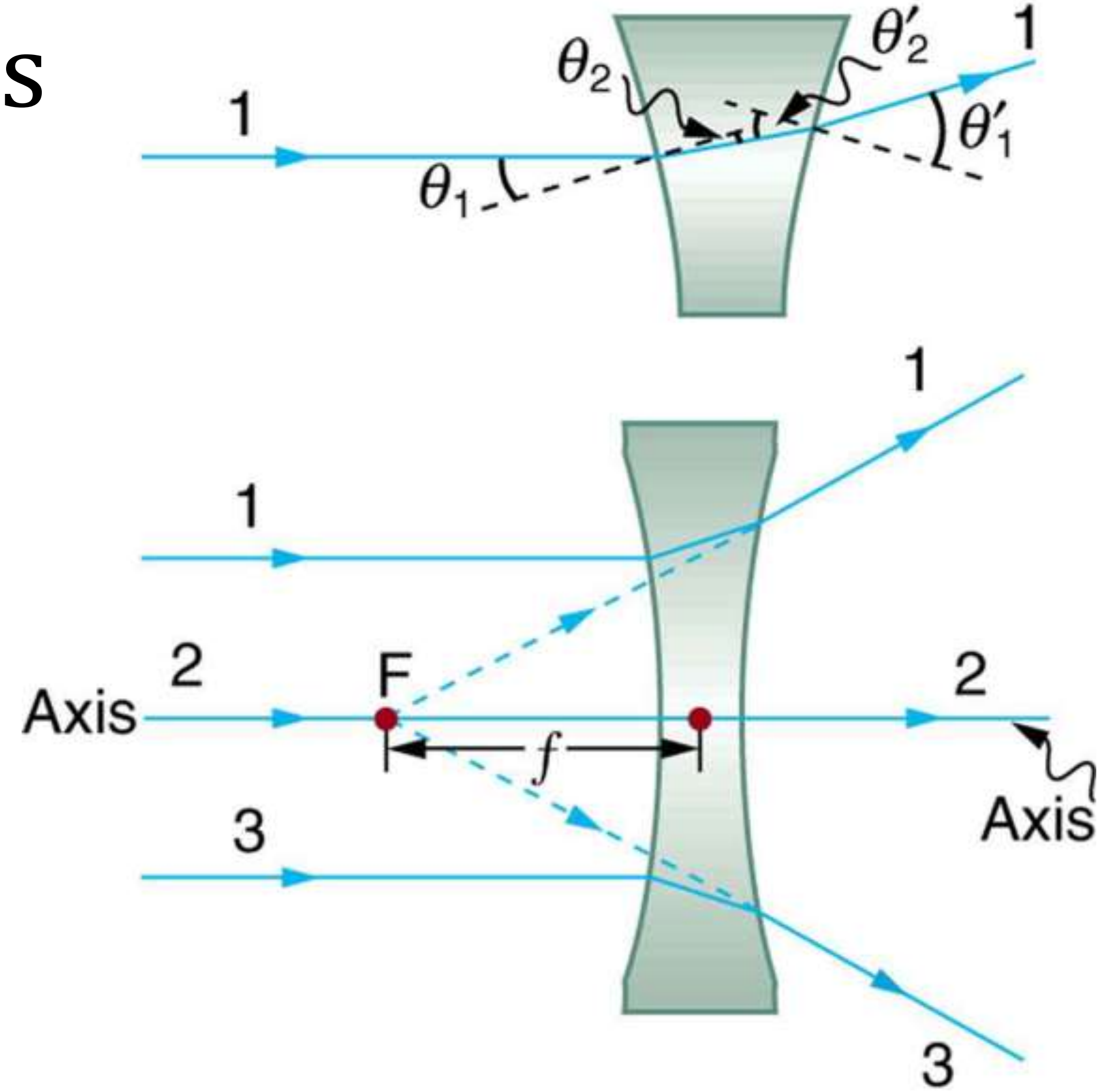
- We can always reverse the direction of the arrows.
- For example: instead of incoming parallel rays focusing at F, we can have diverging rays starting at F and emerging in parallel.





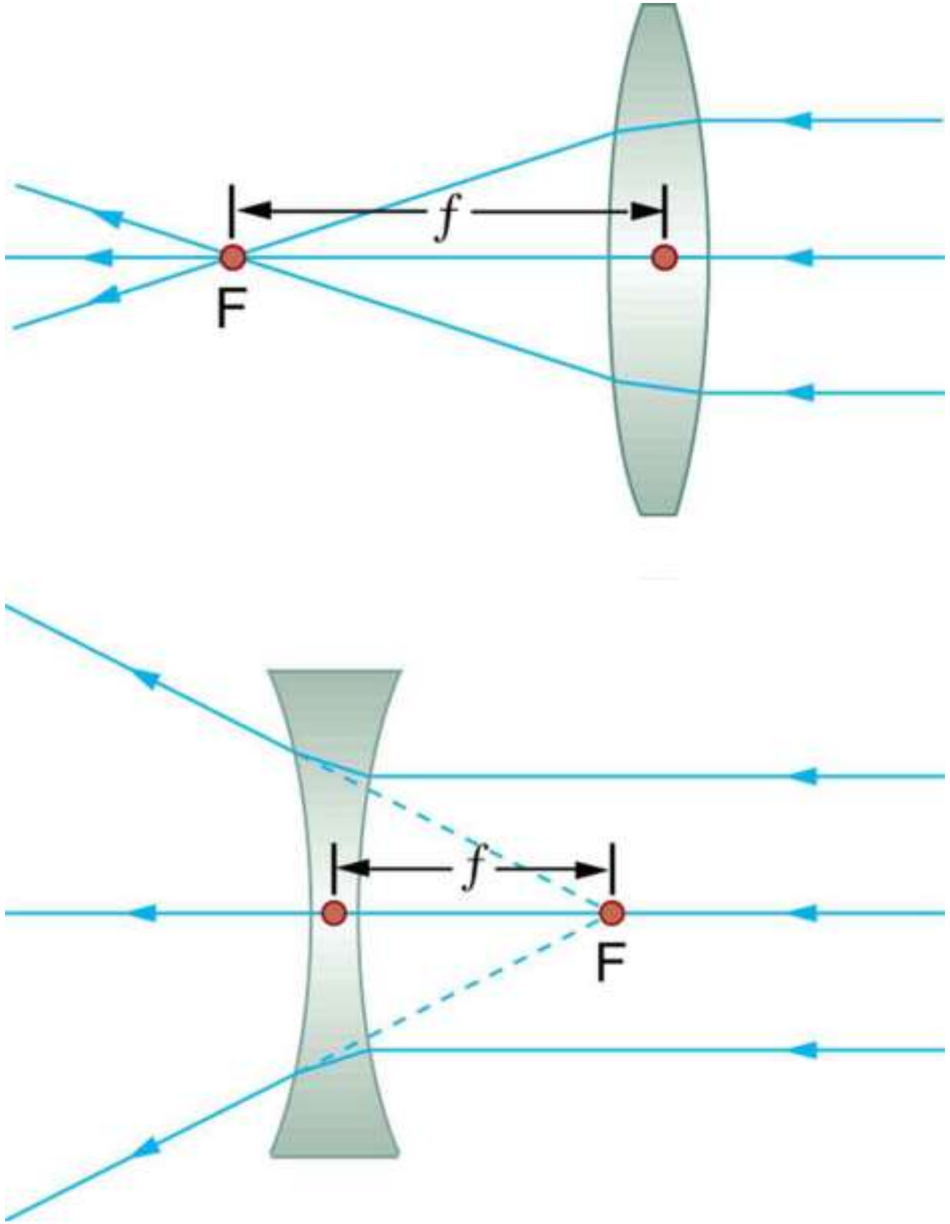
Diverging (concave) lens

- Mnemonic: concave = "there's a cave on both sides of the lens".
- Light rays diverge away from the axis.
- F = **focal point**, the point where the rays **appear to originate from** on the **same** side.
- f = **focal length**, the distance from the center of the lens to F. **Negative** since it's on the opposite side from converging lens.



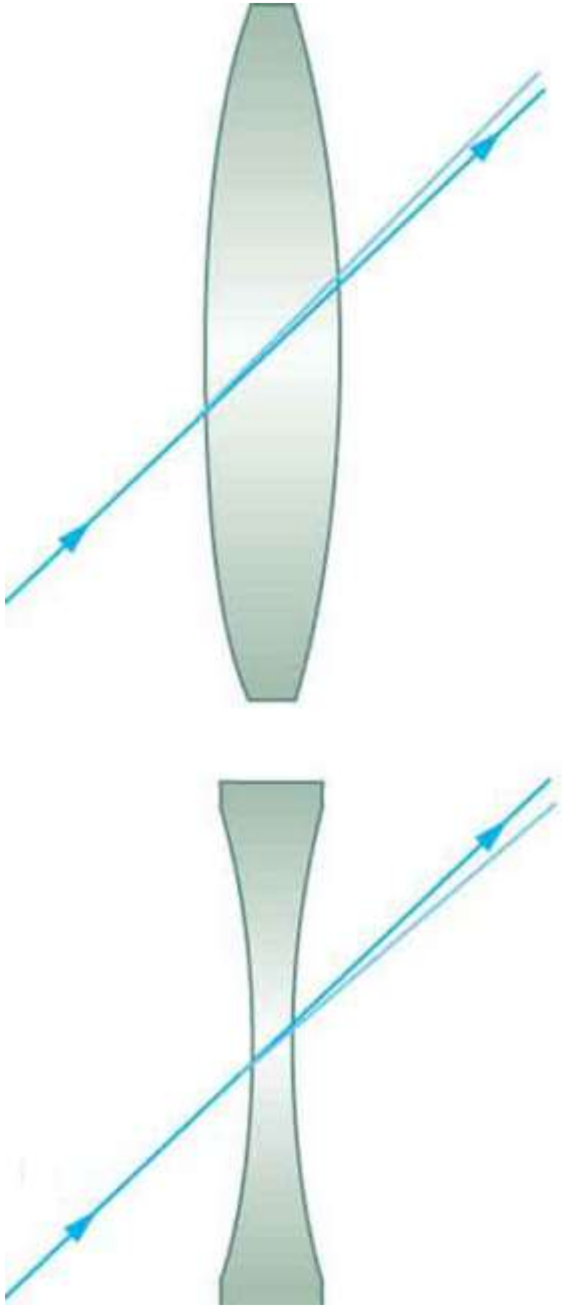
Thin lens

- Thin lens: thickness is negligible, so we can assume that light rays bend only once. This simplifies things.
- A thin symmetrical lens has two focal points, one on either side and both at the same distance f from the lens.



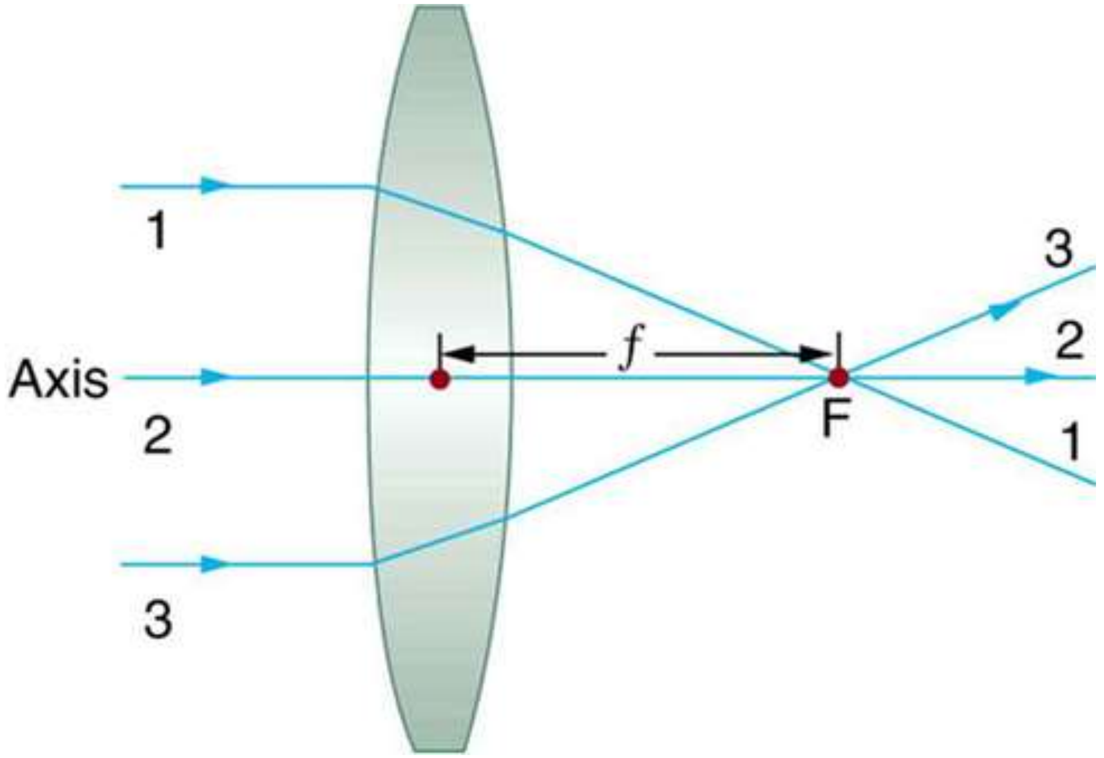
Thin lens

- The light ray through the center of a thin lens is deflected by a negligible amount and is assumed to emerge **parallel** to its original path.
- Again, this simplifies things.



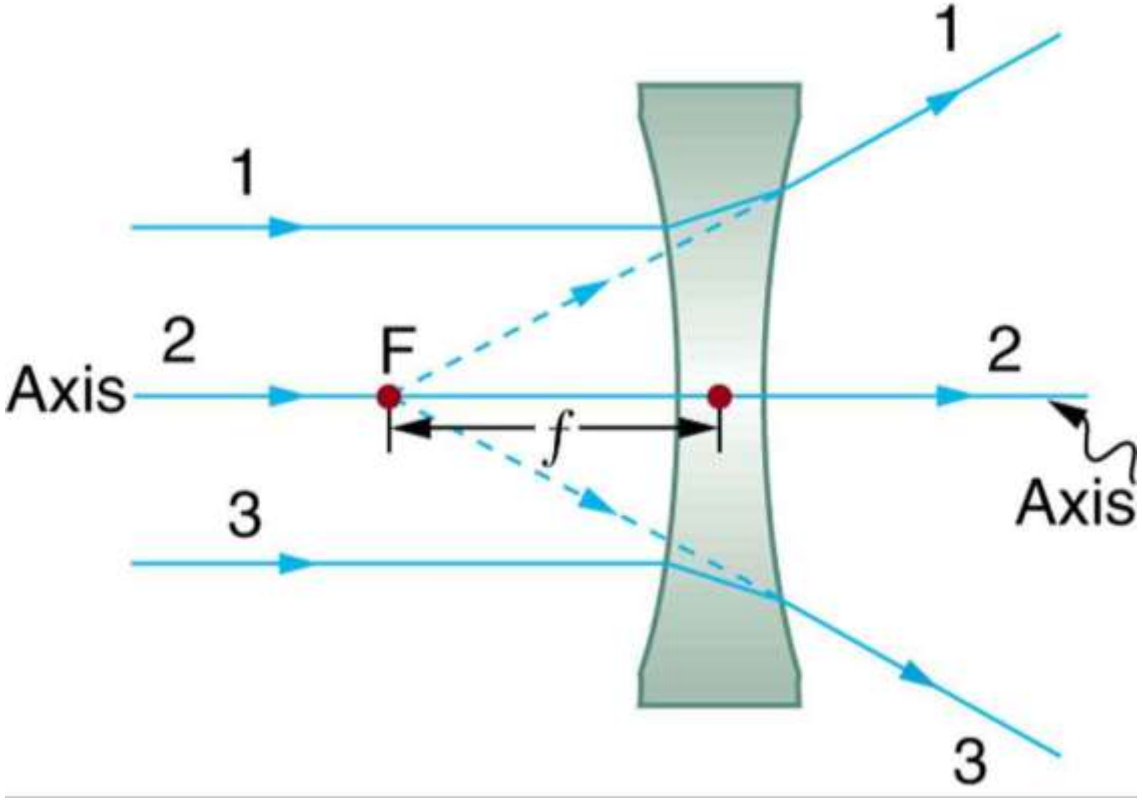
Ray tracing rules for thin lens

1. A ray entering a converging lens parallel to its axis passes through the focal point of the lens on the other side.



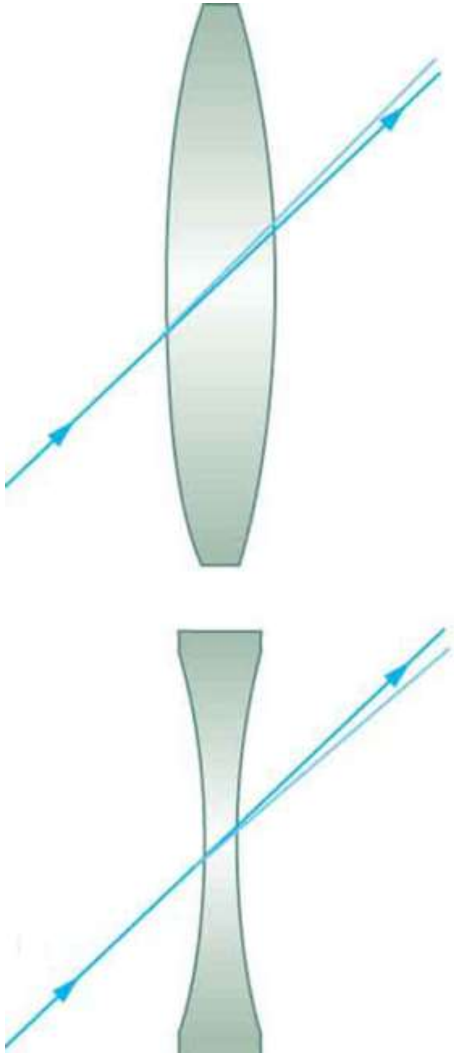
Ray tracing rules for thin lens

2. A ray entering a diverging lens parallel to its axis seems to come from the focal point.



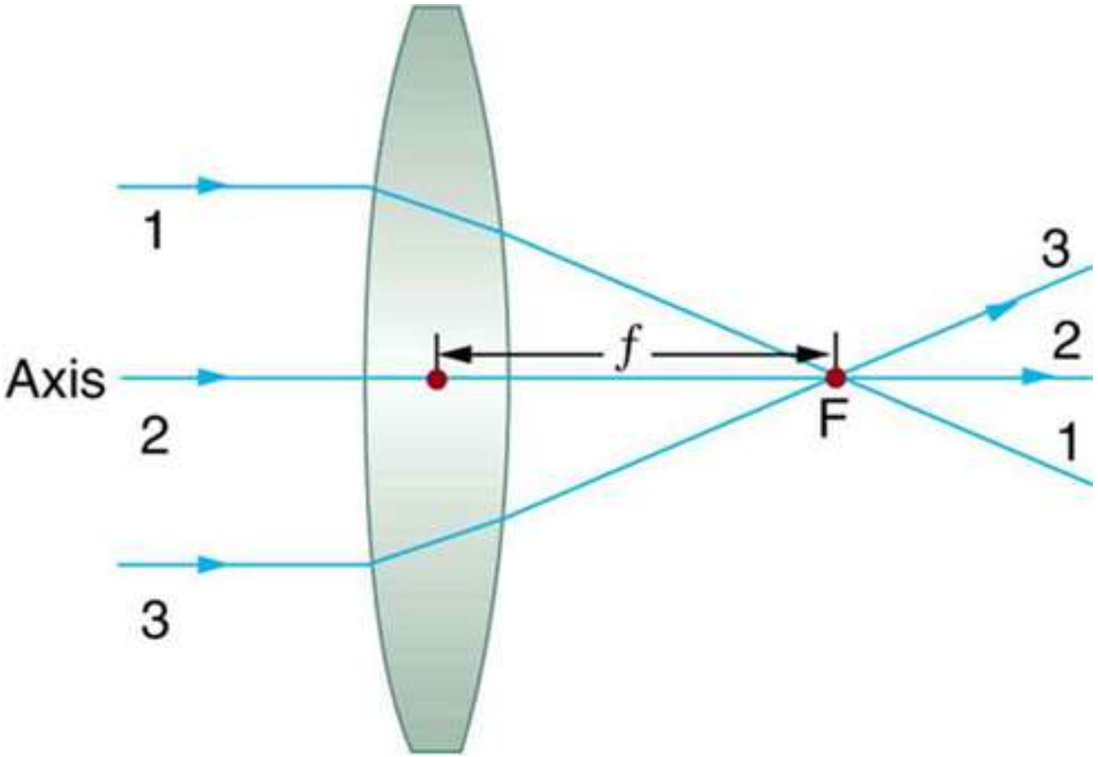
Ray tracing rules for thin lens

3. A ray passing through the center of either a converging or a diverging lens does not change direction.



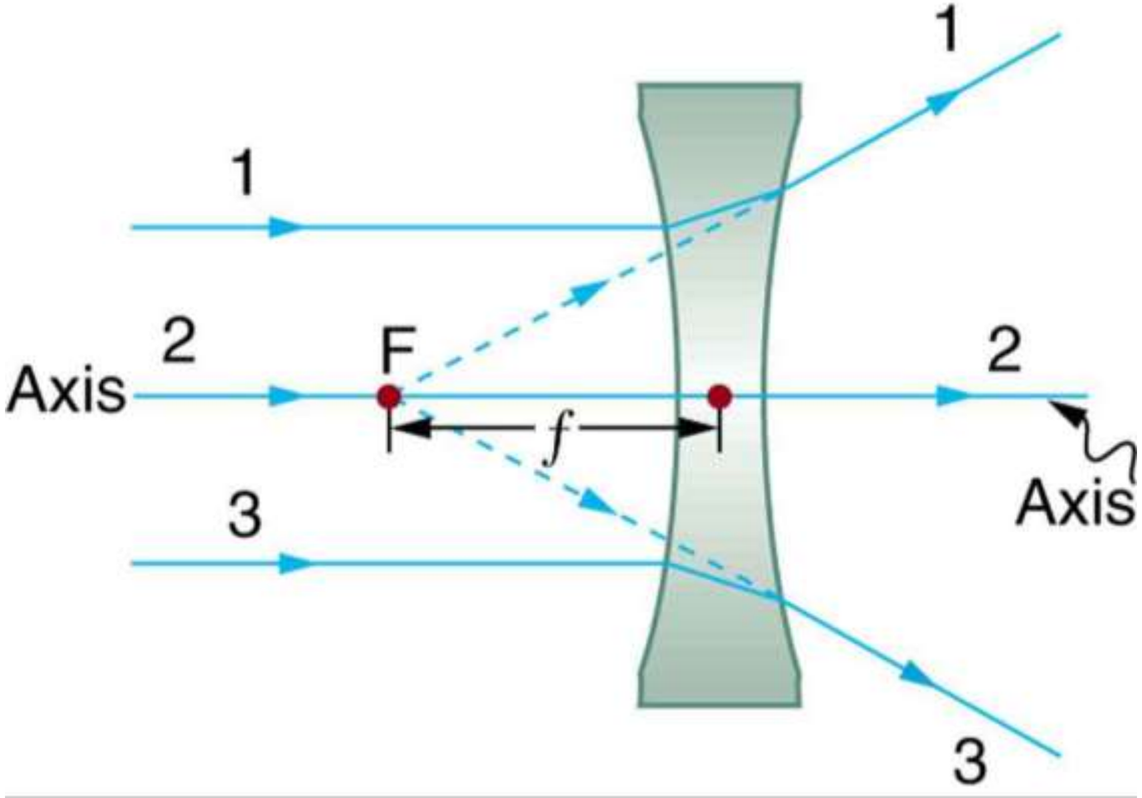
Ray tracing rules for thin lens

4. A ray entering a converging lens through its focal point exits parallel to its axis. (Invert the arrows in the figure)



Ray tracing rules for thin lens

5. A ray that enters a diverging lens toward the focal point on the opposite side exits parallel to the axis. (Invert the arrows in the figure)

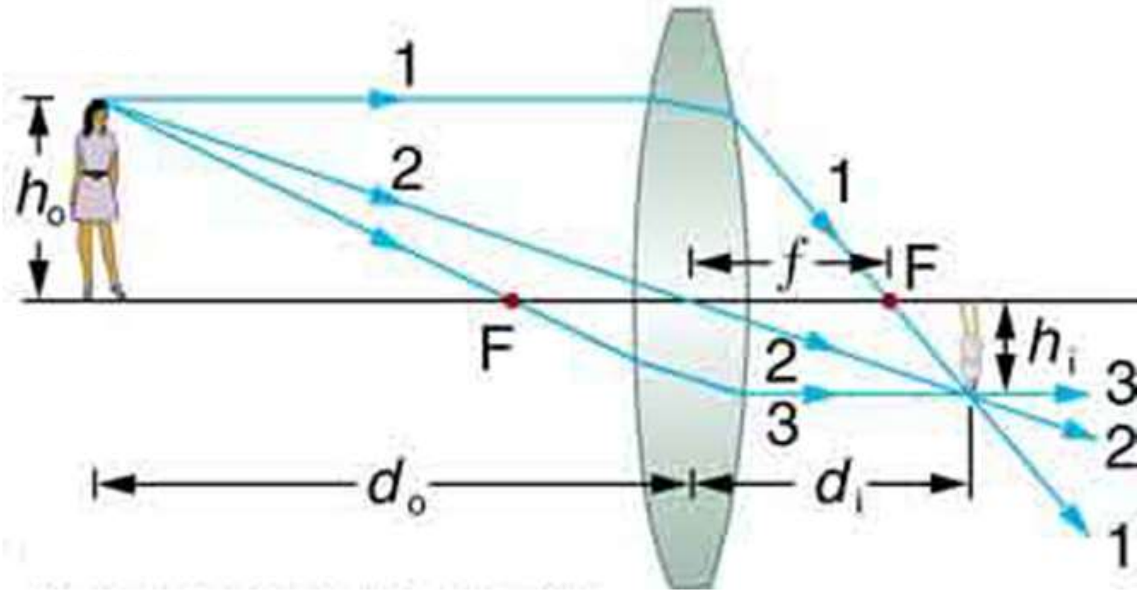


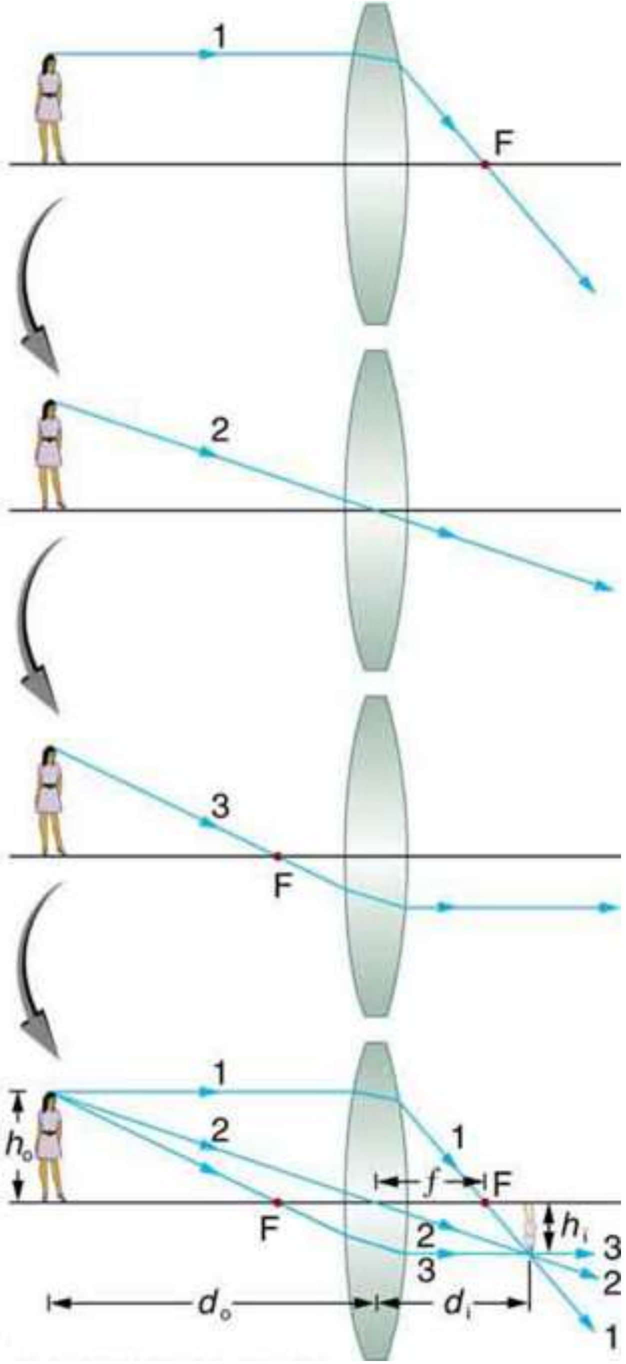
Ray tracing rules for thin lens

1. A ray entering a converging lens parallel to its axis passes through the focal point of the lens on the other side.
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point.
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its axis.
5. A ray that enters a diverging lens toward the focal point on the opposite side exits parallel to the axis.

Image formation by thin lenses

- Draw at least 2 or 3 lines from a single point and follow the rules of ray tracing.
- The image is located at the point where the rays cross.



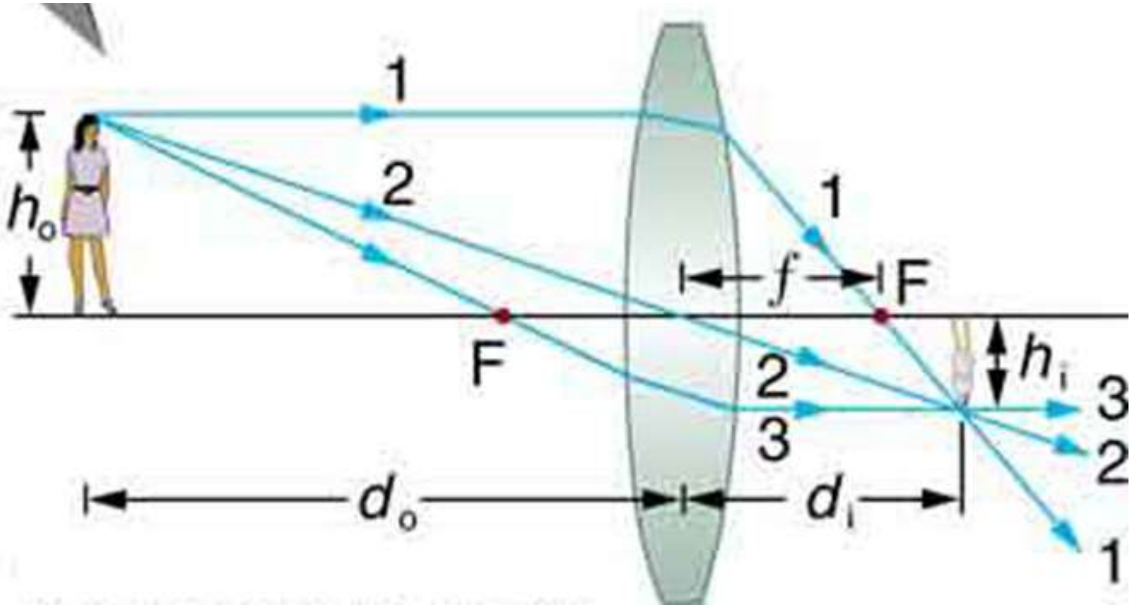


The thin lens equations

- First equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- d_o = distance of the **object** from the center of the lens.
- d_i = distance of the **image** from the center of the lens.
  - **Negative** d_i means the image on the **same side** as the object.
- f = focal length.

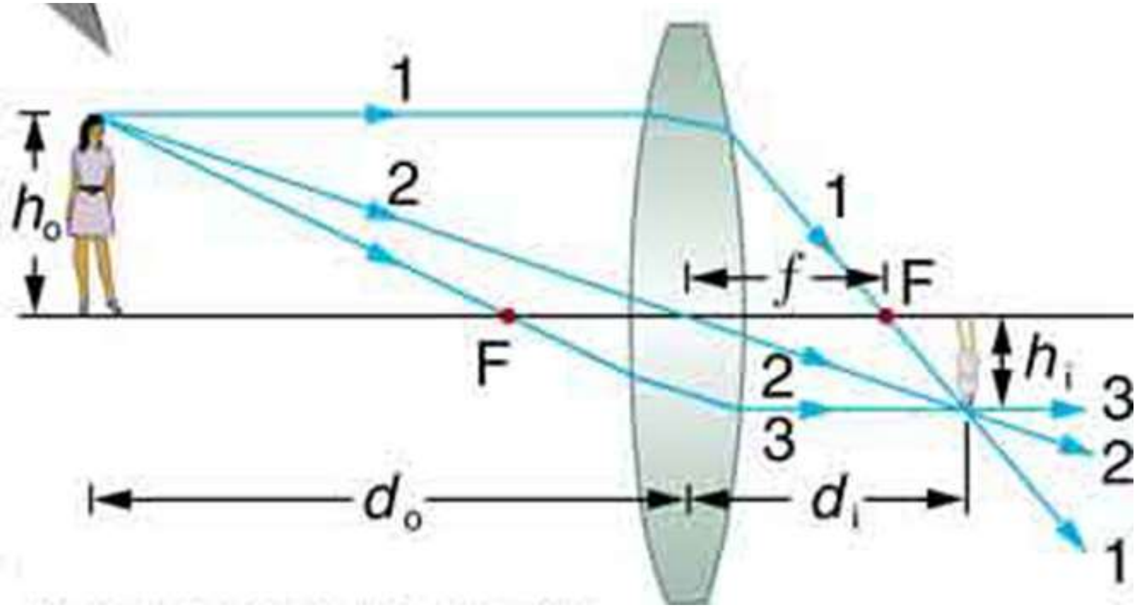


The thin lens equations

- Second equation:

$$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- m = magnification.
- h_o = height of the **object**.
- h_i = height of the **image**.
- **Positive** height = **upright**, **negative** height = **inverted**.
- Negative magnification = image is inverted.



Real Image	Virtual Image
Rays actually converge at a point.	Rays seem to diverge from a point, but never actually reach that point.
Image can be seen from anywhere.	Image can only be seen by looking through the lens.
Can be projected on a screen at the convergence point.	Cannot be projected, since there is nothing at that point, it's only an illusion.
Object looks inverted.	Object is not inverted.
Converging (convex) lens with $d_o > f$.	Converging (convex) lens with $d_o < f$ or diverging (concave) lens with any d_o.
Converging (concave) mirror with $d_0 > f$.	Converging (concave) mirror with $d_0 < f$ or diverging (convex) mirror with any d_0.
Examples: movie projector, camera, eye.	Examples: mirror, microscope.

Virtual demonstration

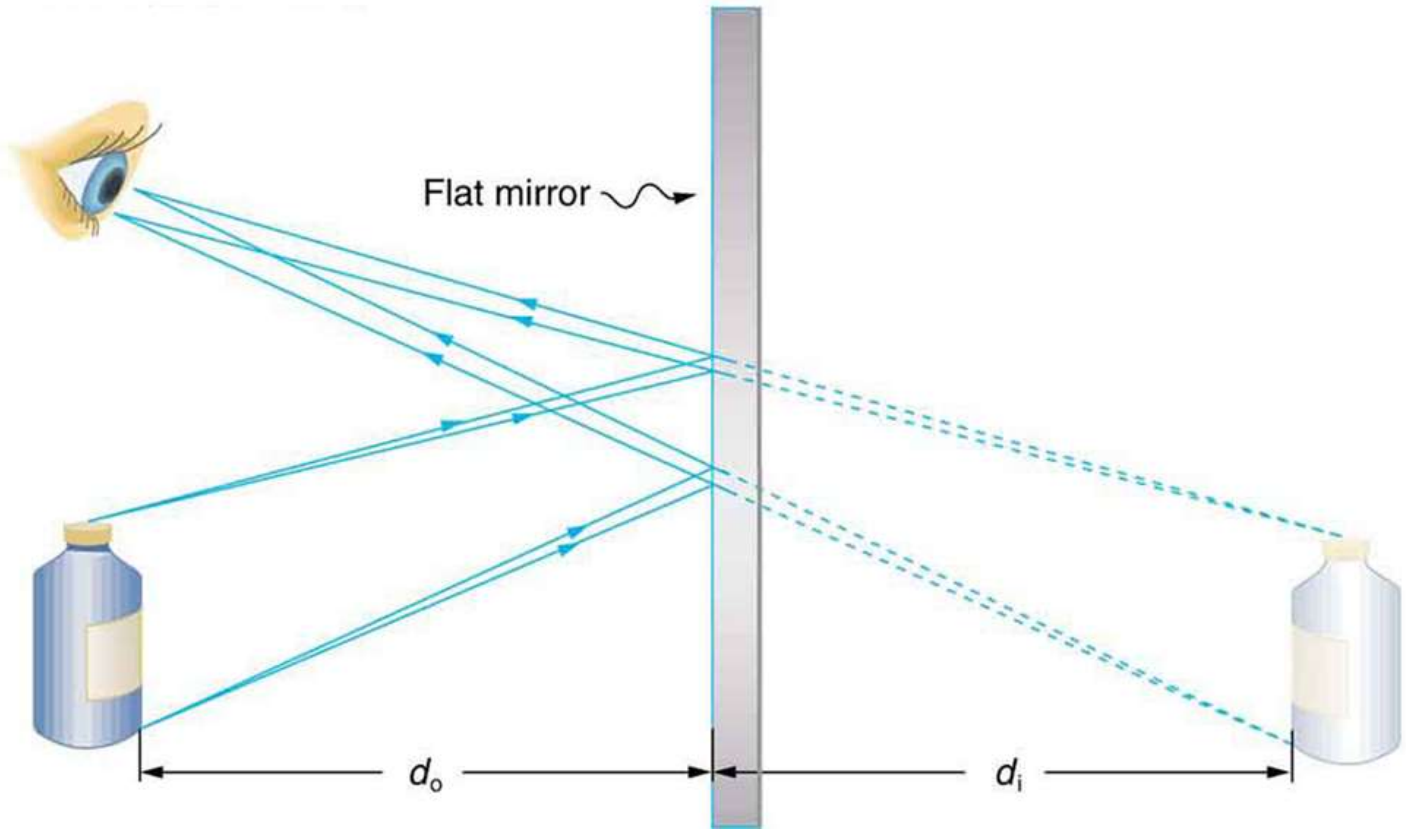
https://phet.colorado.edu/sims/html/geometric-optics/latest/geometric-optics_en.html

Generate real and virtual images for convex and concave lens and mirrors with objects located before and after the focal point.

25.7 Image Formation by Mirrors

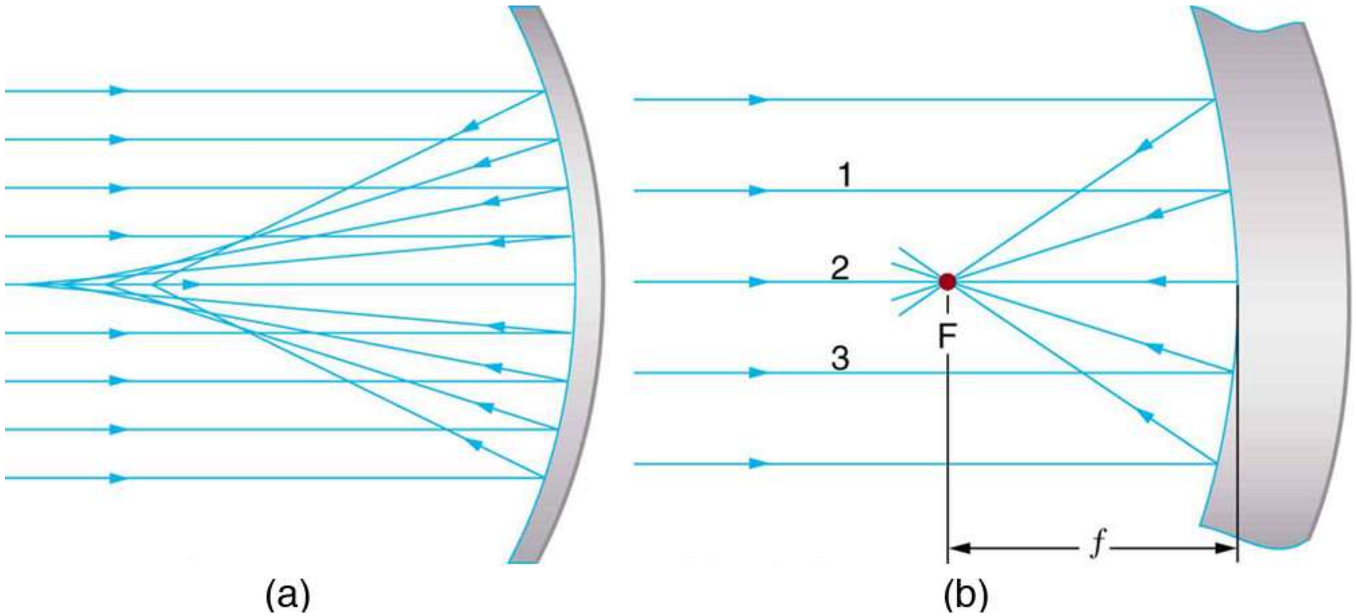
Flat mirror

- Generates virtual image “behind” the mirror.



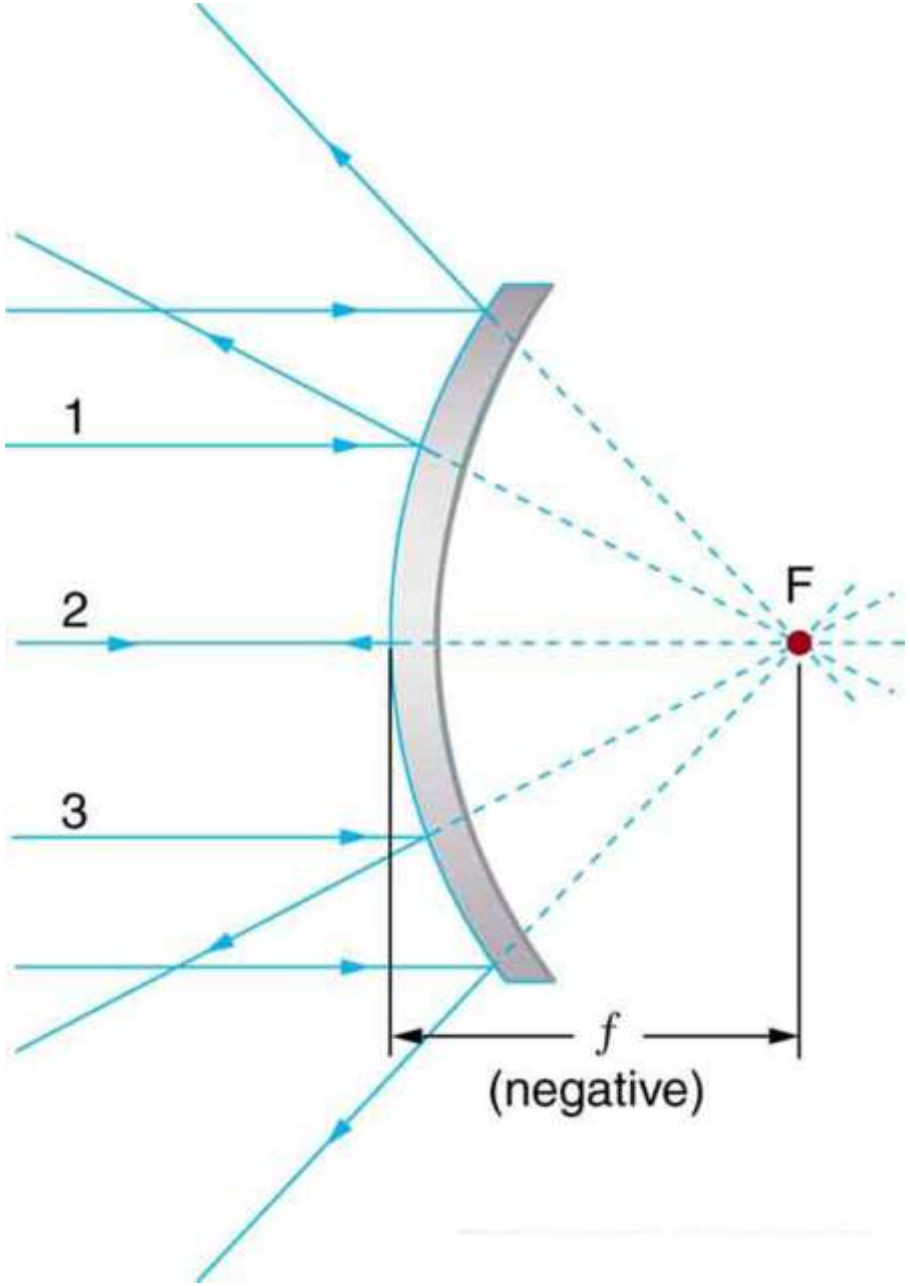
Concave (converging) mirror

- Large spherical mirrors do not have focal points (a), but if the mirror is not too large compared to its radius (b), it has an approximate focal point. (This is similar to the thin lens approximation.)
- $f > 0$ for concave mirrors; focal point is in front of the mirror.
- $d_i > 0$ when image is in front of the mirror (and real). $d_i < 0$ when it's behind the mirror (and virtual).



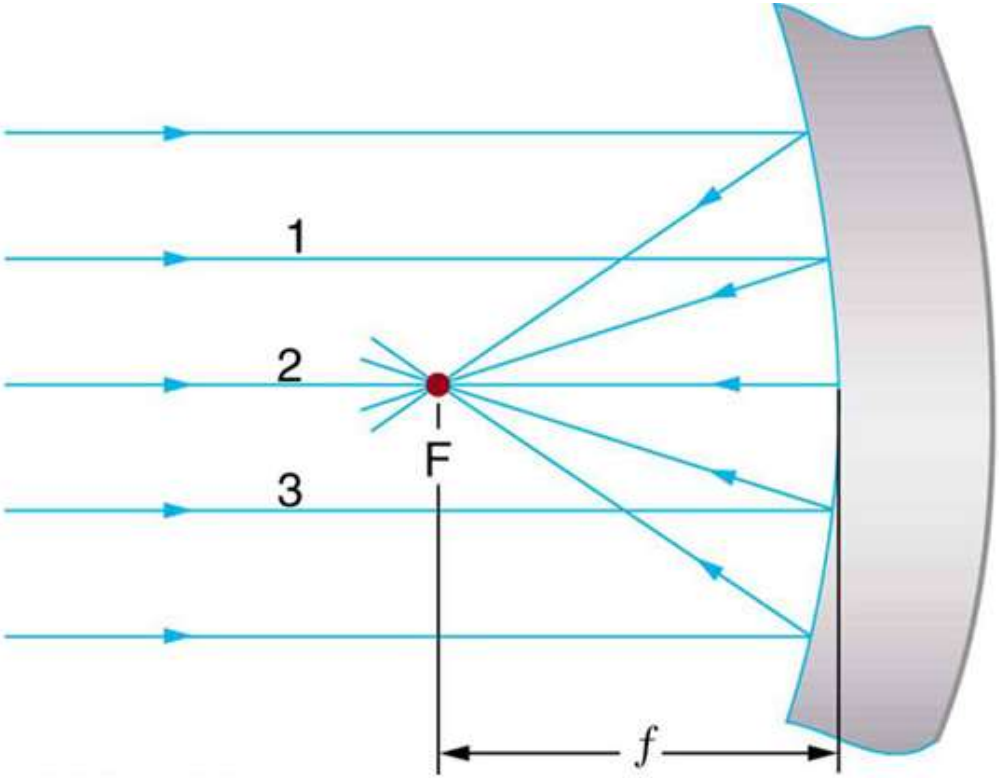
Convex (diverging) mirror

- $f < 0$ for convex mirrors; focal point is behind the mirror.
- $d_i > 0$ when image is in front of the mirror.
- Rays appear to diverge from the focal point. Image is always virtual.



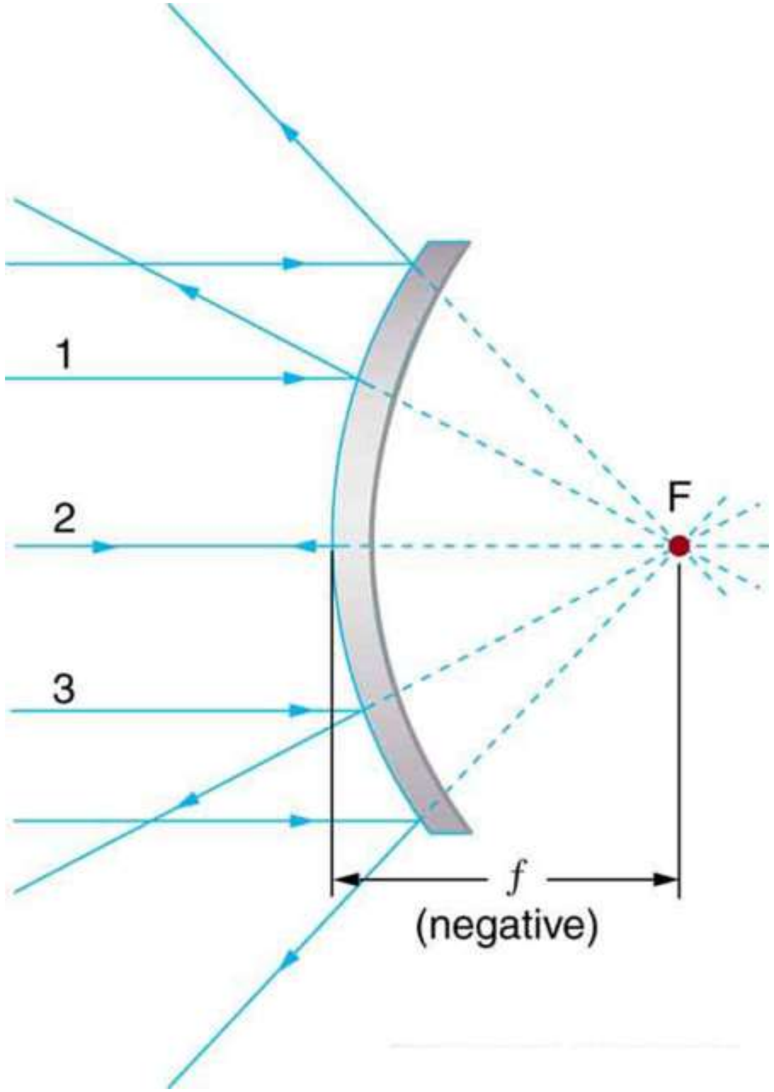
Ray tracing rules for mirrors

1. A ray approaching a concave (converging) mirror parallel to its axis is reflected through the focal point of the mirror on the same side.



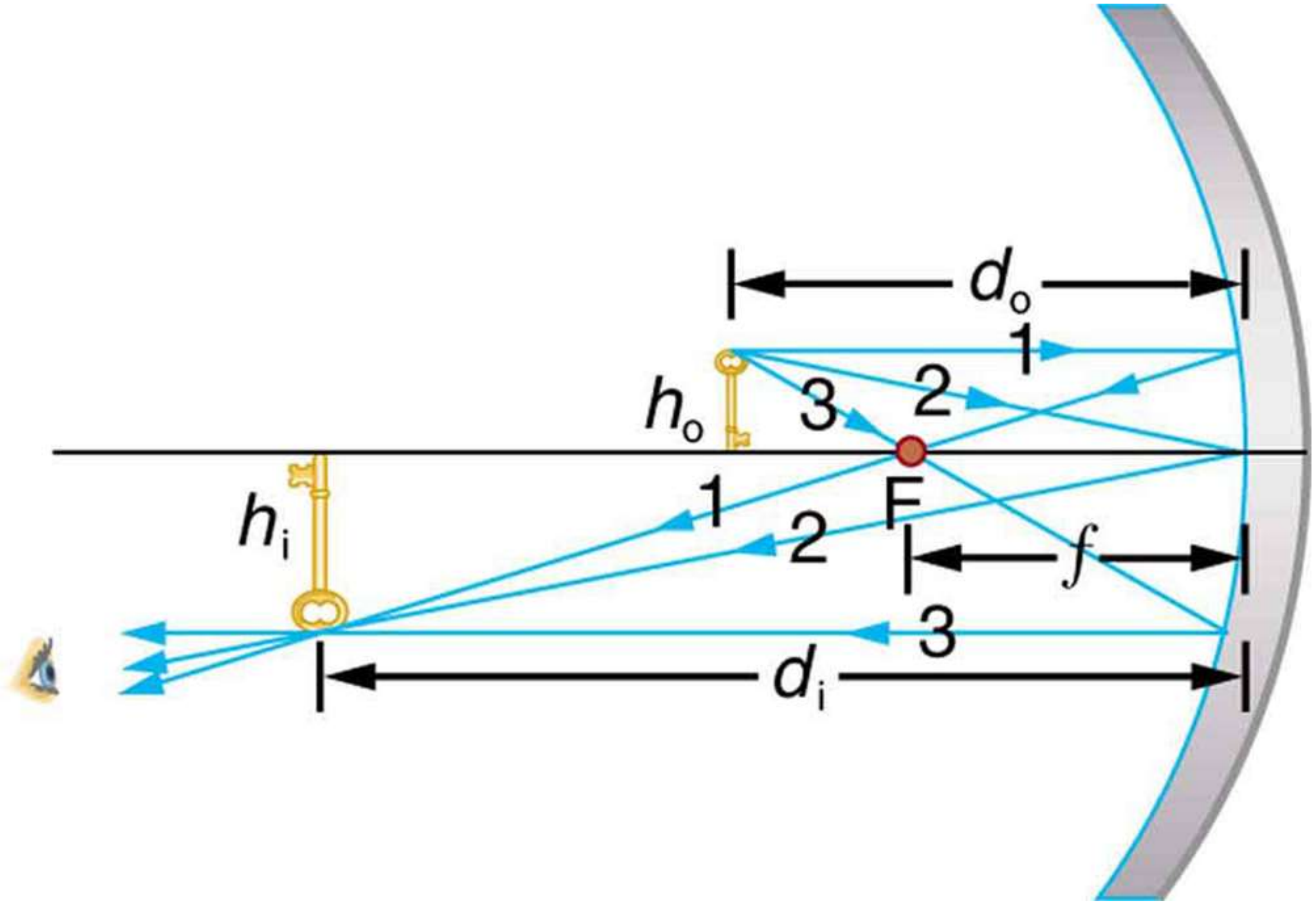
Ray tracing rules for mirrors

2. A ray approaching a convex (diverging) mirror parallel to its axis is reflected so that it seems to come from the focal point behind the mirror.



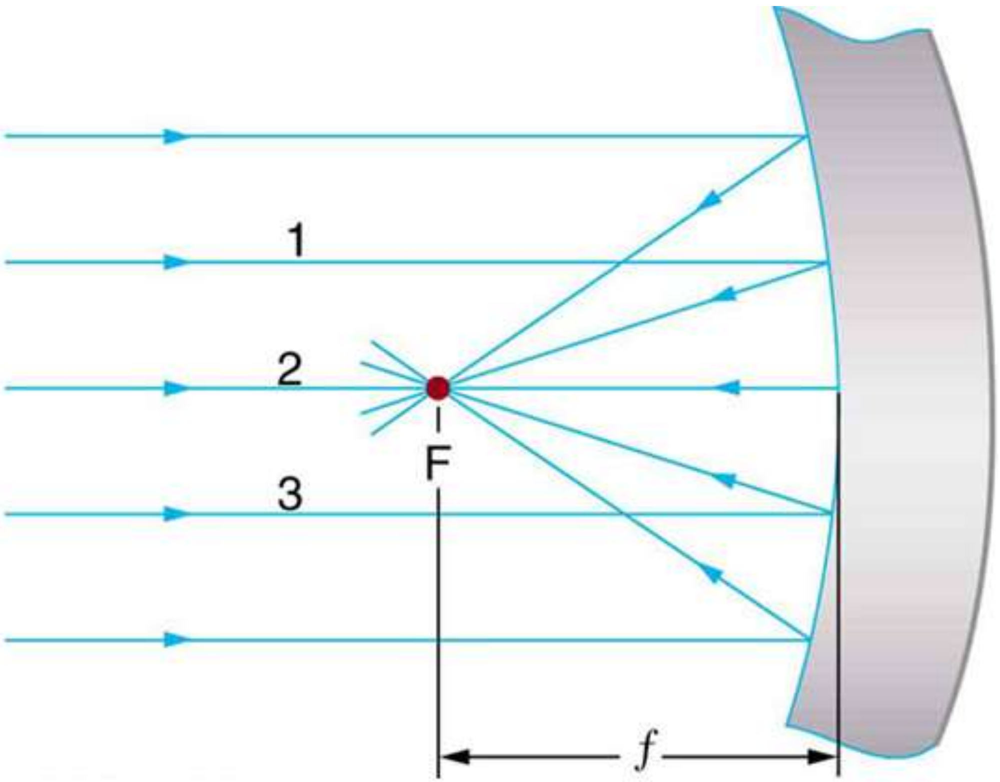
Ray tracing rules for mirrors

3. A ray striking the center of a mirror is followed by applying the law of reflection.



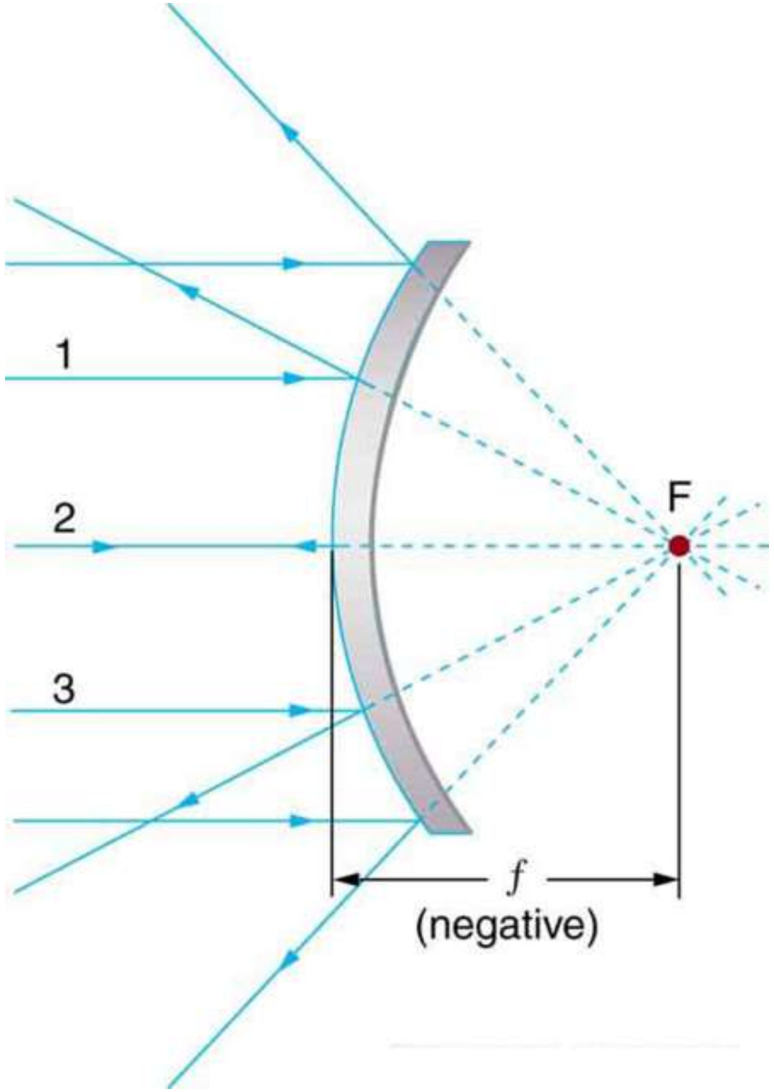
Ray tracing rules for mirrors

4. A ray approaching a concave converging mirror through its focal point is reflected parallel to its axis. (Invert the arrows in the figure)



Ray tracing rules for mirrors

5. A ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis.



Ray tracing rules for mirrors

1. A ray approaching a concave (converging) mirror parallel to its axis is reflected through the focal point of the mirror on the same side.
2. A ray approaching a convex (diverging) mirror parallel to its axis is reflected so that it seems to come from the focal point behind the mirror.
3. A ray striking the center of a mirror is followed by applying the law of reflection.
4. A ray approaching a concave converging mirror through its focal point is reflected parallel to its axis.
5. A ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis.

Demonstrations

1. Virtual demonstration: https://phet.colorado.edu/sims/html/geometric-optics/latest/geometric-optics_en.html

2. Live demonstrations:

Parallel lasers through lens and mirrors

Image through lens and mirrors