PHY 256 Lecture 4

Standard deviation

$$\Delta X = \sqrt{\langle (X - \langle X\rangle)^2 \rangle}$$

$$(X - \langle X\rangle)^2 = X^2 - 2X\langle X\rangle + \langle X\rangle^2$$

$$\langle X^2 - 2X\langle X\rangle + \langle X\rangle^2 \rangle$$

$$= \langle X^2\rangle - 2\underbrace{\langle X\rangle\langle X\rangle}_{\langle X\rangle^2} + \langle X\rangle^2$$

$$\boxed{\Delta X = \sqrt{\langle X^2\rangle - \langle X\rangle^2}}$$

$X = 0$ Heads, $P = \frac{1}{2}$ $X = 1$ tails, $P = \frac{1}{2}$

$$\langle X\rangle = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 1 = \frac{1}{2}$$

$$\langle X^2\rangle = \frac{1}{2}\cdot 0^2 + \frac{1}{2}\cdot 1^2 = \frac{1}{2}$$

$$\Delta X = \sqrt{\langle X^2\rangle - \langle X\rangle^2} = \sqrt{\frac{1}{2} - \frac{1}{4}} = \frac{1}{2}$$

$\Delta X = \frac{1}{2}$ $\Delta X = \frac{1}{2}$

0 $\langle X\rangle = \frac{1}{2}$ 1

$$\langle X\rangle = \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6}\cdot 5 + \frac{1}{6}\cdot 6$$
$$= 3.5 = \frac{7}{2}$$

$$\langle X^2\rangle = \frac{1}{6}\left(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2\right) = \frac{91}{6} \approx 15.2$$

$$\Delta X = \sqrt{\langle X^2\rangle - \langle X\rangle^2} = \sqrt{\frac{91}{6} - \frac{49}{4}} = \sqrt{\frac{35}{12}} \approx 1.7$$

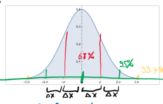
$\Delta X = 1.7$ $\Delta X = 1.7$

1 2 3 4 5 6

$\langle X\rangle = 3.5$

Normal (Gaussian) Distributions

P



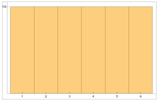
-5σ X 5σ

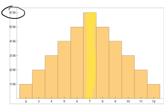
ΔX ΔX ΔX ΔX

$\sigma = \Delta X$

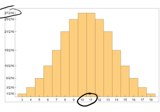
68-95-99.7 rule

Central limit theorem





$7 =$ 1+6, 2+5, 3+4, 4+3, 5+2, 6+1 $\frac{1}{36}$



$3 = 1+1+1$ $\frac{1}{6^3}$

$18 = 6+6+6$ $\frac{1}{6^3}$

Wolfram Mathematica

Python

The foundations of Quantum Theory

Dimensionless and dimensionful constants

Fine structure constant

$\alpha \approx 0.0073$

Speed of light

$c \approx 3.0\times 10^8$ m/s
$\approx 1.1\times 10^7$ miles/minute
≈ 170 au/day
$\approx 3.5\times 10^{-5}$ parsec/hour
≈ 1 ly/year

Planck units

$$c = G = \hbar = \frac{1}{4\pi\varepsilon_0} = k_B = 1$$

$\sqrt{\frac{\hbar G}{c^3}}$ = Planck length $= 1$
$= 1.6\times 10^{-35}$ m

$$A = \frac{\hbar G}{c^3} r\sqrt{j(j+1)} = r\sqrt{j(j+1)}$$

$$\hbar = 1$$

$$X = 2\hbar = 2$$

$$1 = \hbar = \frac{h}{2\pi}$$

Hilbert Spaces, States and operators.

The System Axiom

$\mathbb{C}^2$ $\mathbb{C}^n$

The State Axiom

$|\psi\rangle$ $\|\psi\| = \sqrt{\langle\psi|\psi\rangle} = 1$

$$\frac{|\psi\rangle}{\|\psi\|}$$

$|\psi\rangle$ $e^{i\phi}|\psi\rangle$ $\phi\in\mathbb{R}$

$\cancel{|\psi\rangle = \begin{pmatrix}1\\1\end{pmatrix}}$ $|\psi\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$ $\|\psi\| = 1$

$|\phi\rangle = \frac{1}{\sqrt2}\begin{pmatrix}i\\i\end{pmatrix} = i|\psi\rangle$ $i = e^{i\frac{\pi}{2}}$

$\boxed{|\psi\rangle \sim \lambda|\psi\rangle}$ $\lambda\in\mathbb{C}$

$\|\psi\| = 1$ $\cancel{\begin{pmatrix}1\\1\end{pmatrix}} \sim \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} \sim \frac{1}{\sqrt2}\begin{pmatrix}i\\i\end{pmatrix}$

$$\|e^{i\phi}\psi\| = \|\psi\|$$

$\phi\in\mathbb{R}$ $e^{i\phi}|\psi\rangle$ $|\psi\rangle$

The Operator Axiom

$$A|\psi\rangle = |\phi\rangle$$

Hermitian Operators and Observables

The Observable Axiom

$$A|\psi\rangle = \lambda|\psi\rangle$$

λ ↓ measured value

$\lambda\in\mathbb{R}$

Probability Amplitudes

$|\psi\rangle$ A $|B_i\rangle$ $i = 1,\dots,n$

$$A|B_i\rangle = \lambda_i|B_i\rangle$$

The Probability Axiom

Probability amplitude $\langle B_i|\psi\rangle \in \mathbb{C}$ λ_i

Born Rule

$$p = |\langle B_i|\psi\rangle|^2$$

$|z|^2 = z^* z$ $\langle B_i|\psi\rangle^* = \langle\psi|B_i\rangle$

$$\sum_{i=1}^n |\langle B_i|\psi\rangle|^2 = \sum_{i=1}^n \langle B_i|\psi\rangle^*\langle B_i|\psi\rangle$$
$$= \sum_{i=1}^n \langle\psi|B_i\rangle\langle B_i|\psi\rangle$$
$$= \langle\psi|\left(\sum_{i=1}^n |B_i\rangle\langle B_i|\right)|\psi\rangle$$
$$= \langle\psi|\psi\rangle = 1$$

$$\sum_{i=1}^n |B_i\rangle\langle B_i| = 1$$

$\mathbb{C}^2$ $\sigma_x \equiv \begin{pmatrix}0&1\\1&0\end{pmatrix}$

$\sigma_x\begin{pmatrix}1\\1\end{pmatrix} = 1\cdot\begin{pmatrix}1\\1\end{pmatrix}$ $\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} = |+\rangle$

$\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$ $\frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix} = |-\rangle$ $\langle +|-\rangle = 0$

$\sigma_x\begin{pmatrix}1\\-1\end{pmatrix} = -1\cdot\begin{pmatrix}1\\-1\end{pmatrix}$

$\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} = -1\begin{pmatrix}1\\-1\end{pmatrix}$

$|\psi\rangle = \frac{1}{\sqrt{10}}\begin{pmatrix}1\\3\end{pmatrix}$ $p(+1) = |\langle +|\psi\rangle|^2 = \left|\frac{1}{\sqrt2}(1\ \ 1)\frac{1}{\sqrt{10}}\begin{pmatrix}1\\3\end{pmatrix}\right|^2$

$p(-1) = \frac{1}{5}$ $= \frac{1}{20}4^2 = \frac{16}{20} = \frac{4}{5}$