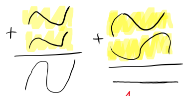
PHY 256 Lecture 5

$A \quad |B_i\rangle \quad i \in \{1, \dots, n\}$

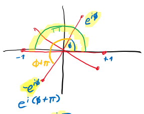
$A|B_i\rangle = \lambda_i |B_i\rangle$

$\langle B_i|\psi\rangle$ prob. amp. $\in \mathbb{C}$

$|\langle B_i|\psi\rangle|^2$ prob $\in [0,1]$



$$+\begin{array}{r}1\\ 1\\ \hline 2\end{array} \qquad +\begin{array}{r}1\\ -1\\ \hline 0\end{array}$$



$e^{i(\phi+\pi)}$

$-1 = e^{i\pi}$

Superposition

$|B_i\rangle$

$$|\psi\rangle = \sum_{i=1}^{n} |B_i\rangle\langle B_i|\psi\rangle$$

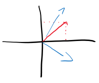
$|\psi\rangle, |B_2\rangle \dots |B_n\rangle$

$$|\psi\rangle = \sum_{i=2}^{n} |B_i\rangle \underbrace{\langle B_i|\psi\rangle}_{=0} + \underbrace{|\psi\rangle\overbrace{\langle\psi|\psi\rangle}^{1}}_{i=1}$$

$|\psi\rangle = |\psi\rangle$

$\langle B_i|\psi\rangle$

$\vec{W}\cdot\vec{V}$



Expectation Values

$A|B_i\rangle = \lambda_i|B_i\rangle$

$\langle B_i|B_j\rangle = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

$$\langle B_i|A|B_j\rangle = \langle B_i|\big(A|B_j\rangle\big)$$
$$= \langle B_i|\lambda_j|B_j\rangle$$
$$= \lambda_j\langle B_i|B_j\rangle$$
$$= \lambda_j\delta_{ij}$$

$$\sum_{i=1}^{n} |B_i\rangle\langle B_i| = 1$$

$$\langle\psi|A|\psi\rangle = \langle\psi|\Big(\sum_{i=1}^{n}|B_i\rangle\langle B_i|\Big)A\Big(\sum_{j=1}^{n}|B_j\rangle\langle B_j|\Big)|\psi\rangle$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\langle\psi|B_i\rangle\langle B_i|A|B_j\rangle\langle B_j|\psi\rangle$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\langle\psi|B_i\rangle\lambda_j\delta_{ij}\langle B_j|\psi\rangle$$
$$= \sum_{i=1}^{n}\lambda_i\langle\psi|B_i\rangle\langle B_i|\psi\rangle$$

$\langle\psi|B_i\rangle = \langle B_i|\psi\rangle^* \qquad zz^* = |z|^2$

$$= \sum_{i=1}^{n}\lambda_i|\langle B_i|\psi\rangle|^2$$

$$\langle\psi|A|\psi\rangle = \langle A\rangle_\psi$$

Summary (discrete systems)

1. The system axiom $\mathcal{H} = \mathbb{C}^n$
2. The state axiom $|\psi\rangle \in \mathcal{H}$
3. The operator axiom $A \in \mathcal{H}$
4. The observable axiom $A = A^\dagger$
5. The probability axiom $\langle B_i|\psi\rangle \quad |\langle B_i|\psi\rangle|^2$

- superposition
- Exp. value $\langle A\rangle_\psi = \langle\psi|A|\psi\rangle$

Two-state systems
Spin $\frac{1}{2}$
Qubits

The Pauli Matrices

$$\sigma_x \equiv \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \sigma_y \equiv \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad \sigma_z \equiv \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\left.\begin{array}{l}\sigma_i^\dagger = \sigma_i\\ \sigma_i^\dagger = \sigma_i^{-1}\end{array}\right] \sigma_i = \sigma_i^{-1} \qquad \sigma_i^2 = \sigma_i$$

σ_x

$|+x\rangle = |+\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$

$|-x\rangle = |-\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$

σ_y

$|+y\rangle = |+i\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$

$|-y| = |-i\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$

σ_z

$|+z\rangle = |0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}$

$|-z\rangle = |1\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \qquad |1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

Spin $\frac{1}{2}$

$s \in \{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \dots\}$

$\{-s, -s+1, \dots, s-1, s\}$

$2s+1$

spin 0: 0

spin $\frac{1}{2}$: $\underbrace{-\frac{1}{2}}_{\text{down}} \quad \underbrace{+\frac{1}{2}}_{\text{up}}$

spin 1: $-1 \quad 0 \quad +1$

spin $\frac{3}{2}$: $-\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$

$S_x \equiv \frac{1}{2}\sigma_x \quad S_y \equiv \frac{1}{2}\sigma_y \quad S_z \equiv \frac{1}{2}\sigma_z$

$\{1, \sigma_x, \sigma_y, \sigma_z\}$

$V \in \mathbb{R}^3 \quad v = (x, y, z) \quad \sqrt{x^2+y^2+z^2} = 1$

$$S_v = xS_x + yS_y + zS_z = \frac{1}{2}\begin{pmatrix}z & x-iy\\ x+iy & -z\end{pmatrix}$$

Qubits

$|\psi\rangle = a|0\rangle + b|1\rangle \qquad |a|^2 + |b|^2 = 1$

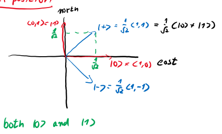
$a = \langle 0|\psi\rangle \quad b = \langle 1|\psi\rangle$

$|0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix} \quad |1\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$

$|0\rangle$ vacuum $|1\rangle$ 1 particle

$|0\rangle$ horizontal $|1\rangle$ vertical

Superposition



both $|0\rangle$ and $|1\rangle$

either $|0\rangle$ or $|1\rangle$

Hidden variable theories
Bell's theorem
de Broglie - Bohm

Classical	Quantum
wave particle	state
and or	superposition